Scheduling with Minimum Unproductive Energy in Job-Shops

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Abstract - **This study considers a job-shop scheduling problem where variable energy must be minimized while considering makespan limitation. For a given production plant, once operations are performed nominally in each machine, the consumption of energy by machines out of operation is the main remaining variable energy that can be minimized. An original formulation of this optimization problem is proposed where the energy and the time spent in unproductive periods is considered. The computational complexity of this problem leads to design of a two-step approach to get an effective solution. The proposed approach is illustrated through a small-scale example.**

Index Terms – job-shop, scheduling, energy minimization, unproductive periods, MILP, separable programming.

I. INTRODUCTION

The improvement of energy efficiency has not played in the past an explicit relevant role in the operation of many manufacturing systems since, minimizing the make-span has been often the priority. Today with the increasing prices of energy and the new environmental protection regulations, the energy-saving issue in job-shops has become an important research field for universities backed by industrial organizations in advanced countries. Given a production plan with its time horizon, the energy spent for its realization in a job-shop is composed of the following elements:

- Energy spent to maintain adequate working conditions in the job-shop (lighting, temperature, humidity, air cleanliness);
- Energy spent in the productive process, composed of the energy spent by the machines and the energy spent by the material handling and transport devices.

The energy spent by the machines is composed of the shutdown and the startup energy, the idle energy when machines are activated at a low regime but not producing and the machining energy during production operation. The first two components can be discribed as unproductive energy.

The relation between industrial management styles, in general Lean management and variants, and energy savings has been of recent concern [1] while new formulations of the job-shop scheduling problem including the energy issue have been proposed. Then, authors such as in [2] and [3] have shown that it is possible to reduce energy manufacturing consumption with a reduced capital investment by rearranging the production process for a given demand through adequate machine selection and operations sequences. Also, in [4] it has been shown that by adjusting power for each operation in a job-shop environment, relevant energy savings can be obtained. However, in a real context where disturbances normally appear, it seems difficult to effectively maintain optimal relative timing of the operating speeds of each machine in the whole workshop, leading to additional delays and costs.

Here, we consider that for each type of operation the machines adopt a nominal mode resulting from various considerations such as product quality, the stresses experienced by the machines, energy and processing time spent per part, etc. In this study it is considered that handling and transport energy costs and delays are constant and depend only of the production plan, the layout of the job shop and the corresponding handling and transport equipment. Then, in this situation the variable energy costs are those linked to the intervals when the machines are not productive, i.e. shut down, startup and idle operation energy consumption. Then minimizing total energy costs in such job shops through an adequate scheduling of operations, reduces to the minimization of the variable energy associated with the unproductive periods of the machines. This problem has been little explored in the specialized literature, although it arises concretely in most production workshops. Frigerio and Matta [5] showed that the gains obtained through the implementation of control policies aimed at reducing the energy consumption of machines during periods of machine inactivity are significant.

The objective of this research is to contribute to energy efficiency in the manufacturing industry, more particularly in high energy-consuming integrated production plants adopting the above assumptions.

In a first step of the study, it is shown how to a given idle time for a specific machine, either mechanical (machining e.g.), thermal (melting, drying e.g.) or hybrid (forming e.g.), a specific local energy policy can be designed to minimize the energy spent during an unproductive period. This results in proposing a generic model relating the duration of an improductive period of a machine to its minimum energy consumption through a local management policy.

Then, is formulated a global scheduling optimization problem with the objective to minimize the variable energy costs associated to production plans while satisfying a make-span constraint. This leads to a complex mixed-integer scheduling optimization problem. Leaving aside computationally intensive approaches such as metaheuristics, which present themselves as black boxes, a comprehensive two-step approach is proposed to solve relatively small instances of this problem. This approach should help to put in place corrective measures in the event of disruptions, thereby promoting the responsiveness of the scheduling process.

The paper is organized as follows: first the general characteristics of the considered problem are displayed, second, local energy policies for unproductive periods are considered and a generic model relating idle periods and energy consumption is introduced. The resulting global optimization problem is formulated and a two steps solution approach is discussed. Finally, an illustration of the proposed modelling and solution approaches is proposed and conclusions are drawn.

II. PROBLEM CHARACTERIZATION AND NOTATIONS

The considered class of production systems is composed of a network of machines, connected by transportation equipment, which process different operations. The final products are obtained at the end of independent sequences of operations which concurrently utilize the workshop's machinery, each machine being used at most one time to perform a job. In our previous paper [6] has been examined in detail the energy consumption of machines during their nonproductive period, according to the idle policy adopted for them, there the cases of electro-mechanical machines and electro-thermal machines have been considered.

Basic assumptions

The considered plant has a set M of m machines. The production plan is composed of n jobs. Each job consists of an independent sequence of operations which are performed using a subset of machines of set M. The operations period starts at time 0 and the maximum time span of production is noted T. A machine can only execute one operation for a given job at a given time and pre-emption is not allowed. To start-up a machine for production, it is supposed that the machine has been set-up: the product to be processed being available has been placed and the tools to be used are ready. -It is supposed that the set-up operation can only be realized when the machine is either in idle mode or shut-down and when the product to be processed is available. - It is supposed that once machine has been set-up and started, production begins immediately. To each machine is attached handling and transport devices which are supposed available when necessary and deliver processed materials. The delays and energy associated with the operation of the handling and transport systems are considered constant for a given product between pairs of machines.

The main objective is to minimize the total production energy E_T needed to perform the n jobs. The total production energy E_T is composed of a constant energy part attached to the processing and the handling and transport of the materials, and of a variable part attached to the unproductive periods of the machines.

Adopted notations

n: index of jobs, $n\in\{1,..., N\}$; k index of operations, $k\in\{1,\ldots,S_i\}$; m: index of machines, $m\in\{1,..., M\}$; N: total number of jobs, M : total number of machines, m_{nk} is the index of the machine allowing to perform operation O_{nk}, the k^{th} operation of job n. M_m is the number of operations which will be performed on machine m, $m=1$ to M, for the considered production plan.

Let t_{nk} be the starting time of operation O_{nk} on machine m_{nk}, τ_{nk}^{su} be its set-up delay and d_{nk} be the processing duration, $n=1$ to n, k=1 to S_n ; T_{nk} be the nominal transfer time of product n from machine m_{nk} to m_{nk+1}, $n=1$ to n, k=1 to S_n. The nominal transfer time is in general the transportation time of the product from one machine to the next.

The results obtained from the analysis of the case of mechanical machines lead to consider that for each situation of unproductive period for a given machine l, an optimal policy exists and depends mainly of the unproductive period length $t_{n\prime k\prime}-(t_{nk}+d_{nk})$ and of the following parameters:

The processing powers of the two successive operations: P_{nk} and $P_{n\prime k\prime}$. The idle power reference for machine m: P_m^{idle}. The values of the different time response parameters. The set-up time: $\tau_{n^{\prime}k^{\prime}}^{su}$ for operation $O_{n^{\prime}k^{\prime}}$ on machine $m_{n\prime k\prime}$.

In the rest of the text, we will note the minimum energy to be spent in an unproductive period of machine $m_{nk}=m_{n\prime k\prime}=$

m between operations O_{nk} and $O_{n'k'}$ by: $\varphi_{mnkn'k'}(t_{nk}, t_{n'k'})$ with:

$$E_{n'k'}^{idle} \leq \varphi_{mnkn'k'}(t_{nk}, t_{n'k'} \leq E_{n'k'}^{hybr} \quad (1)$$

The total energy E_{Tot}^{up} spent during unproductive periods is then bounded:

$$\sum_n \sum_k E_{nk}^{idle} \leq E_{Tot}^{up} \leq \sum_n \sum_k E_{nk}^{hybr} \quad (2)$$

III. FORMULATION OF THE OPTIMAL SCHEDULING PROBLEM

It has been considered that in the case in which the machines of a job shop are operated in a standard way, the amount of energy necessary to perform the different processes resulting from a production plan, can be predicted and taken as a fixed energy amount attached to the production plan. The remaining component of the energy attached to this production plan is relative to the unproductive periods of the different machines. From the previous section it appears that the schedule and timing of the successive operations assigned to a machine has a direct influence on its energy performance and then on the performance of the whole job shop to achieve a production plan. More precisely it appears that the reduction of duration of these unproductive periods leads to a decrease of the minimum energy necessary to ensure the transition from one operation to another on a given machine. So here the objective of minimizing energy and the concern for limiting the production makespan are not antagonistic and the search for a minimum makespan schedule should lead to reduced expenditure of nonproductive energy.

In the considered problem are taken simultaneously into account all the machines of a job-shop to be used in a given production plan and the main objective is to get a schedule of operations that minimize the variable energy necessary to perform the production plan in a given make-span. In the following are described the adopted decision variables, the energy criterion and the constraints to be satisfied by the optimal scheduling.

Decision variables: The previous section has shown that the minimum energies necessary to perform the transitions between the successive operations of the jobs composing the production plan, depend on the scheduling and timing of the different operations on their respective assigned machines. This leads to select as decision variables the start time of each operation and the sequence of operations on each machine:

- t_{nk}: a real variable giving the starting time of operation O_{nk} at machine m_{nk};

- $z_{nn'kk'}$: a boolean variable informing the sequence of operation at each machine where $z_{ii'kk'}$=1 if the k^{th} operation of job n is to be realized just before the k'^{th} operation of job n' on the same machine m ($m_{nk} = m_{n'k'} = m$) and $z_{ii'kk'}$= 0 otherwise.

Criterion: The chosen criterion calculates the variable energy spent to carry out the production plan by implementing a schedule composed of the sequence of operations and their timing. This criterion has to be minimized:

$$\min_{t,z} E_V \quad (3)$$

with $E_V =$
$$\sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{S_n} \sum_{n'=1, n' \neq n}^{N} \sum_{k'=1}^{S_{n'}} \varphi_{mnkn'k'}(t_{nk}, t_{n'k'}) \cdot z_{nnkk'} \quad (4)$$

Constraints:

$$rt_{m_{n1}} \leq t_{n1} \qquad n = 1 \text{ to } N \quad (5)$$

$$t_{nk} + d_{nk} + T_{nk} + \tau_{nk+1}^{su} + \beta_{nk+1}^{idle} \leq t_{nk+1}$$
$$n = 1 \text{ to } N,\ k = 1 \text{ to } S_n - 1 \quad (6)$$

$$t_{nS_n} + d_{nS_n} \leq \min_{m=1 \text{ to } M} \{rt_m\} + T \quad n=1 \text{ to } N \quad (7)$$

$$\forall n\ \forall k \in \{1, \cdots, S_n\},\ :\ \sum \sum_{n',k':m_{nk}=m_{n'k'}} z_{nn'kk'} \leq 1 \quad (8)$$

$$\forall n, \forall k, \forall n', \forall k': z_{nn'kk'} + z_{n'n\,k'k} \leq 1 \quad (9)$$

$$\forall m\ \ \sum_{n=1}^{N} \sum_{k=1}^{S_n} \sum_{n' \neq n} \sum_{k'=1, m_{nk}=m_{n'k'}=l}^{S_{n'}} z_{nn'kk'} = M_m - 1 \quad (10)$$

$$\forall n \neq n', \forall k, k' \text{ with } m_{nk} \neq m_{n'k'}\ :\ z_{ii'kk'} = 0 \quad (11)$$

and $\forall n \neq n',\ \forall k, k'$ with $m_{nk} = m_{n'k'}$:

$$t_{nk} + d_{nk} + \tau_{n'k'}^{su} + \beta_{n'k'}^{idle} \leq t_{n'k'} + V(1 - z_{nnkk'}) \quad (12)$$

$$\text{with} \quad t_{nk} \in R^+ \ i=1 \text{ to } n,\ k= 1 \text{ to } S_n \quad (13)$$

$$z_{nn'kk'} \in \{0,1\}\ n,n'=1 \text{to} N,\ k=1 \text{ to } S_n,\ k'=1 \text{ to } S_{n'} \quad (14)$$

In the above formulation, the constraints (5) and (6) are precedence constraints for each job along its line of production, while constraint (7) is the timespan constraint where T is the make-span. Here S_n is the number of operations of job n, rt_m is the release time of machine m, T_{nk} is the transportation time from machine m_{nk} to machine m_{nk+1}, τ_{nk+1}^{su} and β_{nk+1}^{idle} are unproductive delays as explained in section III. Constraints (9), (10), (11) and (12) are relative to the sequencing of operations at the level of each machine:

Constraints (8) indicates that an operation of a job on a given machine has at most one successor from another job. Constraints (9) assign at most a unique sequence between operations realized on the same machine. Constraints (10) indicate that a machine m which has to perform M_m operations during the production plan will generate $M_m - 1$ transitions between operations of different jobs. Constraints (11) indicates that operations realized on different machines cannot be successor on a same machine. Constraints (12) where V is a very large positive number, enforce a time precedence constraint between two successive operations belonging to different jobs and performed on the same machine. Relations (13) and (14) inform the nature of the two sets of decision variables.

IV. SOLUTION APPROACH

Exact, heuristic and metaheuristic approaches have been already designed to cope with many different classes of job shop scheduling problems (JSSP), see for example [7], [8] and [9]. In this study is introduced a solution strategy composed of two stages: one enumerative where feasible sequences of assigned activities are generated and one where the timing of operations is optimized to minimize the variable energy consumed during the nonproductive periods of the different machines. Problem (12) to (23) is a mixed integer non-linear programming (MINLP) problem where the complicating variables are mainly the precedence variables $z_{nn'kk'}$. By nature, many of these variables will take a zero value and an upper bound of the number of effective *z* variables is given by:

$$\prod_{m=1}^{M} M_m! \tag{15}$$

This makes the counting problem of the schedules satisfying a makespan bound to be P-complete with respect to the computational complexity theory [10]. However, in many practical situations which for example take into account the physical layout of the job shop [11], the scheduling problem can be subdivided in much smaller problems according to the adopted subdivision of the machine pool, turning an enumerative approach computationally feasible. In the extreme case of a layout organized by product for assembly line production, the variables z will become deterministic and the optimization problem decomposes into *N* piecewise linear subproblems. In the case of differentiated production, even with a layout organized by process, the number of jobs that will use the same machine can be very limited, which should keep the combinatorial complexity of the problem manageable.

To each sequence of jobs on each machine can be attached a directed graph without circuits with as many rows as machines and for each job, S_n transverse arcs. Let $O_m = \{o_{mi}, i = 1 \ to \ M_m\}$ be a sequence of operations with machine *m* where o_{mi} is the index of the job whose operation with machine *m* is the i^{th} on it. The weights of the arcs in row *m* are equal to the duration of the previous activity plus the setup time of the following activity with machine *m*. The weight of a transverse arc from machine *m* to machine *m*' along the sequence of operations of a job is equal to the duration of the previous activity plus the transportation time between the two machines and plus the setup time at the new machine.

An example in the case of three machines and three jobs with three operations is given here:

Let $M = \{M_1, M_2, M_3\}$, $S_1 = 3$ with $m_{1,1} = M_1, m_{1,2} = M_2, m_{1,3} = M_3$, $S_2 = 3$ with $m_{2,1} = M_1, m_{2,2} = M_3, m_{1,3} = M_2$, $S_3 = 3$ with $m_{3,1} = M_2, m_{3,2} = M_1, m_{3,3} = M_3$. Then to the sequences of jobs on the three machines being such as $O_1 = \{J_1, J_2, J_3\}$, $O_2 = \{J_3, J_1, J_2\}$ and $O_3 = \{J_2, J_1, J_3\}$, is attached the directed graph without circuits of figure 6 where nodes α and ω are dummy start and end nodes and where the arcs connecting α to the machine rows have a weight equal to 0:

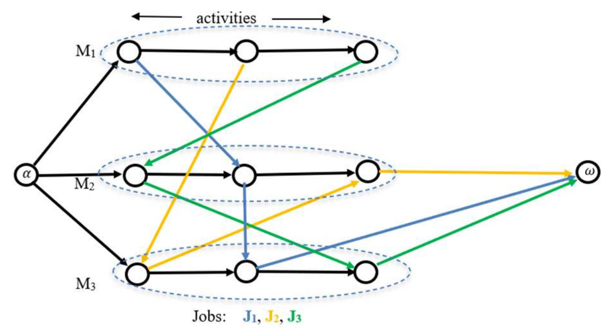

Figure 6: Graphical representation of a sequencing instance in a job shop

Since at start the makespan of the production plan is not known, a desirable makespan will be used in the first stage of the proposed solution strategy. Basic minimum and maximum bounds for the makespan are given by:

$$T_{min} = \max_{m=1 \text{ to } M, n=1 \text{ to } N} \{L_m, W_n\} \tag{16}$$

where L_m is the effective load of machine *m* given by:

$$L_m = rt_m + \sum_{n=1}^{N} \sum_{k=1 \text{ with } m_{ik}=m}^{S_n} (d_{nk} + \tau_{nm}) \tag{17}$$

W_n is the effective workload of job *n* given by:

$$W_n = rt_{n1} + \sum_{k=1}^{S_n} (d_{nk} + T_{nk} + \tau_{nm_{nk}}) \tag{18}$$

and where τ_{nm} is the setup times for the operation of job *n* on machine *m*.

$$T_{max} = \min\{\sum_{m=1}^{M} L_m, \sum_{n=1}^{N} W_n\} \tag{19}$$

If *T* is taken inferior to T_{min} there will not be enough time to perform the whole production plan, if *T* is taken superior to T_{max}, there will be periods in which no machine is operating and hence no job is being processed. An acceptable value *T* of the makespan must be taken in the interval $[T_{min}, T_{max}]$ and must correspond to at least a scheduling. This value should generally be as low as possible to achieve management objectives such as maximizing production turnover or meeting delivery deadlines to customers, let $T_{lim} \leq T_{max}$, be the largest value acceptable from a management perspective for the makespan. It therefore seems interesting to sweep the values of T starting from T_{min} up to T_{lim}, until some schedules allow the production plan to be carried out within this time interval. Then, given a value T for the desired makespan with $T_{min} \leq T \leq T_{lim}$, a sequencing of activities on the m machines of the job shop will be said T-feasible when the completion time of all the jobs is inferior or equal to T.

A straightforward approach of polynomial complexity, can be used to find in the circuit-less graph attached to a scheduling, the earliest time at which the different jobs can be completed [12]. In many situations it will not be necessary to complete the full computation: once the earliest time for completion of an intermediate operation of a job is found to be larger than T, it can be already concluded that the current sequencing of activities is not T-feasible.

By generating all possible sequencings for the machines (216 in the above case) and testing their T-feasibility, the set Z of T-feasible assignments is constructed resulting in the precedence values $\bar{z}_{nn'kk'}$ n, n'=1 to N, k= 1 to S_n, k'= 1 to S_n . Then, for each feasible sequencing, the following piecewise linear programming problem must be solved to minimize the variable energy of the production plan:

$$\min_{t_{nk}\, t_{n'k'}} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{S_n} \sum_{n'=1, n' \neq n}^{N} \sum_{k'=1}^{S_{n'}} \varphi_{mnkn'k'}(t_{nk}, t_{n'k'}) \cdot \bar{z}_{nnkk'} \tag{20}$$

with the remaining timing constraints:

$$rt_{m_{n1}} \leq t_{n1} \quad n\text{=1 to } N \tag{21}$$

$$t_{nk} + d_{nk} + T_{nk} + \tau_{nk+1}^{su} + \beta_{nk+1}^{idle} \leq t_{nk+1}$$
$$n = 1 \text{ to } N,\ k = 1 \text{ to } S_n\text{-}1 \tag{22}$$

and $\forall n \neq n',\ \forall k, k'$ with $m_{nk} = m_{n'k'}$:

$$t_{nk} + d_{nk} + \tau_{n'k'}^{su} + \beta_{n'k'}^{idle} \leq t_{n'k'} + V(1 - \bar{z}_{nn'kk'})) \tag{23}$$

$$\text{with} \quad t_{nk} \in R^{+}\ i = 1 \text{ to } n,\ k = 1 \text{ to } S_n \tag{24}$$

Since the functions $\varphi_{mnkn'k'}(t_{nk}, t_{n'k'})$ are piecewise linear, the problem (20) to (24) is a separable linear programming problem and can be transformed into a classical linear programming problem by introducing additional variables attached to each linear segment of these functions [13]. Dynamic Programming [14] based on the previous directed graph without circuits corresponding to the current schedule of operations, with now an energy objective function, appears also as an interesting option. The solution of this problem will provide, given a scheduling of the jobs on the machines, a feasible timing of operations resulting in a minimum unproductive energy consumption for the considered work plan and also an effective makespan obtained from the expression:

$$T_{eff}(\bar{z}) = \max_{n=1 \text{ to } N} \{t_{nS_n}^{*} + d_{nS_n}\} \tag{25}$$

and by construction: $T_{eff}(\bar{z}) \leq T \quad \bar{z} \in Z$ (26)

Repeating this process over the whole set Z of T-feasible schedules will allow to identify the optimal solutions of this problem.

V. ILLUSTRATIVE EXAMPLE

Here, the previous example with three jobs and three machines is considered with the following delays including processing times, transport times and setup delays:

$$[d_{nm}] = \begin{bmatrix} 3 & 3 & 3 \\ 2 & 4 & 3 \\ 2 & 3 & 1 \end{bmatrix} \tag{27}$$

with the following release times of the machines: $rt_1 = rt_2 = 0$ and rt_3 =5. Here the effective workloads of the jobs are given by $W_1 = 7$, W_2 =10 and $W_3 = 12$. The effective loads of the machines are given by: L_1= 9, L_2= 9 and $L_3 = 6$. Then here T_{min} =12 and T_{max} =24.

Starting with $T = T_{min} = 12$, after generating 216 cases, three different schedules, σ_1, σ_2 and σ_3 lead to a makespan of 12, so, it is not necessary to increase T from T_{min}:

$\sigma_1 = \{(o_{11}, o_{21}, o_{32}), (o_{31}, o_{12}, o_{23}), (o_{22}, o_{13}, o_{33})\}$

to which corresponds the only z values equal to 1: $z_{11211}, z_{12312}, z_{23112}, z_{21223}, z_{32123}, z_{31333}$,

$\sigma_2 = \{(o_{21}, o_{11}, o_{32}), (o_{31}, o_{12}, o_{23}), (o_{22}, o_{13}, o_{33})\}$

to which corresponds the only z values equal to 1: $z_{12111}, z_{11312}, z_{23112}, z_{21223}, z_{32123}, z_{31333}$ a

and $\sigma_3 = \{(o_{21}, o_{11}, o_{32}), (o_{31}, o_{12}, o_{23}), (o_{22}, o_{33}, o_{13})\}$

to which corresponds the only z values equal to 1: $z_{12111}, z_{11312}, z_{23112}, z_{21223}, z_{32323}, z_{31333}$.

Let the setup energy costs on machines 1, 2 and 3, which correspond when $z_{mnn'kk'=1}$ to the cases where $t_{n'k'} = t_{nk} + d_{nk}$, be given by:

$$[\varphi_1] = \begin{bmatrix} - & 1 & 2 \\ 2 & - & 1 \\ 3 & 1 & - \end{bmatrix}, \quad [\varphi_2] = \begin{bmatrix} - & 3 & 4 \\ 2 & - & 1 \\ 1 & 2 & - \end{bmatrix},$$

$$[\varphi_3] = \begin{bmatrix} - & 2 & 1 \\ 3 & - & 2 \\ 2 & 1 & - \end{bmatrix} \tag{28}$$

The solutions of the three piecewise linear problem result in the schedules presented in figures 7.1, 7.2 and 7.3 where the rows correspond to the different machines and the colours to the different jobs:

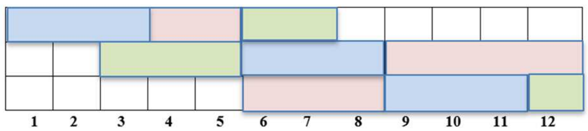

Figure 7.1 Scheduling solution of J_1, J_2 and J_3 for σ_1, variable cost: 10.

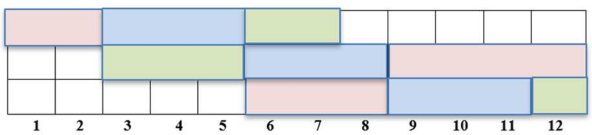

Figure 7.2 Scheduling solution of J_1, J_2 and J_3 for σ_2, variable cost: 12.

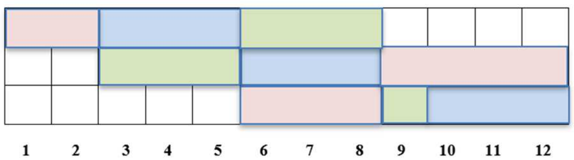

Figure 7.3 Scheduling solution of J_1, J_2 and J_3 for σ_3, variable cost: 12.

It appears that for these schedules which correspond to the minimum bound of the timespan, there is no idle time between operations in the machines and the only variable energy consumption is the setup energy of the machines to process the next operation. Solution 7.1 is then the optimal solution.

VI. CONCLUSION

This article addressed the problem of minimizing energy consumption during unproductive periods for machines in a job shop that must execute a given production plan. The energy consumption required for the transition from one operation to another on each machine with eventually an idle time was analyzed, and locally optimal policies were identified, allowing this energy consumption to be modeled at the job shop level. This led to the formulation of a highly complex MILP-type problem. Instead of embarking on the empirical development of a metaheuristic, as is often the case for various types of JSSPs, an open, two-step approach has been developed. Given an acceptable order of magnitude for the makespan of the production plan, a set of schedules compatible with this objective is identified by associating each generated sequence with a directed, circuit-free graph, which allows for a quick verification of its feasibility. Then for each of the selected sequences, a piecewise linear programming problem is formulated and solved, leading for each schedule to the timing of operations minimizing energy consumption during unproductive periods of the machines. The proposed approach has been illustrated through a small case study, showing its different steps and its effectiveness to provide an optimal energy schedule with a minimum makespan. We believe that, beyond the judicious sequencing and timing of operations on the assigned machines, the energy efficiency of a job shop also depends heavily on adapting its layout to the planned production plan. It seems to us that this will lead to scalable solutions for this type of problem, avoiding the curse of computational complexity.

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