H_∞ AND H_2 CONTROL DESIGN FOR DISCRETE-TIME LPV SYSTEMS SUBJECT TO BOUNDED PARAMETER VARIATIONS

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***Abstract*— This paper investigates H_∞ and H_2 control synthesis for discrete-time linear parameter-varying systems subject to bounded rates of parameter variation. In practical engineering applications, scheduling parameters are typically constrained not only in magnitude but also in their rates of change due to physical limitations. Neglecting these rate bounds can result in overly conservative controller designs. To address this issue, a novel control design method is proposed that employs parameter-dependent Lyapunov functions together with nonlinear gain-scheduling control laws. The proposed approach explicitly incorporates parameter rate bounds into the design conditions. Numerical examples are provided to demonstrate the effectiveness of the method.**

I. INTRODUCTION

Linear parameter-varying (LPV) systems provide a powerful framework for modeling and control of nonlinear and time-varying systems by representing such systems as linear dynamics that depend on measurable, time-varying parameters. Over the past decades, LPV control has been extensively studied due to its capability to bridge the gap between linear control techniques and nonlinear system behavior. As a result, LPV methods have found widespread applications in areas such as aerospace, robotics, process control, and automotive systems, see, e.g., [1], [2].

In many practical applications, scheduling parameters are bounded not only in magnitude but also in their rates of variation due to physical or operational constraints. Ignoring such rate information can lead to conservative designs and degraded performance. As a result, LPV control methods that explicitly account for bounded parameter variation rates have received increasing attention. For instance, Amato et al. [3] proposed a gain-scheduled approach for discrete-time LPV systems that incorporates rate bounds via a polytopic representation and difference inequalities, yielding improved performance over rate-independent designs. However, the approach significantly increases the number of linear matrix inequality (LMI) constraints and decision variables, and thus the computational complexity.

To overcome this limitation, Oliveira and Peres [4] introduced a novel modeling framework for discrete-time LPV systems with bounded parameter variation rates. By embedding the correlation between the scheduling parameters and their rates of variation into an augmented space, the resulting polytopic representation maintains a fixed number of vertices, independent of the rate bounds. This approach enables effective exploitation of rate information while preserving the computational tractability of the LMI conditions. The method has since become a cornerstone in the control design of LPV systems with bounded parameter variations.

In parallel with the development of stability and stabilization conditions, performance-oriented control design for LPV systems – particularly H_∞ and H_2 – has also received considerable attention. Although numerous results are available for LPV systems that neglect parameter variation rates, the incorporation of rate-bounded information into H_∞ and H_2 controller synthesis for discrete-time LPV systems remains relatively limited, especially when seeking to reduce conservatism without increasing computational complexity

Motivated by the above observations, the objective of this paper is to propose a design methodology for H_∞ and H_2 controllers for discrete-time LPV systems with bounded parameter variations. The main contributions of this paper are summarized as follows:

- We propose a class of nonlinearly parameterized state-feedback control laws for H_∞ and H_2 control design.
- To the best of our knowledge, this is the first method that enables the design of continuous state-dependent H_∞ and H_2 control laws for LPV systems while explicitly accounting for bounded parameter variations.

The remainder of the paper is organized as follows. Section II presents the problem formulation and preliminaries. Sections III and IV are dedicated to the H_∞ and H_2 controller design methodologies, respectively. Two numerical examples, including comparisons with existing methods from the literature, are presented in Section V. Finally, conclusions are drawn in Section VI.

Notation: Let $\mathbb{R}$ denote the set of real numbers, and let $\mathbb{R}^{n\times m}$ represent the space of all real matrices of dimension $n \times m$. For a positive integer L, define $\overline{1,L} := 1, 2, \ldots, L$. A matrix P is said to be positive definite if $P \succ 0$. The notation $\mathbb{S}^n$ stands for the set of symmetric positive definite matrices in $\mathbb{R}^{n\times n}$. In a symmetric block matrix, the symbol $(*)$ indicates the transposed symmetric entries. Moreover, 0 and I denote the zero matrix and the identity matrix with compatible dimensions, respectively.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Problem Formulation

We consider the following LPV system

$$\begin{aligned} x(k+1) &= A(k)x(k) + B_w(k)w(k) + B(k)u(k) \\ y(k) &= C(k)x(k) + D(k)u(k) \end{aligned} \tag{1}$$

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where $x \in \mathbb{R}^{n_x}$ is the measured state, $y \in \mathbb{R}^{n_y}$ is the performance output, $u \in \mathbb{R}^{n_u}$ is the control input, $w \in \mathbb{R}^{n_w}$ is the disturbance. The matrices $[A(k) \;\; B_w(k) \; B(k) \; C(k) \; D(k)] \in \Omega$ with

$$\Omega = \text{Co}\left([A_1 \; B_{w1} \; B_1 \; C_1 \; D_1], \ldots, [A_L \; B_{wL} \; B_L \; C_L \; D_L]\right) \tag{2}$$

where $A_i \in \mathbb{R}^{n_x \times n_x}$, $B_i \in \mathbb{R}^{n_x \times n_u}$, $B_{wi} \in \mathbb{R}^{n_x \times n_w}$, $B_i \in \mathbb{R}^{n_x \times n_u}$, $C_i \in \mathbb{R}^{n_y \times n_x}$, $D_i \in \mathbb{R}^{n_y \times n_w}$ are constant matrices, $\forall i \in \overline{1, L}$, L is the number of subsystems, and $\text{Co}(\cdot)$ denotes the convex hull operator. It is well known [5] that the condition $[A(k) \; B_w(k) \; B(k) \; C(k) \; D(k)] \in \Omega$ holds if and only if $\exists p_i(k), \forall i \in \overline{1, L}$ such that

$$\begin{aligned} A(k) &= \sum_{i=1}^{L} p_i(k) A_i, B_w(k) = \sum_{i=1}^{L} p_i(k) B_{wi} \\ B(k) &= \sum_{i=1}^{L} p_i(k) B_i, C(k) = \sum_{i=1}^{L} p_i(k) C_i \\ D(k) &= \sum_{i=1}^{L} p_i(k) D_i \end{aligned} \tag{3}$$

where $p(k) = [p_1(k) \; p_2(k) \; \ldots p_L(k)]^T$ satisfies

$$p(k) \in \Sigma_L := \left\{ p \in \mathbb{R}^L \mid \sum_{i=1}^{L} p_i = 1, \;\; p_i \geq 0 \right\} \tag{4}$$

The vector $p(k)$ may represent time-varying lumped parameters or state space components.

LPV hypothesis: $p(k)$ is available in each time instant k.

The rate of parameter variation $\Delta_i(k) = p_i(k+1) - p_i(k)$ is bounded and satisfies the following condition:

$$-b \leq \Delta_i(k) \leq b, \forall i \in \overline{1, L} \tag{5}$$

where $b \in \mathbb{R}$, $b \in [0, 1]$ is known.

Since $\sum\limits_{i=1}^{L} p_i(k) = 1$, it follows that

$$\sum_{i=1}^{L} \Delta_i(k) = 0 \tag{6}$$

B. Preliminaries

We will use the following lemma to construct a parameter dependent Lyapunov function.

Lemma 1: [6] Given $Q \in \mathbb{S}^{n \times n}$, $G \in \mathbb{R}^{n \times n}$. The following relation holds

$$G^T Q^{-1} G \succeq G + G^T - Q \tag{7}$$

In this paper, we will deal several times with the following double sum positivity problem of the form

$$x^T \left(\sum_{i,j=1}^{L} p_i p_j M_{ij} \right) x \geq 0 \tag{8}$$

where $x \in \mathbb{R}^{n_x}$, $p_i \in \Sigma_L$, $M_{ij} \in \mathbb{R}^{n_x \times n_x}$, $\forall i \in \overline{1, L}, \forall j \in \overline{1, L}$.

Lemma 2: [7] If the following LMI conditions

$$\left\{ \begin{array}{l} M_{ii} \succeq 0, \forall i \in \overline{1, L} \\ \frac{2}{L-1} M_{ii} + M_{ij} + M_{ji} \succeq 0, 1 \leq i \neq j \leq L \end{array} \right. \tag{9}$$

hold, then (8) holds $\forall x \in \mathbb{R}^{n_x}$, $\forall p \in \Sigma_L$.

In the following, we recall the modeling framework in [4] that takes into account the bounded rate of parameter variations. Using Fig. 1, any pair (p_i, Δ_i) belongs to the

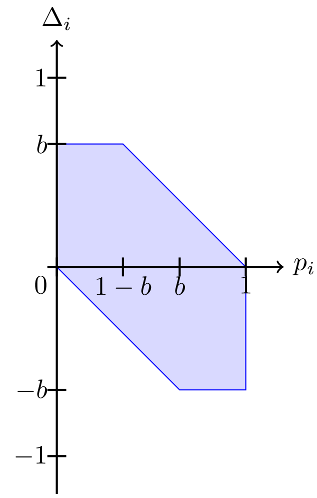

Fig. 1. Region on the plane $\Delta_i \times p_i$ where Δ_i can assume values as a function of p_i (dark region).

polytope Γ_i, $i \in \overline{1, L}$ given by

$$\Gamma_i \triangleq \left\{ \delta \in \mathbb{R}^2 \mid \delta = \sum_{i=1}^{6} \lambda_i h_i, \lambda \in \Sigma_6 \right\} \tag{10}$$

where

$$[h_1, \ldots, h_6] = \begin{bmatrix} 0 & 0 & 1-b & 1 & 1 & b \\ 0 & b & b & 0 & -b & -b \end{bmatrix} \tag{11}$$

i.e. Γ_i is the convex hull of the vertices of the feasible region.

To model the space of (p, Δ), we take the Cartesian product of all Γ_i for $i \in \overline{1, L}$, and then remove any vertices that violate (4) and (6). The resulting polytope is denoted by Γ, i.e.

$$\Gamma \triangleq \left\{ \delta \in \mathbb{R}^{2L} \mid \delta = \sum_{i=1}^{m} \lambda_i h_i, \lambda \in \Sigma_m \right\} \tag{12}$$

where $h_i \in \mathbb{R}^{2L}$ are given vectors, and the number of vertices m grows as a function of L. A key feature of this modeling approach is that both the parameter vector p and its variation Δ are jointly embedded in a higher-dimensional space—referred to as the *γ-space*—of dimension $2L$.

When solving a control problem that depends simultaneously on both p and Δ, we first lift p and Δ to the γ-space. This step can be achieved using the relationship given in (12)

$$\begin{bmatrix} p \\ \Delta \end{bmatrix} = H\gamma \tag{13}$$

where $H = \begin{bmatrix} h_1 & \dots & h_m \end{bmatrix}$, $\gamma \in \Sigma_m$. For example, for $L = 2$, the polytope Γ is constructed by $\Gamma_1 \times \Gamma_2$, vertices h_i are given by

$$\begin{bmatrix} h_1, \dots, h_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1-b & b \\ 0 & 0 & 1 & 1 & b & 1-b \\ -b & 0 & 0 & b & b & -b \\ b & 0 & 0 & -b & -b & b \end{bmatrix} \tag{14}$$

It follows that

$$\begin{aligned} p_1 &= \gamma_1 + \gamma_2 + (1-b)\gamma_5 + b\gamma_6 \\ p_2 &= \gamma_3 + \gamma_4 + b\gamma_5 + (1-b)\gamma_6 \\ \Delta_1 &= -b\gamma_1 + b\gamma_4 + b\gamma_5 - b\gamma_6 \\ \Delta_2 &= b\gamma_1 - b\gamma_4 - b\gamma_5 + b\gamma_6 \end{aligned} \tag{15}$$

Thus

$$\begin{aligned} A(k) &= \hat{A}(\gamma) = \sum_{j=1}^{m} \gamma_j \hat{A}_j \\ A(k+1) &= \tilde{A}(\gamma) = \sum_{j=1}^{m} \gamma_j \tilde{A}_j \end{aligned} \tag{16}$$

where $\hat{A}_1 = A_1$, $\hat{A}_2 = A_1$, $\hat{A}_3 = A_2$, $\hat{A}_4 = A_2$, $\hat{A}_5 = (1-b)A_1 + bA_2$. $\hat{A}_6 = bA_1 + (1-b)A_2$, $\tilde{A}_1 = (1-b)A_1 + bA_2$, $\tilde{A}_2 = A_1$, $\tilde{A}_3 = A_2$, $\tilde{A}_4 = bA_1 + (1-b)A_2$, $\tilde{A}_5 = A_1$, $\tilde{A}_6 = A_2$.

III. H_∞ Non-linear Gain-scheduling Control Design

In this section, we address the problem of H_∞ control synthesis for LPV systems subject to bounded parameter variations.

Consider the following parameter dependent Lyapunov function candidate

$$V(k) = x(k)^T P(k)^{-1} x(k) \tag{17}$$

where $P(k) = \left(\sum_{i=1}^{L} p_i(k) P_i \right)$, $P_i \in \mathbb{S}^{n_x \times n_x}$, $\forall i \in \overline{1, L}$.

It is worth noting that the matrices $A(k), B_w(k)$, $B(k)$, $C(k)$, $D_1(k)$, as well as the Lyapunov matrix $P(k)$ are functions of $p(k)$. As such they are all subject to the bounded parameter variation assumption (5). Consequently, the method [4] can be employed to lift the original parameter space (p, Δ) into the γ-space.

Consider the following nonlinear gain-scheduling state feedback

$$u(k) = F(k) G(k)^{-1} x(k) \tag{18}$$

where $F(k) = \sum_{i=1}^{L} p_i(k) F_i$, $G(k) = \sum_{i=1}^{L} p_i(k) G_i$, with $F_i \in \mathbb{R}^{n_x \times n_u}$, $G_i \in \mathbb{R}^{n_x \times n_x}$.

Our aim in this section is to find the matrix gains F_i, G_i such that the closed-loop system

$$\begin{aligned} x(k+1) &= A_c x(k) + B_w(k) w(k) \\ y(k) &= C_c x(k) \end{aligned} \tag{19}$$

where

$$\begin{aligned} A_c(k) &= A(k) + B(k) F(k) G(k)^{-1} \\ C_c(k) &= C(k) + D(k) F(k) G(k)^{-1} \end{aligned}$$

is robustly asymptotically stable for all $(p(k), \Delta(k)) \in \Gamma$ and has the optimal H_∞ norm bound on the $w-y$ channel.

Recall that m is the number of vertices in the lifted $\gamma-$space. For given matrices G_i, F_i, P_i, A_i, B_i, B_{wi}, C_i, D_i, we denote $\hat{G}_i$, $\hat{F}_i$,$\hat{P}_i$, $\hat{A}_i$, $\hat{B}_i$, $\hat{B}_{wi}$, $\hat{C}_i$, $\hat{D}_i$, $\tilde{P}_i$ as the coefficients of the matrices $\hat{G}(\gamma)$, $\hat{F}(\gamma)$, $\hat{P}(\gamma)$, $\hat{A}(\gamma)$, $\hat{B}(\gamma)$, $\hat{B}_w(\gamma)$, $\hat{D}(\gamma)$, $\tilde{P}(\gamma)$ obtained with change of variable (13) for $G(k)$,$F(k)$,$P(k)$,$A(k)$,$B(k)$,$B_w(k)$, $C(k)$, $D(k)$,$P(k+1)$, respectively.

We have the following theorem.

Theorem 1: Consider the LPV system (1). If there exist matrices $G_i \in \mathbb{R}^{n_x \times n_x}$, $F_i \in \mathbb{R}^{n_u \times n_x}$,$P_i \in \mathbb{S}^{n_x \times n_x}$, $i \in \overline{1, L}$ such that the following LMI conditions hold:

$$X_{jj} \succeq 0, \; j \in \overline{1, m} \tag{20}$$

$$\frac{2}{L-1} X_{jj} + X_{jl} + X_{lj} \succeq 0, 1 \leq j \neq l \leq m, \tag{21}$$

$$X_{jl} = \begin{bmatrix} \hat{G}_j^T + \hat{G}_j - \hat{P}_j & 0 & * & * \\ 0 & I & * & * \\ \hat{A}_j \hat{G}_l + \hat{B}_j \hat{F}_l & \hat{B}_{wj} & \tilde{P}_j & 0 \\ \hat{C}_j \hat{G}_l + \hat{D}_j \hat{F}_l & 0 & 0 & \beta I \end{bmatrix} \tag{22}$$

then the system (1) is robustly asymptotically stable under the bounded parameter variation (5), (6) with the gain-scheduling state feedback (18) and has a H_∞ norm bound on the $w-y$ channel less than β, that is, $\sup_{w \in L_2} \frac{||y||_2}{||w||_2} \leq \beta$.

Proof: Using Lemma 2, the following inequality holds when (20), (21) are satisfied

$$\left(\sum_{j,l=1}^{m} \gamma_j \gamma_l X_{jl} \right) \succeq 0$$

Or equivalently

$$\begin{bmatrix} \hat{G}(\gamma)^T + \hat{G}(\gamma) - \hat{P}(\gamma) & 0 & * & * \\ 0 & I & * & * \\ \hat{A}(\gamma)\hat{G}(\gamma) + \hat{B}(\gamma)\hat{F}(\gamma) & \hat{B}_w(\gamma) & \tilde{P}(\gamma) & 0 \\ \hat{C}(\gamma)\hat{G}(\gamma) + \hat{D}(\gamma)\hat{F}(\gamma) & 0 & 0 & \beta I \end{bmatrix} \succeq 0 \tag{23}$$

Using Lemma 1, (23) holds if

$$\begin{bmatrix} \hat{G}(\gamma)^T \hat{P}^{-1}(\gamma) \hat{G}(\gamma) & 0 & * & * \\ 0 & I & * & * \\ \hat{A}(\gamma)\hat{G}(\gamma) + \hat{B}(\gamma)\hat{F}(\gamma) & \hat{B}_w(\gamma) & \tilde{P}(\gamma) & 0 \\ \hat{C}(\gamma)\hat{G}(\gamma) + \hat{D}(\gamma)\hat{F}(\gamma) & 0 & 0 & \beta I \end{bmatrix} \succeq 0 \tag{24}$$

Pre- and post-multiplying (24) by diag($\hat{G}^{-1}(\gamma)^T$,0,0,0) and diag($\hat{G}^{-1}(\gamma)$,0,0,0), one obtains

$$\begin{bmatrix} \hat{P}^{-1}(\gamma) & 0 & * & * \\ 0 & I & * & * \\ \hat{A}(\gamma) + \hat{B}(\gamma)\hat{F}(\gamma)\hat{G}^{-1}(\gamma) & \hat{B}_w(\gamma) & \tilde{P}(\gamma) & 0 \\ \hat{C}(\gamma) + \hat{D}(\gamma)\hat{F}(\gamma)\hat{G}^{-1}(\gamma) & 0 & 0 & \beta I \end{bmatrix} \succeq 0 \tag{25}$$

By converting back from the γ-space to the $(p\ \Delta)$ space, (25) is equivalent to the following inequality

$$\begin{bmatrix} P(k)^{-1} & 0 & * & * \\ 0 & I & * & * \\ A_c(k) & B_w(k) & P(k+1) & 0 \\ C_c(k) & 0 & 0 & \beta I \end{bmatrix} \succeq 0 \tag{26}$$

Using Shur complement, (26) is satisfied if and only if

$$\begin{gathered}\begin{bmatrix} P(k)^{-1} & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} A_c(k) & B_w(k) \\ C_c(k) & 0 \end{bmatrix}^T \\ \times \begin{bmatrix} P(k+1)^{-1} & 0 \\ 0 & \frac{1}{\beta} I \end{bmatrix} \begin{bmatrix} A_c(k) & B_w(k) \\ C_c(k) & 0 \end{bmatrix} \succeq 0 \end{gathered} \tag{27}$$

Pre- and post-multiplying (27) by $[\mathrm{x(k)}^T\ \ \mathrm{w(k)}^T]$ and its transpose, one gets

$$\begin{aligned} &x(k)^T P(k)^{-1} x(k) - x(k+1)^T P(k+1)^{-1} x(k+1) \\ &+ w(k)^T w(k) - \tfrac{1}{\beta} y(k)^T y(k) \geq 0 \end{aligned}$$

Clearly, this condition implies that the system is asymptotically stable. Hence $\lim\limits_{k\to\infty} x(k) = 0$. It follows that

$$\frac{1}{\beta} \sum_{k=0}^{\infty} y(k)^T y(k) - x(0)^T P(0)^{-1} x(0) \leq \sum_{k=0}^{\infty} w(k)^T w(k)$$

Under the zero initial condition, i.e., $x(0) = 0$, one gets

$$\sum_{k=0}^{\infty} y(k)^T y(k) \leq \beta \sum_{k=0}^{\infty} w(k)^T w(k)$$

Hence the H_∞ norm is less than β. The proof is complete. ■

It is worth noticing that the problem of designing an H_∞ optimal control law for LPV systems with bounded parameter variations was already addressed in [3]. Compared with [3], the proposed approach offers two main advantages. First, when the parameter variation rate is small, the modeling strategy adopted in [3] results in a large number of LMI conditions, which significantly increases the computational complexity of the associated algorithms. Second, [3] employs a discontinuous control law with respect to the state, whereas the proposed approach ensures continuity by construction.

Using Theorem 1, the problem of designing an H_∞ optimal gain-scheduling control law for system (1) can be formulated as

$$\begin{aligned} &\min_{G_i, F_i, P_i, \beta} \beta, \\ &\text{s.t. } (20), (21), (22) \end{aligned} \tag{28}$$

This is a convex semidefinite program, for which several free solvers are available.

IV. H_2 NON-LINEAR GAIN-SCHEDULING CONTROL DESIGN

We first introduce the following lemma to support the method to be developed.

Lemma 3: [8] Consider the following discrete-time LTI syetem

$$\begin{aligned} x(k+1) &= Ax(k) + B_w w(k) + Bu(k) \\ u(k) &= Kx(k) \\ z(k) &= Cx(k) + Du(k) \end{aligned} \tag{29}$$

where matrices A, B, B_w, C, D are of appropriate dimensions.

Let the closed-loop matrices of the system be denoted as $A_{cl} = A + BK$ and $C_{cl} = C + DK$. Suppose that a state-feedback gain K is designed such that A_{cl} is asymptotically stable. Denote by L_o the observability Gramian of (A_{cl}, C_{cl}), i.e., L_o is the solution of the following discrete-time Lyapunov equation

$$A_{cl}^T L_o A_{cl} - L_o + C_{cl}^T C_{cl} = 0. \tag{30}$$

The H_2 norm of the closed-loop system is given by

$$\|H\|_2 = \mathrm{Tr}(B_w^T L_o B_w). \tag{31}$$

In the following we provide a way to extend the result in [8] for the LPV system (1). For this purpose, consider the non-linear gain scheduling state feedback controller (18), and the following parameter-dependent Lyapunov functions

$$V_c(k) = x(k)^T P(k)^{-1} x(k) \tag{32}$$

where $P(k) = \left(\sum\limits_{i=1}^{L} p_i(k) P_i\right)$, $P_i \in \mathbb{S}^{n_x \times n_x}$, $\forall i \in \overline{1, L}$.

The following result holds.

Theorem 2: Consider LPV system (1). If there exist matrices $G_i \in \mathbb{R}^{n_x \times n_x}$, $F_i \in \mathbb{R}^{n_u \times n_x}$, $Z_i \in \mathbb{R}^{n_w \times n_w}$, $P_i \in \mathbb{S}^{n_x \times n_x}$, $\forall i \in \overline{1, L}$ such that the following LMI conditions hold:

$$Y_{jj} \succeq 0,\ j \in \overline{1, m} \tag{33}$$

$$\frac{2}{m-1} Y_{jj} + Y_{jl} + Y_{lj} \succeq 0, 1 \leq j \neq l \leq m, \tag{34}$$

$$Y_{jl} = \begin{bmatrix} \hat{G}_j^T + \hat{G}_j - \hat{P}_j & * & * \\ \hat{A}_j \hat{G}_l + \hat{B}_j \hat{F}_l & \tilde{P}_j & * \\ \hat{C}_j \hat{G}_l + \hat{D}_j \hat{F}_l & 0 & I \end{bmatrix} \tag{35}$$

$$\begin{bmatrix} \hat{Z}_j & \hat{B}_w^T \\ \hat{B}_w & \hat{P}_j \end{bmatrix} \succeq 0 \tag{36}$$

$$Tr(\hat{Z}_j) \leq \eta \tag{37}$$

then the system (1) is robustly asymptotically stable under the bounded parameter variations (5), (6) with the gain-scheduling state feedback (18) and has a H_2 norm bound on the $w - y$ channel less than η.

Proof: Following a similar argument as in the proof of Theorem 1, it can be shown that inequalities (33)–(35) imply

$$A_{cl}^T(k) P(k+1) A_{cl}(k) - P(k) + C_c^T(k) C_c(k) \preceq 0,$$

Hence the system is asymptotically stable, and $P(k)$ plays a role of the controllabity gramian for the closed-loop LPV system.

Note that inequalities (36)–(37) imply

$$\|H\|_2 = \mathrm{Tr}(B_w^T(k)P(k)B_w) \leq \mathrm{Tr}(Z(k)) \leq \eta.$$

The proof is complete. ■

Using Theorem 2, the problem of designing an optimal gain-scheduling control law for system (1) can be formulated as

$$\begin{aligned} &\min_{G_i,F_i,P_i,\eta} \ \eta, \\ &\text{s.t. } (33), (34), (35), (36), (37) \end{aligned} \tag{38}$$

V. Numerical Examples

This section illustrates the potential benefit of the new approaches by simulations of two example systems. We used the CVX toolbox to solve SDP optimization problems. For comparison purpose, we denote the our proposed algorithm, i.e. the H_∞ control law with parameter bounded variation as Hinfpvb, without parameter bounded variation as Hinf, the H_2 control law with parameter bounded variation as H2pbv, without parameter bounded variation as H2.

A. Example 1

This example is taken from [9], and is used to investigate the performance of the H_∞ controller. For this system, the proposed H_∞ control approach is compared with the method in [3] as well as a conventional H_∞ design that does not consider bounded parameter variations.

Consider system (1) with

$$\begin{aligned} A_1 &= \alpha \begin{bmatrix} 1.0 & 0 & -2 \\ 2 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}, \ A_2 = \alpha \begin{bmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -2 & -1 \end{bmatrix} \\ B_1 &= B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B_{w1} = B_{w2} = \begin{bmatrix} 0.1 \\ -0.15 \\ 0.16 \end{bmatrix} \\ C_1 &= C_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, D_1 = D_2 = 1 \end{aligned}$$

The parameter α is 0.66, with a bounded rate of variation $b = 0.25$, i.e.,

$$|p_i(k+1) - p_i(k)| \leq 0.25, \forall i = 1, 2$$

By solving the optimization problem (28), an upper bound of the H_∞ norm is obtained as $\beta = 0.733$, with

$$\begin{aligned} F_1 &= \begin{bmatrix} -0.3691 & -0.2784 & 0.6915 \end{bmatrix} \\ F_2 &= \begin{bmatrix} -0.2817 & -0.2580 & 0.5617 \end{bmatrix} \\ G_1 &= \begin{bmatrix} 0.6605 & 0.3947 & -0.6813 \\ 0.4323 & 0.3840 & -0.3832 \\ -0.2124 & -0.1058 & 0.4278 \end{bmatrix} \\ G_2 &= \begin{bmatrix} 0.5442 & 0.3452 & -0.4034 \\ 0.4819 & 0.3909 & -0.4711 \\ -0.5406 & -0.3722 & 0.7446 \end{bmatrix} \end{aligned} \tag{39}$$

Using the approach proposed in [3], an upper bound of the H_∞ norm is obtained as $\beta = 1.18$. Without considering bounded parameter variations ($b = 1$), the computed upper bound of the H_∞ norm is $\beta = 3.85$.

For the initial condition $x(0) = \begin{bmatrix} 2 & -1 & 0 \end{bmatrix}^T$, Fig. 2 shows the state trajectories as functions of time for the proposed

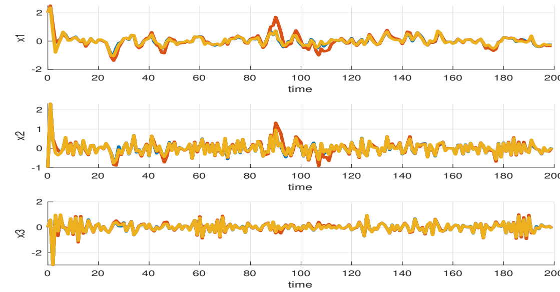

Fig. 2. State trajectories for Hinfpvb (solid blue), Hinf (solid red), and the method in [3] (solid yellow) for Example 1.

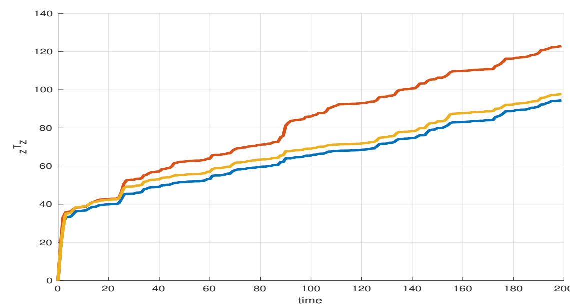

Fig. 3. Accumulated output energy trajectories for Hinfpvb (solid blue), Hinf (solid red), and the method in [3] (solid yellow) for Example 1.

H_∞ controller with parameter bounded variation (Hinfpvb, solid blue), the conventional (without parameter bounded variation) H_∞ controller (Hinf, solid red), and the method in [3] (solid purple) under the Gaussian white noise. Fig. 3 shows the corresponding accumulated output energy.

It can be observed that the proposed method exhibits superior disturbance attenuation capability. It is worth noting that the number of decision variables is the same for our approach and the rate-independent methods. As a result, the proposed method has lower computational complexity compared with the method in [3].

B. Example 2

The second example is adopted from [10] and is used to evaluate the performance of the proposed H_2 controller.

Consider system (1) with

$$\begin{aligned} A_1 &= \begin{bmatrix} 1.0 & -1.4 \\ -1.0 & -0.8 \end{bmatrix}, \ A_2 = \begin{bmatrix} 1.0 & 1.4 \\ -1.0 & -0.8 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 5.9 \\ 2.8 \end{bmatrix}, B_2 = \begin{bmatrix} 3.1 \\ -2.8 \end{bmatrix}, B_{w1} = B_{w2} = \begin{bmatrix} -0.2 \\ 0.16 \end{bmatrix} \\ C_1 &= C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned}$$

We consider the bound rate of variation $b = 0.18$.

By solving the optimization problem (38), we get $\eta = 0.059$. If we do not take into account the bounded rate of parameter variations information, we get $\eta = 0.113$. Therefore, by fully exploiting the information on the bounded

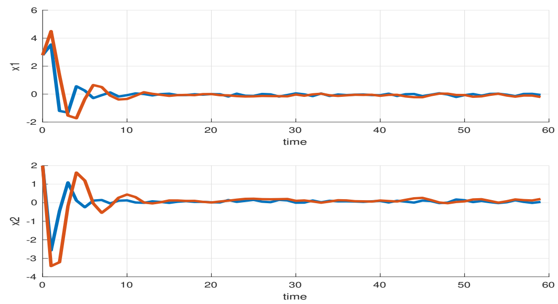

Fig. 4. State trajectories for H2pvb (solid blue), H2 (solid red) for Example 2.

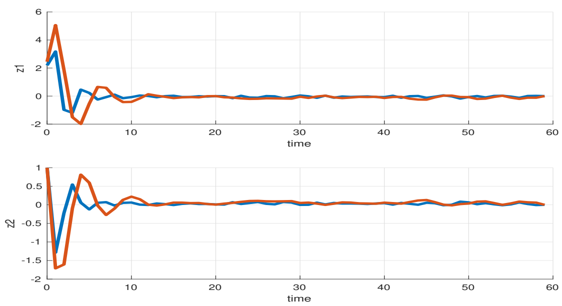

Fig. 5. output trajectories for H2pvb (solid blue), H2 (solid red) for Example 2.

rate of parameter variation, the proposed method achieves a smaller H_2 norm.

Fig. 4 and Fig. 5 show the state and output trajectories as functions of time for the proposed H_2 controller with parameter bounded variation (H2pvb, solid blue), the conventional(without parameter bounded variation) H_2 controller (H2, solid red), Fig. 6 shows the corresponding input trajectories.

VI. Conclusion

This paper has presented a systematic design methodology for H_∞ and H_2 controllers for discrete-time LPV systems with bounded parameter variations. By introducing parameter-dependent Lyapunov functions and employing nonlinear gain-scheduling control laws, the proposed approach explicitly incorporates bounds on parameter variation rates into the controller synthesis process. Theoretical and simulation results show that the proposed approach outperforms state-of-the-art methods.

Future work will focus on extending the proposed framework to output-feedback controller design and to more complex scenarios involving measurement noise and additional parameter uncertainties.

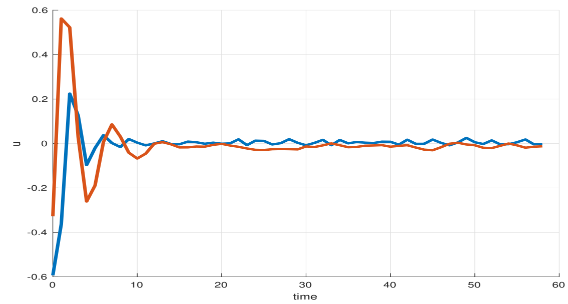

Fig. 6. input trajectories for H2pvb (solid blue), H2 (solid red) for Example 2.

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