Eliminating Binary Variables in Battery Scheduling: LP Reformulations for Energy Arbitrage

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Abstract— **Battery energy storage scheduling for price arbitrage is typically formulated as a Mixed-Integer Linear Program (MILP) to preclude simultaneous charging and discharging. This paper demonstrates that the MILP structure is unnecessarily complex: the original problem can be transformed into an equivalent Linear Program (LP) that solves in polynomial time. Two LP reformulations are presented. The first optimizes net energy variation under ideal efficiency, reducing the number of variables and eliminating binary constraints. The second, more realistic formulation explicitly incorporates distinct charging and discharging efficiencies while preserving a purely linear constraint set. Matrix representations and dimension analyses are provided to facilitate direct implementation with standard LP solvers. Numerical simulations confirm that both formulations produce identical net power flows under ideal conditions, and that the efficiency-aware LP correctly captures losses. Consequently, the computational burden of battery scheduling for real-time energy management is reduced from exponential to polynomial complexity without sacrificing physical accuracy. The reformulations offer a lightweight, solver-agnostic alternative to MILP for day-ahead and intraday bidding, and they lay a foundation for handling degradation, multi-battery coordination, and stochastic extensions.**

I. INTRODUCTION

Battery Energy Storage Systems (BESS) are critical assets for modern power grids, enabling energy arbitrage, peak shaving, and renewable integration. However, managing a BESS requires solving a complex sequential decision-making problem: the operator must decide when to charge or discharge to maximize revenue based on time-varying prices, while respecting physical limits.

Mixed-Integer Linear Programming (MILP) has emerged as the standard formulation for this challenge. MILP naturally captures the fundamental operational constraint of a battery — its inability to charge and discharge simultaneously — through the use of binary variables. This allows for precise modeling of efficiency losses, power rating limits, and state-of-charge boundaries. However, the inclusion of these binary variables introduces significant computational complexity. For a typical 24-hour horizon with 15-minute resolution, the resulting combinatorial search space (2^{96} possibilities) poses a barrier to real-time applications, where optimization must occur within seconds or milliseconds to respond to changing grid conditions. Parallel research directions have explored dynamic modeling of energy storage systems using advanced mathematical frameworks. The comprehensive review of battery energy storage systems and advanced battery management system for different applications is presented in research [1]. It is to be noted that while MILP is great option for finding the best solution for scheduling energy storage over a future period (like a day-ahead), it faces fundamental hurdles when applied to real-time control as MILP problems are classified as NP-hard [2]. Recently there were several attempts to tackle this challenge. Optimization approach that accounts for the nonlinear efficiency characteristics of BESS and inverters, using piecewise linearization techniques was proposed in research [3]. The regularized mixed-integer programming model for the optimal power flow problem with BESS is proposed in [4].

Research [5] proposed an approach based on Volterra (evolutionary) integral equations of the first kind, formulating the storage operation as an inverse problem. This evolutionary model naturally accounts for the memory effects inherent in battery systems, where past charging/discharging history influences future state and performance. The Volterra framework offers several advantages: robustness to forecast errors through regularization techniques, ability to incorporate nonlinear efficiency characteristics dependent on SoC, and natural consideration of temporal degradation (state-of-health). Furthermore, the connection between the classical ampere-hour integral model and Volterra equations provides a solid theoretical foundation for dynamic storage analysis. For more details on the Volterra models theory readers may refer to book [8].

The objective of the present research is to reduce the traditional MILP to LP while accurately taking into account the effect of efficiency for charging and discharging processes.

The remainder of this paper is organized as follows. Section II provides a detailed mathematical description of the original MILP problem, section III gives a refined mathematical description and software implementation of the first MILP modification (ideal efficiency case), section IV presents the mathematical description and software implementation of the second MILP modification (realistic efficiency case), section V shows calculation results comparing both modifications. and section VI outlines tasks for future

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stages.

II. TRADITIONAL MILP APPROACH TO BESS

MILP has become a cornerstone methodology for optimizing the operation and planning of energy storage systems. Its popularity stems from its ability to model the discrete decisions (e.g., charging/discharging status, on/off states of generators) and continuous power flows that characterize modern energy systems, all within a rigorous mathematical framework. Key applications of MILP for ESS optimization involves price-based operations, cost minimization, and load scheduling. The development of Energy Management Systems (EMS) that schedule ESS dispatch in response to time-varying energy prices and local load demands is one of core applications [6]. The primary objective is to minimize the total cost of energy consumed from the grid, often by shifting load –charging the storage when prices are low and discharging during peak price periods to avoid expensive grid purchases.

A significant evolution in MILP applications is the integration of uncertainty, addressing the intermittent nature of renewable energy sources (RES) like solar and wind, as well as volatile market prices. Researchers have developed stochastic and robust MILP frameworks (see e.g. [7]) to ensure schedules are not only optimal for a single forecast but are resilient across a range of possible future scenarios.

Traditional approach to battery energy storage systems management employs the following objective function maximization

$$\max \sum_t \big(P_{dch}(t) - P_{ch}(t)\big)\, EP(t), \tag{1}$$

subject to

$$P_{ch}(t)\,\eta_{ch} - P_{dch}(t)\,\eta_{dch} \le Q_{bes,\max} - Q_{bes}(t-1), \tag{2}$$

$$P_{dch}(t)\,\eta_{dch} - P_{ch}(t)\,\eta_{ch} \le Q_{bes}(t-1) - \sum_{k=t}^{t+T_{\text{backup}}} D(k)\,\eta_{dch}, \tag{3}$$

$$P_{ch}(t)\,\eta_{ch} \le \Delta Q_{ch,\max}\, X_{ch}(t), \tag{4}$$

$$P_{dch}(t)\,\eta_{dch} \le \Delta Q_{dch,\max}\, X_{dch}(t), \tag{5}$$

$$X_{ch}(t) + X_{dch}(t) \le 1, \quad X_{ch}(t), X_{dch}(t) \in \{0,1\}, \tag{6}$$

$$Q_{bes}(t) = Q_{bes}(t-1) + P_{ch}(t)\,\eta_{ch} - P_{dch}(t)\,\eta_{dch}. \tag{7}$$

The description of employed functions and parameters are given in Table I.

The original problem is mixed-integer due to the binary variables $X_{ch}(t)$ and $X_{dch}(t)$ in constraints. However, as shown in Section IV, only the net power difference $P_{dch}(t) - P_{ch}(t)$ has physical meaning, which allows elimination of the binary variables.

III. THE FIRST MILP MODIFICATION (IDEAL EFFICIENCY)

This modification reduces the MILP to a simplified LP problem by optimizing the net charge variation $\Delta Q_{bes}(t)$ instead of individual charging/discharging powers. It assumes ideal efficiency ($\eta_{ch} = \eta_{dch} = 1$).

TABLE I
FUNCTIONS AND PARAMETERS DESCRIPTION

Data	Description
$EP(t)$	future Energy Price with 15 minutes interval
$D(t)$	future Demand with 15 minutes interval
$P_{ch}(t) \ge 0$	battery charge power during t-th time interval
$\eta_{ch} < 1$	battery charge efficiency
$X_{ch}(t) \in \{0,1\}$	discrete variable, 1 if battery charges, else 0
$P_{dch}(t) \ge 0$	battery discharge power during t-th time interval
$\eta_{dch} > 1$	inverted battery discharge efficiency
$X_{dch}(t) \in \{0,1\}$	discrete variable, 1 if battery discharges, else 0
$Q_{bes}(t)$	total battery charge after t-th time interval
$\Delta Q_{ch,\max} > 0$	maximum allowable battery charge over 15 min
$\Delta Q_{dch,\max} > 0$	maximum allowable battery discharge over 15 min
$Q_{bes,\max} > 0$	maximum allowable battery capacity
T_{backup}	required number of backup time intervals

To simplify the problem, we proposed to modify the objective function. This modification will reduce the MILP problem to the LP problem. In particular, we propose to move from optimizing charging ($P_{ch}(t)$) and discharging ($P_{dch}(t)$) powers of the battery, to optimizing the variable $\Delta Q_{bes}(t)$ corresponding to increasing ($\Delta Q_{bes}(t) > 0$) or decreasing ($\Delta Q_{bes}(t) < 0$) battery charge during time interval equal to 15 minutes. This replacement will reduce the number of variables to be optimized, eliminate discrete variables (transition from MILP to LP), and simplify the final LP problem. Considering the efficiency of modern batteries, the transition from power optimization to charge optimization will not introduce a significant error, and if we consider $\eta_{ch} = \eta_{dch} = 1$, then the results of power and charge optimization will be identical:

$$\min \sum \Delta Q_{bes}(t) EP(t) = \min \sum (P_{ch}(t)\eta_{ch} - P_{dch}\eta_{dch}) EP(t) = \min \sum (P_{ch}(t) - P_{dch}(t)) EP(t) = \min \sum \Delta P_{bes}(t) EP(t). \tag{8}$$

Input data description is given in Tables II.

TABLE II
INPUT DATA FOR THE FIRST MILP MODIFICATION

Data	Description
$EP(t)$	Energy Price in future 24 hours, 15 min interval
$D(t)$	Demand in future 24 hours, 15 min interval
$Q_{bes,0}$	battery initial charge value
$\Delta Q_{ch,\max} > 0$	max allowable charge per 15 min
$\Delta Q_{dch,\max} < 0$	max allowable discharge per 15 min
$Q_{bes,\max} > 0$	maximum battery capacity
$Q_{bes,\min} \ge 0$	minimum battery capacity
T_{backup}	required number of backup intervals

A. Objective Function

The objective is transformed to:

$$\min \sum_{t=m}^{N} \Delta Q_{bes}(t)\, EP(t), \tag{9}$$

where

$$\Delta Q_{bes}(t) = P_{ch}(t)\eta_{ch} - P_{dch}(t)\eta_{dch}$$

represents net charge variation (positive for charging, negative for discharging). The parameter m is determined by backup power requirements.

B. Modified Constraints

To form a complete set of constraints, it is necessary to fulfill the backup power supply requirements in order to . Based on this, the following parameters are determined: m is measurement number from which the algorithm starts to perform the optimization procedure, $T_{backupCorr}$ is an adjusted backup time based on the predicted load schedule, $Q_{bes,backup}$ is battery initial charge value considering backup supply.

a) Backup power supply constraints: At the initial time, the following must hold:

$$Q_{bes,0} \geq \sum_{i=1}^{T_{\text{backup}}} D(i)\, \eta_{dch}. \tag{10}$$

If not, the first m periods are used to charge at maximum rate until sufficient reserve is accumulated.

Also, for all k $(1 \leq k \leq N - T_{\text{backup}} + 1)$:

$$\sum_{i=k}^{k+T_{\text{backup}}-1} D(i)\, \eta_{dch} \leq Q_{bes,\max}. \tag{11}$$

If violated, T_{backup} may be reduced or the problem is infeasible.

b) Rate limits:

$$\Delta Q_{dch,\max} \leq \Delta Q_{bes}(t) \leq \Delta Q_{ch,\max}, \quad m \leq t \leq N. \tag{12}$$

c) State of charge limits:

$$Q_{bes,\min}(t) \leq Q_{bes,0}^{\text{backup}} + \sum_{i=m}^{t} \Delta Q_{bes}(i) \leq Q_{bes,\max}, \quad m \leq t \leq N. \tag{13}$$

where $Q_{bes,0}^{\text{backup}}$ is the initial charge adjusted for backup.

C. Matrix Representation

The LP is written in standard form:

$$\min(f_1 \cdot x_1) \quad \text{such that} \quad \begin{cases} A_1 x_1 \leq b_1, \\ A_{eq1} x_1 = b_{eq1}, \\ lb_1 \leq x_1 \leq ub_1, \end{cases} \tag{14}$$

where f_1, x_1, b_1, b_{eq1}, lb_1, ub_1 are vectors, A_1 and A_{eq1} are matrices. Objective function's coefficients vector is defined as

$$f_1 = \begin{bmatrix} EP(t) & EP(t+1) & \cdots & EP(N) \end{bmatrix}, \tag{15}$$

$$x_1 = \begin{bmatrix} \Delta Q_{bes}(t) & \Delta Q_{bes}(t+1) & \cdots & \Delta Q_{bes}(N) \end{bmatrix}^T, \tag{16}$$

$$A_1 = \begin{bmatrix} A_{1\_1} \\ A_{2\_1} \end{bmatrix}, \quad A_{1\_1} = \text{tril}(ones(n)), \quad A_{2\_1} = -A_{1\_1}, \tag{17}$$

$$b_1 = \begin{bmatrix} (Q_{bes,\max} - Q_{bes,0}^{\text{backup}})\, \mathbf{1}_{n\times 1} \\ (Q_{bes,0}^{\text{backup}} - Q_{bes,\min})\, \mathbf{1}_{n\times 1} \end{bmatrix}, \tag{18}$$

$$lb_1 = \Delta Q_{dch,\max}\, \mathbf{1}_{n\times 1}, \quad ub_1 = \Delta Q_{ch,\max}\, \mathbf{1}_{n\times 1}. \tag{19}$$

The latter are lower and upper bounds. Here $n = N - m + 1$, $m \leq t \leq N$, $1 \leq m \leq N+1$, $t, m \in \mathbb{N}$.

IV. THE SECOND MILP MODIFICATION (REALISTIC EFFICIENCY)

This modification reduces the original MILP to an LP while accurately accounting for charging/discharging efficiencies $\eta_{ch} < 1$, $\eta_{dch} > 1$. The key idea is that only the net power difference $P_{dch}(t) - P_{ch}(t)$ has physical meaning, allowing elimination of binary variables.

Input data description is given in Tables III.

TABLE III
INPUT DATA FOR THE SECOND MILP MODIFICATION

Data	Description
$EP(t)$	Energy Price, 15 min interval
$D(t)$	Demand, 15 min interval
$Q_{bes,0}$	initial battery charge
$\Delta Q_{ch,\max} > 0$	max charge per interval
$\Delta Q_{dch,\max} < 0$	max discharge per interval
$Q_{bes,\max} > 0$	max capacity
$Q_{bes,\min} \geq 0$	min capacity
T_{backup}	required backup intervals
$\eta_{ch} < 1$	charging efficiency
$\eta_{dch} > 1$	inverted discharging efficiency

A. Objective Function

The objective retains the original form:

$$\min \sum_{t=m}^{N} \left(P_{ch}(t) - P_{dch}(t)\right) EP(t). \tag{20}$$

with $P_{ch}(t) \geq 0$, $P_{dch}(t) \geq 0$. This vector contains $2(N-m)$ elements. Parameter m is determined by the requirements of energy reserve for the backup power supply during the T_{backup} time interval.

B. Modified Constraints

a) Backup power supply constraints: Identical to (10) and (11).

b) Rate limits with efficiency:

$$\Delta Q_{dch,\max} \leq P_{ch}(t)\eta_{ch} - P_{dch}(t)\eta_{dch} \leq \Delta Q_{ch,\max}, \quad m \leq t \leq N. \tag{21}$$

c) State of charge limits:

$$Q_{bes,\min}(t) \leq Q_{bes,0}^{\text{backup}} + \sum_{i=m}^{t} \left(P_{ch}(i)\eta_{ch} - P_{dch}(i)\eta_{dch}\right) \leq Q_{bes,\max}, \quad m \leq t \leq N. \tag{22}$$

C. Matrix Representation

The LP is:

$$\min(f_2 \cdot x_2) \quad \text{such that} \quad \begin{cases} A_2 \cdot x_2 \leq b_2, \\ A_{eq2} \cdot x_2 = b_{eq2} \\ l_b 2 \leq x_2 \leq u_b 2, \end{cases} \tag{23}$$

with

$$f_2 = \begin{bmatrix} EP(t) & -EP(t) & \cdots & EP(N) & -EP(N) \end{bmatrix} \tag{24}$$

$$x_2 = \begin{bmatrix} P_{ch}(t) & P_{dch}(t) & \cdots & P_{ch}(N) & P_{dch}(N) \end{bmatrix}^T, \tag{25}$$

where $m \le t \le N, t \in \mathbb{T}$, $1 \le m \le N+1, m \in \mathbb{N}$. Here parameter m is defined using backup within time T_{backup}. The constraint matrix A_2 has four blocks:

$$A_2 = \begin{bmatrix} A_{1\_2} \\ A_{2\_2} \\ A_{3\_2} \\ A_{4\_2} \end{bmatrix}_{4n\times 2n}, \tag{26}$$

where $n = N - m + 1$.

The matrix A_{1_2} has the following block structure

$$A_{1\_2} = \begin{bmatrix} \eta_{ch} & -\eta_{dch} & 0 & 0 & \cdots & 0 & 0 \\ \eta_{ch} & -\eta_{dch} & \eta_{ch} & -\eta_{dch} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta_{ch} & -\eta_{dch} & \eta_{ch} & -\eta_{dch} & \cdots & \eta_{ch} & -\eta_{dch} \end{bmatrix}$$

and A_{3_2} is block-diagonal:

$$A_{3\_2} = \begin{bmatrix} \eta_{ch} & -\eta_{dch} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \eta_{ch} & -\eta_{dch} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \eta_{ch} & -\eta_{dch} \end{bmatrix}.$$

The constraint vector b_2 is:

$$b_2 = \begin{bmatrix} b_{1\_2} \\ b_{2\_2} \\ b_{3\_2} \\ b_{4\_2} \end{bmatrix}_{4n\times 1} \tag{27}$$

with

$$b_{1\_2} = (Q_{bes,\max} - Q_{bes,0}^{\text{backup}})\,\mathbf{1}_{n\times 1}, \tag{28}$$

$$b_{2\_2} = (Q_{bes,0}^{\text{backup}} - Q_{bes,\min})\,\mathbf{1}_{n\times 1}, \tag{29}$$

$$b_{3\_2} = \Delta Q_{ch,\max}\,\mathbf{1}_{n\times 1}, \tag{30}$$

$$b_{4\_2} = -\Delta Q_{dch,\max}\,\mathbf{1}_{n\times 1} \quad (\text{since } \Delta Q_{dch,\max} < 0). \tag{31}$$

Lower bounds: $lb_2 = \mathbf{0}_{2n\times 1}$. No upper bounds or equality constraints.

V. VALIDATION

To verify equivalence under ideal conditions, both modifications were implemented in MATLAB R2025b with $\eta_{ch} = \eta_{dch} = 1$. The optimization horizon is 24 hours with 15-minute resolution (96 intervals). Load and price data were generated using a demand/price generator.

Figures 1 and 2 show the results. As expected, the net power flow $P_{ch}(t) - P_{dch}(t)$ from the second modification exactly matches $\Delta Q_{bes}(t)$ from the first, confirming the theoretical equivalence.

When realistic efficiencies are used, the second modification produces physically accurate results, while the first (which ignores losses) would be incorrect. This demonstrates the necessity of the second modification for practical applications.

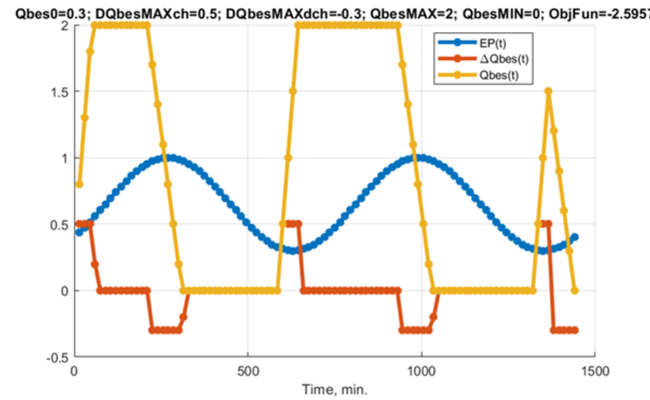

Fig. 1. First Modification Results Plot: (a) Electricity price $EP(t)$, (b) Net charge variation $\Delta Q_{bes}(t)$, (c) State of charge $Q_{bes}(t)$.

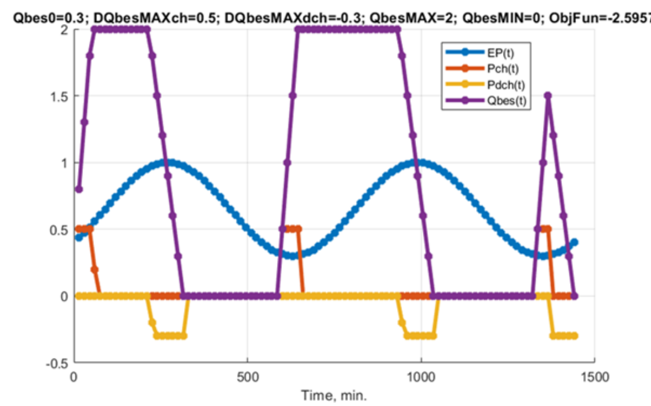

Fig. 2. Second Modification Results Plot: (a) Electricity price $EP(t)$, (b) Charging power $P_{ch}(t)$, (c) Discharging power $P_{dch}(t)$, (d) State of charge $Q_{bes}(t)$.

VI. CONCLUSION

This research successfully addresses the computational challenges associated with the real-time optimization of Battery Energy Storage Systems. By demonstrating that the canonical Mixed-Integer Linear Programming formulation can be reformulated into equivalent Linear Programming problems, this work eliminates the need for binary variables that typically model the mutually exclusive charging and discharging states. This transformation reduces the computational complexity from exponential to polynomial time, thereby enabling real-time operational strategies.

Two distinct modifications are presented. The first, a simplified approach, optimizes the net charge variation under the assumption of ideal efficiency. The second, more significant modification retains the linear structure while accurately accounting for realistic charge and discharge efficiencies, making it suitable for practical applications where energy losses are non-negligible. Complete mathematical formulations and matrix representations are provided for both to facilitate direct implementation with standard LP solvers.

Numerical simulations confirm the theoretical equivalence of the two formulations under ideal efficiency conditions, validating the reformulation strategy. Furthermore, the results demonstrate the necessity of the second modification for generating physically accurate solutions in real-world scenarios with efficiency losses. By enabling polynomial-time solutions, these LP reformulations lay the groundwork for the development of advanced BESS control strategies that can respond dynamically to changing grid conditions. Future work will focus on extending these models to incorporate degradation costs, manage multi-battery systems, and integrate the uncertainty of renewable generation through stochastic programming.

Future work will focus on incorporating battery degradation costs, extending to multi-battery systems with shared constraints, integrating renewable generation uncertainty using stochastic programming and deevelopment of the distributed optimization algorithms for large-scale storage fleets.

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