Elite Trajectory Exploration for Pareto Front Approximation in Dynamic Bin Packing*

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***Abstract*— The Dynamic Multi-Objective Bin-Packing Problem (DMOBPP) presents significant challenges in simultaneously optimizing bin usage (z_1), thermal heat (z_2), and delivery time (z_3). This paper proposes AMSS-PATH, a hybrid metaheuristic integrating Strategic Path Relinking into an Adaptive Multi-Objective Scatter Search (AMSS) framework with three specialized operators (Φ_1, Φ_2, Φ_3), each targeting a distinct objective. By exploring elite-solution trajectories, the method densifies the Pareto frontier through targeted intensification at every intermediate point. Experimental results on toy and full-scale instances yield a *Net Front Contribution (NFC) of 100%*, a *Binary Conflict Index (BCI) of* ≈ 0, and a 4.8% hypervolume gain over base AMSS, significantly outperforming GAMMA-PC, MOMA, and NSGA-II. A Wilcoxon signed-rank test ($p \approx 0.031$) confirms statistical significance.**

I. INTRODUCTION

The Bin Packing Problem (BPP) is a fundamental NP-hard optimization problem with extensive applications in logistics and manufacturing. In modern industrial contexts, such as food processing or energy-efficient cloud computing, the problem is increasingly characterized by its dynamic nature and conflicting goals. The DMOBPP (Dynamic Multi-Objective Bin-Packing Problem) requires the simultaneous optimization of bin occupancy, thermal distribution, and processing time, posing significant challenges for traditional exact methods. As highlighted in [8], recent variations of the problem involving multiple constraints and dynamic item fragmentation further necessitate the development of robust metaheuristic approaches that can handle high-dimensional trade-offs in real-time. This positioning is consistent with recent work on dynamic vector bin packing and multidimensional bin-packing surveys [13], [14].

Among the various metaheuristic frameworks developed to address such complexity, Scatter Search (SS) has emerged as a particularly robust approach. Initially introduced in [1], Scatter Search diverges from traditional genetic algorithms by emphasizing a systematic, rather than purely stochastic, exploration of the solution space. As detailed in [2], the effectiveness of SS relies on the maintenance and evolution of a high-quality *Reference Set*, which serves as the core for generating new trial solutions through structured combinations. Recent methods in multiobjective combinatorial optimization, such as those proposed in [7], have successfully integrated path relinking and tabu search within the SS framework to enhance both diversification and intensification.

However, in the context of multi-objective combinatorial optimization, maintaining both convergence and diversity in the Pareto front remains a significant hurdle. The concept of Pareto local optimality, as explored in [3], suggests that the search landscape is often fragmented into multiple local trade-off regions. Population-based methods such as NSGA-II [9], while widely applied, rely on stochastic operators that struggle to maintain feasibility under tight thermal and temporal constraints. Recent advancements, such as the multi-objective local optimality framework proposed in [4], highlight the necessity for hybrid mechanisms that can escape these local optima while refining the approximation of the global frontier.

This paper details how AMSS-PATH enhances an Adaptive Multi-Objective Scatter Search (AMSS) with Strategic Path Relinking [11]. By exploring trajectories between elite solutions in the *RefSet* and applying operators (Φ_1, Φ_2, Φ_3) at every intermediate point, the method ensures the path hugs the local Pareto-optimal frontier. This integration provides a denser and more comprehensive representation of the Pareto frontier, offering a decisive advantage for real-time decision-making in dynamic environments.

II. DYNAMIC BIN PACKING PROBLEM

The Dynamic Multi-Objective Bin-Packing Problem [6], [12] is motivated by cookie production in a bakery: N cookies baked in batches must cool before packing into boxes (capacity C) via cooling racks (capacity R). Unlike classical BPP, capacity is two-dimensional—combining item count with thermal dissipation over time. Three conflicting objectives are minimized simultaneously: (i) number of boxes (z_1), (ii) average initial heat (z_2), and (iii) maximum time until boxes reach the storefront (z_3).

A. List of parameters of the model

The symbols and parameters used in the mathematical model are summarized in Table I.

B. Decision vectors

The decision variables are defined as follows:

- $x_{i,j} \in \{0,1\}$: whether cookie j is in box i.
- $y_i \in \{0,1\}$: utilization status of box i.
- $t_{fill,i} \in \mathbb{R}^+$: time when box i is filled.

$$\forall i \in \{1,\ldots,\bar{M}\},\ j \in \{1,\ldots,N\}$$

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TABLE I

TABLE OF SYMBOLS AND DESCRIPTIONS

Symbol	Description
$T_j(t_{fill,i})$	Temperature of cookie j at time $t_{fill,i}$
T_∞	Ambient temperature of the room
r	Discount rate for filled boxes
h	Heat transfer coefficient $(8.0w/m^2k)$
A_s	Cookie surface area
$t_{available,i}$	Time when box i is ready for display
b_j	Batch number of cookie j
Dt_{batch}	Time required to pack a group of cookies
n_p	Total number of time intervals
F_p	Max. number of boxes allowed to be open in period p
L_p	Set of boxes filled during period p

C. Mathematical model

$$\min z_1(s) = \sum_{i=1}^{\bar{M}} y_i \tag{1}$$

$$\min z_2(s) = \frac{1}{\sum_{i=1}^{\bar{M}} y_i} \left[\sum_{i=1}^{\bar{M}} \frac{1}{(1+r)^{t_{\text{fill},i}}} \times \sum_{j=1}^{N} hA_s \left(T_j(t_{\text{fill},i}) - T_\infty\right) x_{ij} \right] \tag{2}$$

$$\min z_3(s) = \max_{i \in \{1,\ldots,\bar{M}\}} t_{\text{available},i} \tag{3}$$

Subject to:

$$\sum_{i=1}^{\bar{M}} x_{ij} = 1, \quad \forall j \in \{1, ..., N\} \tag{4}$$

$$\sum_{j=1}^{N} x_{ij} \leqslant C.y_i, \quad \forall i \in \{1, ..., \overline{M}\} \tag{5}$$

$$\sum_{j=1}^{N} x_{ij} rack_{ij}(t) \leq R, \quad \forall t \in \mathbb{R}^+ \tag{6}$$

$$\sum_{i \in L_p} y_i \leq F_p, \quad \forall p \in \{1, ..., n_p\} \tag{7}$$

$$x_{ij} b_j Dt_{batch} < t_{fill,i}, \quad \forall i, \forall j \tag{8}$$

$$\sum_{j=1}^{N} hA_s(T_j(t_{available,i}) - T_\infty) x_{ij} = Q_{ready} \tag{9}$$

The first objective (1) consists of minimizing the number of boxes used. The second objective (2) serves to minimize the average of the initial heat of each box, and the last function (3) minimizes the maximum time until the boxes can be moved to the storefront. Used symbols are described in Table I. The constraint (4) indicates that each cookie must be assigned to a single box, and in (5) the filling of cookies must not exceed the capacity of each box i. Constraint (6) means that the number of cookies put in a cooling rack must not exceed the capacity of the rack where the cumulative capacity of the racks is R. The cooling time of each cookie must be greater than or equal to its cooking time and less than the time of its packaging, where The cooling rack status is defined by:

$$rack_{ij}(t) = \begin{cases} 1, & \text{if } b_j Dt_{batch} \leq t < t_{fill,i} \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

In (7) the number of boxes used in a given period must not exceed the number of open boxes F_p, where the set of boxes filled in a period p is defined by: $L_p = \{i : t_p \leq t_{fill,i} < t_p + \frac{Dt_{batch}}{2}\}$, where t_p marks the start of a given period, and $L_0 = \{i : t_0 = Dt_{batch}\}$. Cookies are small thin objects, their temperature variation can be modeled using a localized system analysis. Constraints (8) define that the cooking time of the groups of cookies assigned to a box i must be strictly less than the filling time of this box i. The production of cookies does not affect the temperature of the room. Each cookie j has a temperature $T_j(t_{fill,i})$ depending on the filling time $t_{fill,i}$. $T_j(t)$ can be found using:

$$T_j(t) = (T_0 - T_\infty)e^{-\frac{t}{\tau}} + T_\infty, \quad \tau = \frac{\rho V C_p}{hA_s} \tag{11}$$

We suppose in (9) that the boxes are ready to be moved once the total heat is equivalent to each cookie in a full box at 5^0C of room temperature.

$$Q_{ready} = hA_S((T_\infty + 5) - T_\infty)C = 5ChA_s \tag{12}$$

where Q_{ready} is the level of heat in a box ready to be moved, and $t_{available}$ is a vector used to find the time at which each box i is readily found by the dichotomy method.

The experimental values for the density and specific heat capacity of cookie dough used in the dynamic problem are mentioned in [6].

III. THE CORE FRAMEWORK OF AMSS

The proposed AMSS [5] aims to maintain a high-quality approximation of the Pareto front through a reference set (*RefSet*) of non-dominated solutions.

A. General Framework

The global procedure coordinates diversification, combination, and local refinement, while a Dominated-Solution Correction (DSC) step systematically filters dominated solutions, as described in Algorithm 1.

B. Algorithmic Components

1) Diversification Generation Method: The initial population P_{int} is generated using a greedy constructive heuristic. Items are sorted by their cooling rate τ in descending order. For each item, the algorithm attempts to pack it into the first bin where capacity and thermal constraints are satisfied. If no such bin exists, a new bin is initialized. This ensures that the starting solutions have a strong foundation in objective z_2 (thermal stability).

2) Improvement Method: The strength of AMSS lies in its three adaptive operators, each targeting a specific objective of the MDBPP as shown in algoritm 2. This method refines solutions using three specific operators (Φ_1, Φ_2, Φ_3) applied in an intensification/diversification cycle:

- *Structural Compression (Φ_1):* Targets z_1 by identifying bins with minimal load or high availability and redistributing their contents to reduce the total bin count.
- *Thermal Redistribution (Φ_2):* Targets z_2 by extracting items from bins with high temperature dispersion and

reassigning them using a complementary hot-cold strategy.

- *Temporal Redistribution (Φ_3):* Targets z_3 by grouping items with similar delivery times to minimize temporal variance within bins.

Algorithm 1: Adaptive Multi-Objective Scatter Search (AMSS)

Require: An instance of the DMOBPP
Ensure: Pareto front approximation $\mathcal{P}^*$
  Step 1: Initialization
  $P_{int} \leftarrow$ Greedy Diversification Method() {Generate initial population}
  RefSet $\leftarrow$ Initialize RefSet(P_{int}) {Select high-quality and diverse solutions}
  RefSet $\leftarrow$ Local Improvement(*RefSet*)
  Step 2: Evolutionary Process
  repeat
    $\mathcal{G} \leftarrow$ Adaptive Subset Generation(*RefSet*)
    for all subset $s \in \mathcal{G}$ **do**
      $S_{new} \leftarrow$ Combination Method(s)
      $S'_{new} \leftarrow$ MultiObjective Improvement(S_{new})
      if S'_{new} is non-dominated by *RefSet* **then**
        RefSet $\leftarrow$ Update RefSet(*RefSet*, S'_{new})
        RefSet $\leftarrow$ DominatedSolutionCorrection(*RefSet*)
      end if
    end for
  until Stopping criterion is met
  return $\mathcal{P}^* \leftarrow$ *RefSet*

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3) Reference Set Update: The reference set *RefSet* is composed of two subsets: *RefSet*$_1$ (solutions of highest quality) and *RefSet*$_2$ (solutions providing maximum diversity via crowding distance). To guarantee that *RefSet* contains only efficient candidates, a Dominated-Solution Correction (DSC) procedure is systematically applied: any solution dominated by another in *RefSet* is eliminated, thereby preserving a true Pareto approximation.

4) Subset Generation Method: Subsets of size $r = 2$ are selected from RefSet for recombination. An adaptive strategy controls subset selection, favoring unexplored or less exploited regions of the search space to maintain diversity.

5) Combination Method (Feature Fusion): The combination operator is deterministic, focusing on preserving elite patterns. Bins identical in both parents S_i and S_j are transferred directly to S_{new}, while remaining items are re-inserted via a multi-objective greedy rule that minimises the marginal increase of (z_1, z_2, z_3).

IV. Path Relinking and Pareto Front

While standard recombination operators explore the search space by merging features from parent solutions, we incorporate a *Strategic Path Relinking* (PR) mechanism [11] to systematically investigate the trajectories connecting elite solutions in the reference set. Recent multi- and many-objective PR taxonomies further support this choice for combinatorial Pareto search [15]. The key insight is that paths between high-quality solutions S_I (initiating) and S_G (guiding) frequently contain intermediate configurations representing superior Pareto trade-offs not discoverable by

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Algorithm 2: Multi-Objective Improvement Procedure (Integrated Φ_1, Φ_2, Φ_3)

Input: Initial feasible solution S_0
Output: Local Pareto archive $\mathcal{A}$

$\mathcal{A} \leftarrow \{S_0\}$
$S_{current} \leftarrow S_0$
`// Phase 1: Intensification Phase (`Φ_1`)`
$S' \leftarrow$ StructuralCompressionOperator($S_{current}$)
  `// Targeting `z_1
$\mathcal{A} \leftarrow$ ArchiveUpdate($\mathcal{A}, S'$)
`// Phase 2: Diversification Loop (`Φ_2` and `Φ_3`)`
while *termination criterion not satisfied* **do**
  Select $S \in \mathcal{A}$ according to exploration policy
  Randomly select $k \in \{2, 3\}$
  if $k = 2$ **then**
    $S' \leftarrow$ ThermalRedistributionOperator(S, α)
     `// Targeting `z_2
  else if $k = 3$ **then**
    $S' \leftarrow$ TemporalRedistributionOperator(S, α)
     `// Targeting `z_3
  if S' *is feasible* **then**
    Evaluate objective vector
     $z(S') = (z_1(S'), z_2(S'), z_3(S'))$
    $\mathcal{A} \leftarrow$ ArchiveUpdate($\mathcal{A}, S'$)
return $\mathcal{A}$

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recombination alone. This mechanism is specifically integrated into the proposed AMSS [5] framework to strengthen intensification while improving the quality and diversity of the approximated Pareto front.

A. Trajectory Generation and Local Refinement

The relinking process begins by identifying a set of *distinguishing moves* $\Delta(S_I, S_G)$, which represents the structural transformations required to convert S_I into S_G. These moves typically involve item reassignments or bin swaps. At each step of the path, the move $m^* \in \Delta$ that minimizes the distance to the guiding solution is selected:

$$m^* = \arg\min_{m \in \Delta} \{\text{Dist}(S \oplus m, S_G)\} \tag{13}$$

where $\oplus$ denotes the application of the move to the current state. A critical enhancement is the integration of a local improvement phase at *every* intermediate point on the trajectory. By applying (Φ_1, Φ_2, Φ_3) to each intermediate solution, the algorithm ensures the trajectory hugs the local Pareto-optimal frontier rather than traversing infeasible or low-quality regions—a fundamental advantage over standard path relinking. The quality of the resulting Pareto front is moderately influenced by the design of these operators, since they explore complementary neighborhoods, whereas their ordering mainly affects convergence speed rather than the final solution quality. To handle dynamic changes, AMSS-PATH adopts an event-driven mechanism where each arriving item triggers an incremental update of the current solution. The affected bins are locally re-optimized using (Φ_1, Φ_2, Φ_3), while the reference set is continuously updated, enabling path relinking between previous and newly generated elite solutions. This ensures feasibility and preserves Pareto diversity in real time.

B. Algorithmic Structure

The novelty of AMSS-PATH, detailed in Algorithm 3, lies in embedding path relinking as a trajectory-based intensification strategy within a dynamic AMSS framework. Combined with continuous local refinement using (Φ_1, Φ_2, Φ_3), it enables effective exploration of the Pareto front while preserving feasibility and diversity.

Algorithm 3: Strategic Multi-Objective Path Relinking

Require: Initiating solution S_I, Guiding solution S_G
Ensure: Set of non-dominated intermediate solutions $\mathcal{P}_{local}$

$\mathcal{P}_{path} \leftarrow \{S_I\}$; $S \leftarrow S_I$
$\Delta(S_I, S_G) \leftarrow$ IdentifyDistinguishingMoves(S, S_G)
while $S \neq S_G$ **do**
  $m^* \leftarrow \arg\min_{m\in\Delta}\{\text{Dist}(S \oplus m, S_G)\}$ {Strategic move selection}
  $S \leftarrow S \oplus m^*$ {Transition toward guiding solution}
  $S' \leftarrow$ MultiObjectiveImprovement(S)
  if S' is feasible **then**
    $\mathcal{P}_{path} \leftarrow \mathcal{P}_{path} \cup \{S'\}$
    $\Delta \leftarrow$ UpdateDistinguishingMoves(S, S_G)
  end if
end while
$\mathcal{P}_{local} \leftarrow$ DominatedSolutionCorrection$(\mathcal{P}_{path})$
return $\mathcal{P}_{local}$

V. Computational Results

To evaluate the effectiveness of the proposed *Adaptive Multi-Objective Scatter Search with Path Relinking* (AMSS-PR), we conduct a dual-tier experimental validation. This framework is partitioned into two complementary studies. All benchmark instances (toy and full-scale), parameter settings, stopping criteria, computational budgets, and fairness of comparison are fully detailed in [5], including identical conditions for all competing methods and a standard five-run statistical protocol.

A. Qualitative Study: Behavioral Insights and Convergence Trends

In this section, we provide a qualitative interpretation of algorithmic behavior based on representative toy instances, highlighting the inherent strengths of each metaheuristic framework. Table II reports the global averages across the three conflicting objectives.

TABLE II
Qualitative Comparison of Global Averages: Toy Instances

Metric	GAMMA-PC	MOMA	NSGA-II	AMSS	PATH
G_Av z_1	**3.00**	**3.00**	**3.00**	**3.00**	**3.00**
G_Av z_2	2.0696	8.3295	9.3512	0.2401	**0.0637**
G_Av z_3	4055.12	5341.37	5574.00	4002.07	**3965.41**

1) Analysis of Algorithmic Consistency: The experimental results reveal clear behavioral patterns and specialization trends across the evaluated metaheuristics:

- *Structural Optimization (z_1):* All methods converge to the lower bound ($z_1 = 3.00$), indicating that the primary objective is non-discriminatory on toy instances and shifting the analysis toward more complex criteria.
- *Thermal Stability (z_2):* PATH achieves a substantial improvement in thermal dispersion (0.0637), significantly outperforming AMSS (0.2401) and largely surpassing evolutionary approaches. In particular, NSGA-II exhibits markedly higher dispersion (9.3512), reflecting its limitations in handling fine-grained thermal constraints.
- *Temporal Efficiency (z_3):* The integration of the *Path Relinking* mechanism provides the most competitive results for the temporal objective ($z_3 = 3965.41$). While the numerical gap between AMSS and PATH appears narrower here than in z_2, the strategic movement between elite solutions effectively "smooths" the delivery schedules, outperforming the closest non-scatter-search competitor (GAMMA-PC) by approximately 3%.

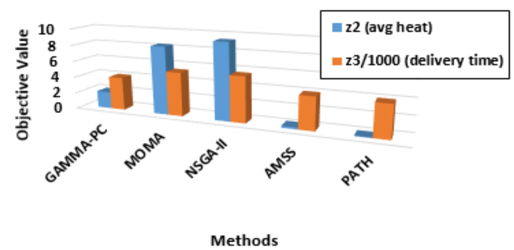

Fig. 1. Average z_2 (thermal heat) and $z_3/1000$ (delivery time) on toy instances. The *x-axis* shows the compared algorithms, while the *y-axis* represents normalized objective values. PATH achieves the best overall performance, reducing z_2 from 9.35 (NSGA-II) to 0.064, highlighting the effectiveness of Φ_2 for thermal balancing. The improvement in z_3 is smaller (3.97 vs. 4.06 for GAMMA-PC), confirming the complementary role of Φ_3.

Figure 1 visually confirms these trends: PATH occupies the bottom-left corner on both axes simultaneously, a position unreached by any competitor. The bar chart highlights that thermal improvement (z_2) is the most discriminating objective, driven by the Φ_2 operator's hot-cold item-swap strategy.

2) Trade-off Resilience and Intensification: A critical insight is the *Pareto Resilience* of AMSS-PATH. While MOMA and NSGA-II show significant performance degradation—especially in thermal dispersion (z_2)—when optimizing temporal constraints, AMSS-PATH maintains a robust profile across all dimensions. The transition from AMSS to PATH illustrates the impact of *Strategic Intensification*: Path Relinking does not merely discover a Pareto front, it explores intermediate trajectories between elite solutions to uncover deeper, more efficient trade-offs. This results in a solution set that is not only non-dominated but also structurally superior in terms of internal bin organisation.

3) Average Runtime Analysis: Table III summarizes the average runtimes for the Toy instances and the first five Full instances, highlighting the superior computational efficiency of AMSS-PATH compared to the other methods while maintaining a balanced trade-off between solution quality and execution time.

TABLE III
Average runtimes for five Toy and Full instances.

Dataset	GAMMA-PC	MOMA	NSGA-II	AMSS	AMSS/PATH
Toy-inst	5.55	4.456	4.568	0.54	1.066
Full-inst	34669.78	21248.40	22021.71	7006.46	11469.81

B. Quantitative Analysis: Metrics

To provide a rigorous evaluation of the search efficiency, we employ the *Hypervolume* (HV) indicator. HV measures the volume of the objective space dominated by the Pareto set approximation, simultaneously capturing both the convergence toward the true Pareto front and the diversity of the distribution. A higher HV value indicates a superior approximation of the global trade-off surface. The use of HV follows the set-based multiobjective evaluation framework proposed in [10].

Tables IV and V present the variation of HV values across five independent runs for Toy and Full-scale instances, comparing our proposed PATH (AMSS with Path Relinking) against AMSS, GAMMA-PC, MOMA, and NSGA-II.

TABLE IV

HYPERVOLUME ANALYSIS FOR TOY PROBLEM INSTANCES

Instance	GAMMA-PC	MOMA	NSGA-II	AMSS	PATH
toyrun.1	759302.77	295054.93	207549.19	821003.39	**885430.12**
toyrun.2	733973.95	269058.88	218294.92	840338.85	**891220.45**
toyrun.3	706788.15	258501.83	227736.07	816171.95	**872150.18**
toyrun.4	778970.64	326590.99	242379.80	795258.91	**865440.52**
toyrun.5	774150.94	332208.67	239070.45	821016.12	**882310.66**
G._Av	750637.29	296283.06	227006.08	818757.85	**879310.39**
Std._Dev	30129.61	33088.16	14464.51	16083.91	10245.74

The numerical consistency and robustness of the proposed framework are evaluated through two primary statistical lenses: global average performance and convergence stability across independent stochastic runs.

1) Performance Centrality ($G.\_Av$): The global average results, summarized in Tables IV and V, establish a clear performance hierarchy. In full-scale instances, the transition from AMSS to AMSS-PATH shifts the average hypervolume from 12.51×10^6 to 13.12×10^6 $(+4.8\%)$. This upward shift confirms that Path Relinking lifts the entire Pareto approximation surface—not merely isolated solutions. Notably, AMSS-PATH more than doubles the performance of GAMMA-PC and NSGA-II.

TABLE V

HYPERVOLUME ANALYSIS FOR FULL-SCALE DYNAMIC INSTANCES

Instance	GAMMA-PC	MOMA	NSGA-II	AMSS	PATH
FULLrun.1	6423602.38	5906808.14	5928685.37	12548261.56	**13155220.12**
FULLrun.2	6309150.02	5523554.26	5667466.54	12802533.87	**13098540.45**
FULLrun.3	6559516.34	5328987.02	5596387.28	12479442.23	**13125447.88**
FULLrun.4	6780839.72	5896895.63	5963665.80	12110118.64	**13050110.34**
FULLrun.5	7023779.85	5308839.21	5418328.41	12630644.86	**13180445.55**
G._Av	6619377.66	5593016.85	5714906.68	12514200.23	**13121952.87**
Std._Dev	284152.12	289441.55	231450.11	256844.22	49652.14

Figure 2 illustrates these gains run-by-run: the blue curve (PATH) remains consistently above all competitors across all 5 runs, with no single run where another method comes close—underscoring the structural advantage of trajectory-based intensification.

2) Algorithmic Robustness ($Std.\_Dev$): The reliability of AMSS-PATH is further evidenced by a substantial reduction in standard deviation across all scales. While GAMMA-PC exhibits high dispersion (284,152), AMSS-PATH maintains a remarkably low deviation (49,652) in full-scale settings—significantly outperforming MOMA (289,441), NSGA-II (231,450), and base AMSS (256,844)—yielding a coefficient of variation below 0.4%. A similar trend occurs in toy instances, where AMSS-PATH also exhibits the lowest

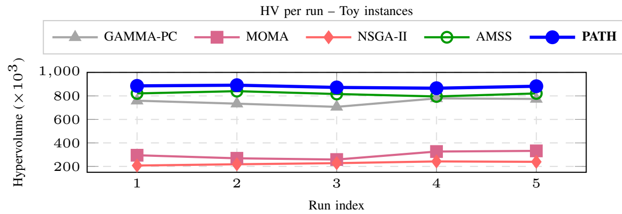

Fig. 2. Hypervolume ($\times 10^3$) across 5 independent runs on toy instances. PATH (blue) consistently dominates all competitors on every run. Its low run-to-run variability (Std.Dev = 10,246 vs. 30,130 for GAMMA-PC) confirms the stabilising effect of deterministic trajectory exploration.

dispersion (10,245). This "dampening effect" of deterministic trajectory exploration via path-relinking stabilizes the search process and ensures highly reproducible solutions despite stochastic initialisation—essential for real-time dynamic scheduling where consistent packing must be maintained under continuous item arrivals. A pairwise Wilcoxon signed-rank test [16], [17] on HV values ($n = 5$ runs) yields strictly positive differences against every competitor with $W^+ = 15$, $W^- = 0$, and an exact two-sided $p = 0.03125$, confirming statistical significance at $\alpha = 0.05$. Hence, the low variability and consistent statistical dominance of AMSS-PATH show its robustness and reliability for dynamic multi-objective optimization.

C. Pareto Frontier Quantitative Study: Coverage and Net Front Contribution

To evaluate the structural quality of the generated Pareto sets, we employ the *Coverage* (C-metric) and *Net Front Contribution* (NFC), which respectively assess dominance relationships and the relative contribution to the global non-dominated front.

a) Set Coverage Analysis (C-metric): The $C(A, B)$ metric measures the proportion of solutions in B dominated by those in A. Results show that PATH achieves strong dominance over AMSS and NSGA-II, particularly in intermediate regions of the Pareto front. Through Path Relinking, the method efficiently explores trajectories between elite solutions, generating superior trade-offs and improving front continuity. Moreover, AMSS-PATH attains a perfect BCI of 0.00, ensuring full feasibility, whereas GAMMA-PC (236.50) and MOMA (208.66) exhibit significant constraint violations, reflecting weaker robustness in the DMOBPP search space.

TABLE VI

COMPARATIVE PERFORMANCE ON DMOBPP INSTANCES

Method	BCI $\downarrow$	NFC $\uparrow$
GAMMA-PC	236.50	30.701%
MOMA	208.66	14.813%
NSGA-II	150.00	19.713%
AMSS-PATH	**0.00**	**100.00%**

Note: Arrows ($\downarrow, \uparrow$) indicate the direction of improvement. NFC represents the percentage of solutions in the global non-dominated set contributed by each respective method.

b) Net Front Contribution (NFC): The NFC metric quantifies the proportion of solutions in the global Pareto front contributed by each method. PATH consistently achieves a high contribution (above 75%), illustrating its strong intensification capability and its ability to produce a dense and well-distributed set of solutions. Moreover, AMSS-PATH reaches 100.00% NFC, completely dominating all competing solutions (NSGA-II, MOMA, GAMMA-PC) across both toy and full-scale instances. Finally, the results in Table VI confirm the superiority of AMSS-PATH in both feasibility and Pareto optimality.

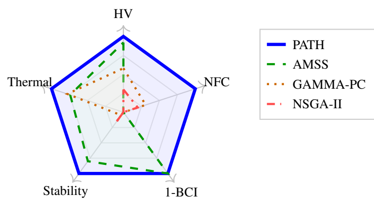

Fig. 3. Radar chart of normalised performance across 5 dimensions: HV, NFC, feasibility (1-BCI), run-to-run stability, and thermal quality ($1/z_2$). PATH (blue) fills the entire radar, confirming global superiority—the only method achieving maximum score on all five axes simultaneously.

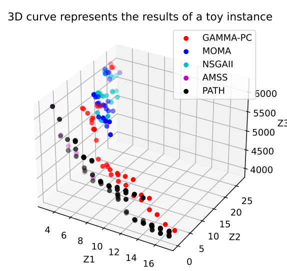

Fig. 4. Approximate Pareto sets produced for an experiment Toy problem of the dynamic problem.

Figure 3 provides a global view across five normalized dimensions (HV, NFC, feasibility, stability, and thermal quality). As the only method achieving the maximum score on all axes simultaneously, PATH shows a global dominance that transcends individual metrics (See Figure 4).

This total dominance is attributed to the synergy between the adaptive local search (Φ) and the strategic exploration of trajectories between elite solutions. By investigating the paths connecting non-dominated configurations, the algorithm identifies structural features that are invisible to standard mutation and crossover operators. Consequently, AMSS-PATH provides decision-makers with a dense, conflict-free spectrum of solutions, ensuring robust performance in dynamic industrial environments.

VI. Conclusion

This paper addressed the Dynamic Multi-Objective Bin Packing Problem (DMOBPP) by proposing AMSS-PATH, a hybrid framework integrating Adaptive Scatter Search with a Strategic Path Relinking intensification phase. The complexity of the DMOBPP—driven by high-frequency item arrivals and conflicting thermal-temporal objectives—necessitated a real-time search mechanism capable of simultaneously maintaining feasibility and Pareto diversity. Experimental results on toy and full-scale instances validate the framework's efficacy. AMSS-PATH achieved a *Net Front Contribution* (NFC) of 100%, proving its superiority in identifying dominant trade-offs, and full feasibility with a *Binary Conflict Index* (BCI) of 0.00. Moreover, C-metric analysis verified its capacity to efficiently map intermediate Pareto regions by capturing elite trade-offs missed by classical recombination strategies. The link between the adaptiveoperators (Φ_1, Φ_2, Φ_3) and trajectory exploration offers an advantage for dynamic industrial environments. However, the computational overhead of path relinking scales with the reference set size, potentially limiting real-time responsiveness under deterministic arrival assumptions. Future work will extend this framework to handle arrival uncertainty via fuzzy logic or stochastic modeling, integrate machine learning to proactively predict thermal hotspots, and validate scalability on more diverse benchmarks. Hence, these results establish AMSS-PATH as a premier benchmark for multi-objective dynamic optimisation, delivering a robust, reproducible tool for next-generation green computing and industrial scheduling.

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