Traffic Light Timing Design by Machine Learning

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Abstract— **The traffic flow control problem in urban road networks in the case of sudden change of its quantitative characteristics is considered. Traffic flow is usually characterized by density, velocity, etc. These characteristics may change due to road accidents or other temporary events that increase or decrease demand on transportation. Dealing with such situations is not stationary and requires high quality online solutions made by human experts or traffic engineers. It is proposed to consider such a problem as a control synthesis problem and to find a control law that will assist humans in decision making. The searched control is a function of the deviation of the current traffic flow characteristics from that calculated before. The problem is formulated as a control synthesis. To solve the control synthesis problem of traffic flows, a symbolic regression approach is used. It allows to find a mathematical expression of control function, that returns changes of the traffic light phases duration. The traffic light timing obtained is optimal even for states that were not included in the training set. A statement of the control synthesis problem for traffic light timing is presented. In the computational experiment, a test example for an urban network of two intersections is presented.**

I. INTRODUCTION

Millions of cars travel through the cities 24/7. This flow mainly obeys drivers will, needs, behavior, etc. which basically means that it lacks the ability to be fully controlled. To avoid chaos, enthusiasts of intelligent transportation systems are still trying to cope with the situation deriving new mathematical models, developing control methods, maintaining road infrastructure and inventing algorithms and software tools.

The mathematical representation of traffic flows in an urban road network is often considered analogous to the motion of a fluid within a confined vessel. A road subnetwork is regarded as a reservoir which contains a certain number of vehicles, the difference between the incoming and outgoing flows. The traffic flow behavior is considered analogous to fluid behavior. The primary objective of traffic flow studies was to determine the conditions required to ensure a specific traffic speed on a given road section. As a result of these studies, it was established that the speed of a traffic flow on a road section is related to the density of traffic in that road section [1], [2], [3]. Models of this type are classified as macroscopic.

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From the perspective of control theory, a limitation of such traffic flow models is the absence of explicit control actions capable of influencing changes in the desired characteristics. Currently, the only operational control mechanism for traffic flow within an urban road network is the system of traffic lights at signalized intersections. By selecting specific timing of the traffic lights this system affects the quantitative characteristics of the traffic flow, or rather, its state.

Optimal traffic flow control within an urban road network consists of switching the active signal phases at all signalized intersections in the network at specific time intervals, depending on the current number of vehicles present on all entry and internal road sections.

One of fundamental analytical approaches to calculating traffic signal timing parameters at isolated intersections is the Webster method introduced in [4]. The author derived a formula to calculate the average delay experienced by a vehicle on an approach to a fixed-cycle traffic signal. By minimizing this delay, the formulas for the optimal cycle length and the distribution of green time among the phases were obtained.

Among numerical optimization approaches, application of genetic algorithms (GA) to signal timing started by Foy et al. [5] in 1992 when GA was used to optimize the phase sequence and phase splits for four intersections and found near optimal signal timings minimizing total delay. In further works, GA was successfully used for the simultaneous optimization of cycle length, offsets, splits, and phase sequence [6], transit signal priority problem [7], [8], for optimizing traffic signal timing plan in non-stady-state conditions of the flow caused by trains [9], etc.

Another approach applied in practice is adaptive traffic signal control (ATSC). It automatically adjusts the duration of the traffic lights for each direction at the intersection based on current traffic state [10], [11], [12]. ATSC adapts to traffic in several cycles, and reduces delays and queue lengths, though it has high costs and complicated tuning.

The further development of various mathematical models [13] and optimization algorithms resulted in traffic simulation and optimization tools, for example, Transyt-7F, SUMO, PTV Vissim, AnyLogic, etc. [14].

In this paper, the traffic flow model of an urban road network constructed on the basis of controlled networks [15]. The urban road network is represented as a directed graph, in which the road segments between intersections serve as nodes, and the maneuvers performed at intersections serve as arcs. If an intersection is signalized, the active signal phases permit or prohibit specific maneuvers, thereby excluding or including the corresponding arcs within the directed graph.

The proposed model allows the formulation and solution of the optimal traffic control problem [16] according to the chosen performance criterion.

The solution to the optimal traffic control problem for a given initial network state yields a control program that can be implemented in real-world conditions as an optimal signal timing plan. For optimal traffic signal timing it is proposed to use a modified GA which is called a variational GA [17], since it uses a special principle of coding possible solutions [18].

According to traffic signal control regulations, the sequence of active phase switching remains unchanged during the control process, and the duration of each active phase is bounded by both upper and lower limits. In signalized control, the specific characteristics of maneuver performance at each intersection must be taken into account, particularly the duration of the maneuver, which depends on the physical geometry of the intersection, as well as quantitative characteristics reflecting the distribution of the traffic flow at the intersection among alternative directions.

Currently, in most cases, the control at signalized intersections is set as fixed over a specific time interval; that is, it functions as a program control with active signal phase durations that remain constant throughout this interval. This program control is referred to as a coordination plan and is calculated in advance during the system design stage based on expected traffic flows at different times of day, days of the week, etc.

If necessary, real-time adjustment of the coordination plan is performed manually by a traffic engineer, who modifies the duration of specific active signal phases based on the magnitude of deviation of the quantitative traffic flow characteristics on the roads adjacent to the intersection from the values used in the calculation of the given coordination plan.

Since a traffic signal control program consists of the sequential switching of active signal phases at specific time intervals—whereby both the sequence and the duration of the active phases remain unchanged throughout the entire coordination plan—this coordination plan, or program-based signal control, can be described by a vector of constant parameters. The number of parameters for each signalized intersection is determined by the number of active signal phases at that intersection, and the value of each parameter is defined by the duration of the specific active phase that this parameter describes.

Coordination plans are, as a rule, calculated based on the current state of the traffic flow within the urban road network. This state is defined by certain average values of the traffic flow on all entry and internal road sections. The traffic flow values for which the coordination plans are calculated can be represented as a vector of constant parameters. The dimension of the vector is determined by the number of entry and internal road sections, while the values of these components define the quantitative characteristics of the traffic on these sections.

If, for each current traffic state, an optimal coordination plan for all signalized intersections in the network can be assigned in correspondence with that state, then it can be asserted that for any urban road network equipped with traffic signal control, there exists a function that maps the current traffic state onto the corresponding coordination plans for all signalized intersections of the network

$$f : State\ Space \rightarrow Control\ Space$$

Thus, the synthesis problem for traffic flow control in an urban road network featuring signalized intersections reduces to identifying a function that maps the current traffic state vector onto the optimal coordination plan for every controlled intersection in the network. This paper addresses the solution to this very problem.

This paper presents a formal statement of the traffic control synthesis problem in an urban road network. To solve the control synthesis problem, the study employs symbolic regression, which approximates the training dataset with a mathematical expression of a multivariate function [19], [20], [21]. The training dataset is obtained by solving the optimal traffic flow control problem for various initial flow states. The computational experiment provides an example of solving optimal control problems in order to obtain a training dataset. Subsequently, the control synthesis problem is solved, and a function is derived for constructing the optimal coordination plan depending on the state of the traffic flow in the network.

The rest of the paper is organized as follows. The control synthesis problem statement is presented in Section II. The symbolic regression method is briefly described in Section III. The solution of the control synthesis problem for two intersections is solved in Section IV. Discussion of results of computational experiments is presented in Section V.

II. PROBLEM STATEMENT

The network of urban roads is described by an oriented graph in which each section of the road is a node. The maneuver of transport from one section of the road to another is carried out at the intersection and in the graph is described by an arc. If the intersection is regulated, then it is equipped with a traffic light, the working phases of which allow or prohibit certain maneuvers. The state of the transport flow is determined by the state vector

$$\mathbf{x}(t) = [x_1(t) \dots x_L(t)]^{\mathrm{T}}, \tag{1}$$

where L is a number of road sections in the network, value of the vector $\mathbf{x}(t)$ defines quantity of transport in the network at moment t. The traffic flow is measured by medium-sized cars. For example, a truck can be presented as 3.5 midget cars.

A traffic flow state vector includes input N_0, internal N_1, and output N_2 network sections,

$$N = N_0 + N_1 + N_2.$$

To control the traffic flow, it is necessary to have the information on the state of entry and internal sections, therefore the state vector will be limited to components that provide information about transport on these sections of the roads

$$\tilde{\mathbf{x}} = [\tilde{x}_1 \dots \tilde{x}_{N_0+N_1}]^{\mathrm{T}}. \quad (2)$$

To control the transport flow, we use the control vector

$$\tilde{\mathbf{u}} = [\tilde{u}_1 \dots \tilde{u}_m]^{\mathrm{T}}, \quad (3)$$

where m is a number controlled intersections in the urban network.

Each controlled intersection has a traffic light with a certain number of working phases. Thus, the control vector is more accurately written in the following form

$$\mathbf{u} = [u_{1,1} \dots u_{1,r_1}\ u_{2,1} \dots u_{2,r_2} \dots u_{m,1} \dots u_{m,r_m}]^{\mathrm{T}}, \quad (4)$$

where r_i is a number of the work phase in the controllable intersection i, $i = 1, \dots, m$.

Values of components of this control vector indicate duration of working phases of traffic lights.

Therefore, to control the traffic flow in the urban road network, we have a control vector with

$$M = \sum_{i=1}^{m} r_i \quad (5)$$

components

$$\mathbf{u} = [u_1 \dots u_M]^{\mathrm{T}}. \quad (6)$$

Suppose, we have a set of initial states

$$\mathrm{X}_0 = \{\mathbf{x}^{0,1}, \dots, \mathbf{x}^{0,K}\}, \quad (7)$$

where

$$\mathbf{x}^{0,i} = [x_1^{0,i} \dots x_{N_0+N_1}^{0,i}]^{\mathrm{T}},\ i \in \{1, \dots, K\}. \quad (8)$$

For each initial state of the set (7), optimal control is calculated - these are the durations of all working phases of traffic lights in all controlled sections of the network

$$\mathrm{U}^* = \{\mathbf{u}^{*,1}, \dots, \mathbf{u}^{*,K}\}, \quad (9)$$

where

$$\mathbf{u}^{*,i} = [u_1^{*,i} \dots u_M^{*,i}]^{\mathrm{T}}. \quad (10)$$

This control provides an optimal redistribution of the flow in the network for some time and is a plan for coordinating network traffic lights for a transport flow whose state at the initial time belongs to the interval

$$\mathbf{x}(0) \in \mathbf{x}^{0,i} \pm \Delta_i,\ i \in \{1, \dots, K\}, \quad (11)$$

where

$$\Delta_i = [\delta_1^i \dots \delta_{N_0+N_1}^i]^{\mathrm{T}}. \quad (12)$$

The control synthesis problem for traffic in an urban road network is to find the function of mapping a set of initial states of traffic flows to a set of optimal coordination plans

$$f(\mathbf{x}^0) : \mathrm{X}_0 \to \mathrm{U}^*. \quad (13)$$

To obtain a training set, it is necessary to find the optimal coordination plans for switching the working phases of traffic lights. Since the control in the network is the duration of each working phase of the traffic light, it is necessary to use a mathematical model of the movement of the traffic flow in the network, which includes the control vector of traffic flow, to calculate the optimal control. For this purpose, we use a mathematical model built on the basis of the theory of controlled networks [16].

This model represents a network of urban roads in the form of an oriented graph. Each graph node is associated with a road section. All maneuvers between road sections are made at intersections. Maneuvers between road sections in the network are indicated by arcs between nodes. The directed graph of the road network is formally described by the adjacency matrix

$$\mathbf{A} = [a_{i,j}],\ i.j = 1, \dots, L, \quad (14)$$

where L is a number of road sections in a network,

$$a_{i,j} = \begin{cases} 1, \text{ if there is a maneuver } i \to j \\ 0, \text{ otherwise} \end{cases} \quad (15)$$

In order to determine how traffic is redistributed over the road sections of the network, you need to have additional information about the properties of the road network.

The mathematical model is dynamic and the change in the state of the traffic flow is determined at each cycle of the working phase of the traffic light. To calculate the change in the state of the traffic flow, you need to know the throughput of all maneuvers in the network. Throughput information is contained in the throughput matrix

$$\mathbf{B} = [b_{i,j}],\ i.j = 1, \dots, L, \quad (16)$$

where $b_{i,j}$ is a throughput capacity of maneuver from road section i to road section j for one the tact. If the $a_{i,j} \neq 0$, then $b_{i,j} \neq 0$.

We should also know how the flow is distributed at the intersection, if there are alternative directions for this road section. This information is contained in a distribution matrix

$$\mathbf{D} = [d_{i,j}],\ i.j = 1, \dots, L, \quad (17)$$

where $d_{i,j} \neq 0$, if $a_{i,j} \neq 0$, $i, j \in \{1, \dots, L\}$.

The sum of the values of all elements in any non-zero row of the distribution matrix $\mathbf{D}$ is equal to one

$$\sum_{j=1}^{L} d_{i,j} = 1, \text{ if for row } i\ \exists a_{i,j} \neq 0,\ i \in \{1, \dots, L\}.$$

In order to describe at what regulated intersection and at what working phase of the traffic light the maneuver is performed, we use two more matrices, a control matrix and a matrix of working phases of the traffic light. The elements of control matrix, where the adjacency matrix has a non-zero element, indicate the number of regulated intersection

$$\mathbf{C} = [c_{i,j}],\ i.j = 1, \dots, L, \quad (18)$$

where $c_{i,j} \neq 0$, if $a_{i,j} \neq 0$, $i, j \in \{1, \dots, L\}$.

The working phase matrix consists of sets. Non-empty sets contain the numbers of the working phases under which this maneuver is allowed

$$\mathbf{F} = [f_{i,j}],\ i.j = 1, \dots, L, \quad (19)$$

where $f_{i,j} = \emptyset$, if $a_{i,j} = 0$, $i, j \in \{1, \dots, L\}$, otherwise

$$f_{i,j} = \{f_1^{i,j} \dots, f_{n_{i,j}}^{i,j}\},\ i, j \in 1, \dots, L, \tag{20}$$

$f_k^{i,j}$ is the number of working phase, when the maneuver from section i to section j exists, $k \in \{1, \dots n_{i,j}\}$.

The working phases of traffic lights are switched sequentially at fixed time intervals. Duration of working phases of traffic lights determine traffic flow control. Vector (6) is not sufficient to determine the current control state. A vector of control actions is still needed, which shows what working phases of traffic lights are active at regulated intersections at the moment.

The vector of control actions at step k has the following description

$$\mathbf{v}(k) = [v_1(k) \dots v_m(k)]^{\mathrm{T}}, \tag{21}$$

where $v_i(k) \in \{1, \dots, r_i\}$, $i \in \{1, \dots, m\}$.

Depending on the number of the working phase at the regulated intersection, certain maneuvers between sections are allowed or prohibited. To describe the permitted maneuvers, we use a matrix of configurations

$$\mathbf{A}(\mathbf{v}(k)) = [a_{i,j}(\mathbf{v}(k))],\ i, j = 1, \dots, L, \tag{22}$$

where

$$a_{i,j}(\mathbf{v}(k)) = \begin{cases} 0, \text{ if } a_{i,j} = 0 \text{ or } v_{c_{i,j}}(k) \notin f^{i,j} \\ 1, \text{ otherwise} \end{cases}. \tag{23}$$

Using the configuration matrix, we change the structures of the matrices $\mathbf{B}$ and $\mathbf{D}$

$$\mathbf{B} \odot \mathbf{A}(\mathbf{v}(k)),\ \mathbf{D} \odot \mathbf{A}(\mathbf{v}(k)),$$

where $i, j = 1, \dots, L$, $\odot$ is an Hadamard product of matrices,

$$\mathbf{G} \odot \mathbf{H} = [g_{i,j} h_{i,j}],\ i, j = 1, \dots, L,$$

$\mathbf{G}$, $\mathbf{H}$ are matrices of dimension $L \times L$.

We consider flow changes at each time k. The time interval between steps $k-1$ and k is considered to be the smallest non-reduced time interval.

To describe the changes of traffic flow state on all sections at step k, we introduce a unit vector

$$\mathbf{1}_L = [\overbrace{1 \dots 1}^{L}]^{\mathrm{T}}. \tag{24}$$

Then the change of traffic flow after one step is

$$\mathbf{x}(k) = \mathbf{x}(k-1) - \mathbf{\Delta}_x \mathbf{1}_L + \mathbf{\Delta}_x^{\mathrm{T}} \mathbf{1}_L, \tag{25}$$

where

$$\mathbf{\Delta}_x = [\Delta_{i,j}],\ i, j = 1, \dots, L, \tag{26}$$

$\Delta_{i,j}$ is the number of traffic that moved from the section i to the section j in one interval,

$$\mathbf{\Delta}_x^{\mathrm{T}} = [\Delta_{j,i}],\ j, i = 1, \dots, L, \tag{27}$$

$\Delta_{j,i}$ is the number of traffic that arrived to section j from section i per interval. The matrix $\mathbf{\Delta}_x$ is determined by the equation

$$\begin{aligned} \mathbf{\Delta}_x \ = \ & ((\mathbf{x}(k-1)\mathbf{1}_L^{\mathrm{T}}) \odot \mathbf{A}(\mathbf{v}(k)) \odot \mathbf{D} - \\ & ((\mathbf{x}(k-1)\mathbf{1}_L^{\mathrm{T}}) \odot \mathbf{A}(\mathbf{v}(k)) \odot \mathbf{D} \dot{-} \\ & \mathbf{A}(\mathbf{v}(k)) \odot \mathbf{B}))\mathbf{1}_L^{\mathbf{1}}, \end{aligned} \tag{28}$$

where $\dot{-}$ is subtraction operation for positive integers

$$a \dot{-} b = \begin{cases} a - b, \text{ if } a \geq b \\ 0, \text{ otherwise} \end{cases},$$

$$(\mathbf{x}(k-1)\mathbf{1}_L^{\mathrm{T}}) \odot \mathbf{A}(\mathbf{v}(k)) \odot \mathbf{D} = \mathbf{W},$$

$$\mathbf{W} = [w_{i,j}],\ i, j = 1, \dots, L,$$

$w_{i,j}$ is the amount of traffic that wanted to leave section i and move to section j at step k,

$$\mathbf{A}(\mathbf{v}(k)) \odot \mathbf{B} = \mathbf{P},$$

$$\mathbf{P} = [p_{i,j}],\ i, j = 1, \dots, L,$$

$p_{i,j}$ is the amount of traffic that can move from section i to section j at step k,

$$(\mathbf{x}(k-1)\mathbf{1}_L^{\mathrm{T}}) \odot \mathbf{A}(\mathbf{v}(k)) \odot \mathbf{D} \dot{-} \mathbf{A}(\mathbf{v}(k)) \odot \mathbf{B} = \mathbf{S},$$

$$\mathbf{S} = [s_{i,j}],\ i, j = 1, \dots, L,$$

$s_{i,j}$ is the amount of traffic that remained on the section i, after departure of some traffic to section j at step k,

$$\mathbf{W} - \mathbf{S} = [w_{i,j} - s_{i,j}],\ i, j = 1, \dots, L,$$

$w_{i,j} - s_{i,j}$ is the amount of traffic that actually left the road section i and arrived the road section j at the moment k,

$$\mathbf{W}^{\mathrm{T}} - \mathbf{S}^{\mathrm{T}} = [w_{j,i} - s_{j,i}],\ j, i = 1, \dots, L,$$

$w_{j,i} - s_{j,i}$ is the amount of traffic that actually arrived on the road section j from the road section i at moment k. As a result, we obtain

$$\mathbf{\Delta}_x = \mathbf{W} - \mathbf{S}. \tag{29}$$

Let K^+ be the number of minimum intervals during which the traffic light timing contains constant durations of working phases. Then to calculate the optimal coordination plan the following control quality criteria are used

$$J_1 = \sum_{i \in I_2} x_i(K^+) \to \max, \tag{30}$$

where I_2 is a set of the output road sections,

$$J_2 = \sum_{i \in I_2} x_i(K^+) - \sum_{i \in I_0} x_i(K^+) \to \max, \tag{31}$$

where I_0 is a set of the entry road sections.

We obtain the training sample for the synthesis problem after solving the OCP for the model (25) for various initial states with the quality criterion (30) or (31).

III. SYMBOLIC REGRESSION METHOD

To solve the control synthesis problem a symbolic regression method of network operator was applied [22]. It uses directed graphs to present mathematical expressions. The source nodes of the graph contain the arguments of the mathematical expression. The remaining nodes correspond to functions with two arguments. The edges of the graph correspond to functions with one argument. Due to topological sorting, graphs are stored as integer upper triangular matrices in PC memory

$$\Psi = [\psi_{i,j}], i, j = 1, \dots, L, \psi_{i,j} = 0, i < j. \quad (32)$$

The network operator matrix (32) is constructed based on the adjacency matrix of the graph. Zero elements on the main diagonal ($\psi_{j,j} = 0$) correspond to source nodes, and non-zero elements on the main diagonal ($\psi_{j,j} > 0$) correspond to indices of functions with two arguments from the alphabet of functions. Non-zero elements above the main diagonal ($\psi_{i,j} \neq 0, i > j$) correspond to indices of functions with one argument. To obtain a mathematical expression for the network operator matrix, the following relationship is used

$$z_j^{(i)} \leftarrow \begin{cases} s_{\psi_{j,j}}(z_j^{(i-1)}, f_{\psi_{i,j}}(z_j^{(i-1)})), \text{ if } \psi_{i,j} \neq 0 \\ z_j^{(i-1)}, \text{otherwise} \end{cases}, \quad (33)$$

where $\mathbf{z}$ is a vector of size L; L is the number of rows in the network operator matrix; $s_{\psi_{j,j}}$ is a function with two arguments that is placed on the diagonal of matrix Ψ (32); $f_{\psi_{i,j}}$ is a function with one argument that is placed in element $\psi_{i,j}$. The initial values of the components of vector $\mathbf{z}^{(0)}$ are the arguments of the mathematical expression and the unit elements of the functions with two arguments. The algorithm sequentially follows the elements on the main diagonal. The value of a certain component in $\mathbf{z}$ is changed according to (33).

Symbolic regression allows to find mathematical expressions using evolutionary techniques. All symbolic regression methods encode mathematical expressions with a special code, and then search for the desired expression using a special genetic algorithm in which crossover and mutation operations are performed taking into account the form of encoding. To encode a mathematical expression, an alphabet of elementary functions predetermined by the researcher is used.

As a rule, an alphabet of elementary functions includes functions without arguments, or arguments and parameters of the desired mathematical expression, functions with one argument and functions with two arguments.

The effectiveness of using symbolic regression to find the mathematical expression is determined by the preservation of the inheritance property. It is the case when performing a crossover operation. It is essential the the offspring solutions inherit good fitness from their parents and still be feasible solutions to the problem solved.

Studies have shown that the most well-known method of symbolic regression, such as GP, does not retain inheritance properties when performing a crossover operation. For example, performing a crossover operation on two identical parent codes may result in two different offspring codes that do not look like each other and do not look like parent codes, since the crossover operation involves code exchanges of randomly selected parent code sub-trees. Thus, as a result of the crossover operation, new codes of possible solutions are generated and the search for the best of possible solution becomes simply random.

The property of inheritance is maximally preserved by symbolic regression methods, which use the principle of small variations of the basis solution [18]. According to this principle, one basic possible solution is encoded by the method of symbolic regression, which, in the opinion of the researcher, is the closest to the desired possible solution. The remaining possible solutions are encoded by ordered sets of codes of small variations of the basis solution. The number of elements in a set of small variation codes is called the depth of variation. All operations of the genetic algorithm, crossover and mutation, are performed on sets of codes of small variations.

Each generation in the genetic algorithm is determined by the number of crossover and mutation operations performed. After a certain number of generations, which is called the eapoch, the basic solution is replaced by the best solution found at that moment.

IV. COMPUTATIONAL EXPERIMENT

Consider the road network of two regulated intersections and its graph, presented in Fig. 1.

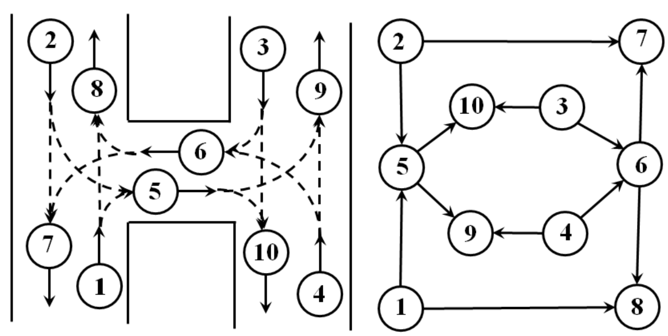

Fig. 1. Road network of two regulated intersections and its graph

The adjacency matrix of the graph shown on the Fig. 1 is

$$\mathbf{A} = \begin{bmatrix} 0&0&0&0&1&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&1 \\ 0&0&0&0&0&1&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}.$$

For this network, six optimal coordination plans were calculated for different initial states on the entry road sections. The symbolic regression method solved the problem of approximating the function, which for four values of the traffic flow on the entry road sections finds the calculated optimal coordination plans for all six experiments. The symbolic regression method of network operator was used.

The search of mathematical expression of control function was performed by the following criterion

$$J = \max_{i,j} ||\mathbf{u}_j^{*,i} - \tilde{\mathbf{u}}_j^i|| \to \min, \quad (34)$$

where $i = 1, \dots, S$, j is a component of control vector (10), $\mathbf{u}_j^{*,i}$ is the control obtained via OCP, $\tilde{\mathbf{u}}_j^i$ is the result of control synthesis problem taking into account the constraints on control.

For the solution presented below the value of criterion (34) is no more than 1 control step.

The function that was found by the network operator is as follows

$$\tilde{u}_1 = \tanh(z_{17})\vartheta(z_{16}), \quad (35)$$

$$\tilde{u}_2 = 0.5u_1 \mathrm{sgn}(z_{11})\vartheta(q_2x_2)\sqrt{q_4}\mu(q_1), \quad (36)$$

$$\tilde{u}_3 = 2\tanh(u_2)u_1\sqrt{q_2}, \quad (37)$$

$$\tilde{u}_4 = \sqrt[3]{u_3} + \rho_{19}(z_{17}) + \rho_{19}(z_{16}), \quad (38)$$

$$\tilde{u}_5 = \tanh(u_4) + \rho_{19}(z_{15}), \quad (39)$$

$$\tilde{u}_6 = \arctan(u_5)\mu(z_{11})\mu(z_{10} + \arctan(x_1^2))\rho_{18}(q_1), \quad (40)$$

$$z_{10} = \sqrt[3]{x_1^2 q_3 x_3 \sqrt[3]{x_1}}, z_{11} = \mathrm{sgn}(q_2x_2)q_4 \arctan(x_4)x_3,$$

$$z_{14} = \rho_{17}(\mu(z_{11})) + z_{10} + \arctan(x_1^2) + \mu(x_4) + x_3^2,$$

$$z_{15} = 8z_{14}\mu(z_{11})z_{11}x_4, z_{16} = 0.5z_{15}z_{14}\tanh(q_3),$$

$$z_{17} = z_{16} + 0.5x_4 + x_3^3,\ \rho_{17}(\alpha) = \mathrm{sgn}\ln(|\alpha| + 1),$$

$$\mu(\alpha) = \begin{cases} \alpha, \text{ if } |\alpha| \leqslant 1, \\ \mathrm{sgn}(\alpha), \text{ otherwise}, \end{cases} \quad \vartheta = \begin{cases} 1, \text{ if } \alpha > 0, \\ 0, \text{ otherwise}, \end{cases}$$

$$\rho_{18}(\alpha) = \mathrm{sgn}(\alpha)(\exp(|\alpha|) - 1),$$

$$\rho_{19}(\alpha) = \mathrm{sgn}(\alpha)\exp(-|\alpha|),$$

$q_1 = 1.046$, $q_2 = 5.53$, $q_3 = 4.05$, $q_4 = 1.45$.

V. CONCLUSIONS

The problem of traffic flow control synthesis was considered. The main idea is to obtain a function that performs traffic light timing design for the traffic flow state domain. To solve this problem, it is proposed to initially solve the optimal control problem many times for different initial states, and then approximate the set of obtained solutions using a symbolic regression method. An example of a solution to the problem of synthesizing traffic flow control for a road network having ten road sections and two controlled intersections with three working phases of traffic lights is given.

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