

# Stabilizing Adaptive Performance Control of Mobile Systems with Guaranteed Collision Avoidance

Marcel Rüger<sup>1</sup> and Olaf Stursberg<sup>1</sup>

**Abstract**—This paper proposes a control framework for trajectory tracking of mobile systems with linear time-invariant (LTI) dynamics subject to input constraints as well as moving obstacles. Building on the concept of adaptive performance functions, a performance bound is introduced that flexibly expands whenever input saturation or collision avoidance temporarily dominate the control action. A dedicated collision avoidance term is incorporated into the control law, and a geometric feasibility condition based on a tangent criterion establishes the minimum control input required to guarantee obstacle avoidance. Stability analysis proves forward invariance of the prescribed performance envelope and boundedness of all closed-loop signals. The procedure is illustrated for an example.

**Index Terms**—Constrained control, collision avoidance, mobile systems, stability, performance functions.

## I. INTRODUCTION

Safety-critical motion control of autonomous systems requires the simultaneous satisfaction of tracking objectives, actuation limits, and geometric separation constraints. Applications such as autonomous aerial vehicles, robotic manipulators operating in dynamic environments, and autonomous ground vehicles share the need to follow prescribed trajectories while reliably avoiding moving obstacles and respecting physical bounds of input [1], [2]. Achieving these objectives with formal guarantees and without resorting to centralized or computationally intensive schemes remains a central open problem in the control of constrained dynamical systems.

Early approaches to trajectory tracking with safety constraints relied on potential field methods [3], in which repulsive potentials centered on obstacles are superimposed on an attractive field towards the goal. While conceptually simple and computationally not very demanding, potential field methods suffer from local minima and provide no formal transient performance guarantees. Optimization-based frameworks such as Model Predictive Control (MPC) include safety constraints explicitly over the prediction horizon [4], [5], [6], [7], but their computational demand scales unfavorably with prediction horizon length and obstacle complexity, limiting real-time applicability in fast-changing environments.

A different line of research addresses safety through barrier Lyapunov functions and Control Barrier Functions (CBFs), which encode safety constraints directly into the control law via forward invariance of a safe set [8], [9].

These approaches provide safety guarantees but typically require precise knowledge of the system dynamics and do not explicitly regulate transient tracking performance. Extensions to systems with input constraints have been proposed [10], yet the interplay between actuator saturation, transient performance, and obstacle avoidance remains difficult to handle.

Prescribed performance control (PPC), introduced in [11], offers a systematic approach to guaranteeing transient and steady-state tracking performance through time-varying performance bounds. By mapping the constrained error through a smooth transformation onto an auxiliary function which diverges at the boundary, the method renders the performance envelope forward invariant without requiring the solution of an optimization problem online. Extensions to input-constrained systems, however, reveal a fundamental trade-off: if the actuator saturates, the prescribed envelope can no longer be maintained by the original performance function. This limitation was addressed in [12], where an adaptive performance function for SISO nonlinear systems was introduced that expands the performance envelope upon saturation, restoring feasibility while preserving the performance guarantee. A related approach for single-integrator systems with input constraints and static obstacle avoidance was developed in [13], where experimental validation on a nano-quadrotor demonstrated practical applicability in a single-agent setting. The extension to multi-agent systems with leader-follower architectures was explored in [14], establishing containment control under performance and actuation constraints. The extension to the homogeneous, fully decentralized case was recently addressed in [15], but only for the very simple agent dynamics, in which the state derivatives are equal to the control inputs.

The present work extends prescribed performance control to the notion of collision avoidance, considering the case of arbitrary LTI dynamics of an agent subject to moving obstacles as well as input constraints. The main contributions compared to existing literature are three-fold: (1) An adaptive performance function is introduced which expands the admissible space upon actuator saturation and upon conflicting avoidance actions, while recovering nominal convergence once both effects vanish. The error dynamics formulated with respect to steady-state behavior under constrained inputs establishes well-posedness of the adaptive control law near the reference. (2) A geometric feasibility condition is derived that characterizes the minimum control range required to guarantee collision avoidance. (3) Stability analysis proves forward invariance of the region admitted by the performance function as well as boundedness of all closed-loop signals.

<sup>1</sup>Marcel Rüger and Olaf Stursberg are with the Control and System Theory Lab, Dept. of Electrical Engineering and Computer Science, University of Kassel, Germany, {marcel.rueger, stursberg}@uni-kassel.de

Simulation results for an example in a 3D state space illustrate the trajectory tracking with exclusion of collision and consistent performance, even during phases dominated by avoidance maneuvers, or input saturation.

The paper is organized such that Sec. II formulates the system model including input constraints together with the performance and safety requirements. The control law and the adaptive performance function are presented in Sec. III, followed by the derivation of the geometric feasibility condition in Sec. IV. Section V provides the stability analysis, and simulation results demonstrating the effectiveness of the proposed method are described in Sec. VI. Conclusions and an outlook are contained in Sec. VII.

## II. PROBLEM FORMULATION

### A. System Model and Input Constraints

Consider mobile systems modeled by the class of linear time-invariant (LTI) systems:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

with state  $x(t) \in \mathbb{R}^n$  at time  $t \in \mathbb{R}$ , vector of control inputs  $u(t) \in \mathbb{R}^m$ , as well as state and input matrices  $A$  and  $B$  of appropriate dimensions. The control input is restricted to  $\|u(t)\| \leq u_{\max}$  for all  $t \geq 0$ .

Let an initial state  $x(0) \in \mathbb{R}^n$  and a reference trajectory of desired states  $x_d(t) \in \mathbb{R}^n$ ,  $t \in [0, \infty[$  be given.

The following assumption is imposed throughout.

*Assumption 1:* For the system (1), let the matrix  $A$  be Hurwitz, the pair  $(A, B)$  be controllable, and the matrix  $B$  be invertible. Furthermore, it is assumed that the reference trajectory  $x_d(t)$  is provided such that the trajectory of inputs  $\bar{u}(t) = -B^{-1}Ax_d(t)$  for realizing  $x_d(t)$  as steady-state  $\bar{x}$  is admissible according to  $\|\bar{u}(t)\| \leq u_{\max}$  for all  $t \geq 0$ .

The tracking error is defined by:

$$e(t) = x_d(t) - x(t). \quad (2)$$

By decomposing the control input into its steady-state part  $\bar{u}(t)$  and a deviation  $u_\delta(t)$ :

$$u(t) = \bar{u}(t) + u_\delta(t), \quad (3)$$

the error dynamics is obtained to:

$$\dot{e}(t) = Ae(t) + Bu_\delta(t). \quad (4)$$

Note that for  $u_\delta(t) \rightarrow 0$ , the error satisfies  $e(t) \rightarrow 0$  and the condition  $\|\bar{u}(t)\| \leq u_{\max}$  from Assumption 1 holds.

### B. Performance Requirement

A prescribed scalar performance bound function  $\rho(t) > 0$  is associated with each component  $e_i(t)$  of (2):

$$|e_i(t)| < \rho(t), \quad i \in \{1, \dots, n\}, \quad \forall t \geq 0. \quad (5)$$

This bound is characterized by its convergence rate  $\lambda > 0$ , an initial value  $\rho_0$  and its prescribed steady-state value  $\rho_\infty > 0$ , subject to:

$$\rho_0 > |e_i(0)|, \quad \lim_{t \rightarrow \infty} \rho(t) = \rho_\infty. \quad (6)$$

The normalized error:

$$\xi_i(t) = \frac{e_i(t)}{\rho(t)} \in (-1, 1) \quad (7)$$

quantifies how close each error component operates to its current performance bound. Obviously, the condition (5) is equivalent to  $\xi_i(t) \in (-1, 1)$  for all  $t \geq 0$ .

### C. Safety Requirement and Obstacle Model

A moving obstacle with time-varying position  $x_o(t) \in \mathbb{R}^n$  and minimum admissible separation radius  $r_o > 0$  is considered. This radius accounts for the size of the obstacle and the agent, such that  $r_o$  represents the minimum admissible distance between their centres.

The distance between the mobile system and the obstacle is:

$$d(x(t), x_o(t)) = \|x(t) - x_o(t)\| - r_o, \quad (8)$$

and collision avoidance requires:

$$d(x(t), x_o(t)) > 0, \quad \forall t \geq 0. \quad (9)$$

The obstacle is detected once it enters a sensing region of radius  $\delta_r > r_o$  centred at  $x(t)$ . The geometric relation between the quantities  $x(t)$ ,  $x_o(t)$ ,  $r_o$ ,  $d$ , and  $\delta_r$  is illustrated in Fig. 1.

In general, of course, more than one obstacle may be contained in the sensing radius of the mobile system with position  $x(t)$ . For simplicity of explanation, the following description is restricted, however, to just one obstacle in the vicinity of  $x(t)$ . Furthermore, the most obvious interpretation of collision avoidance for  $x_o(t)$  and  $x(t)$  is that both vectors only contain position coordinates, while these vectors can, in general, contain additional components (such as velocities).

The obstacle motion  $x_o(t) \in \mathbb{R}^n$  can be understood as the solution of an autonomous ODE:

$$\dot{x}_o(t) = \chi(x_o(t)) \quad (10)$$

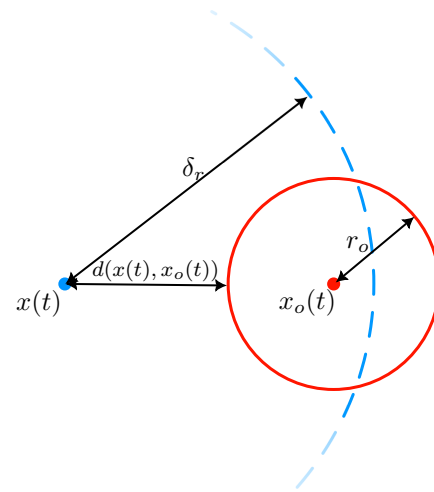


Fig. 1. Geometric quantities: position of the mobile system  $x(t)$ , obstacle center  $x_o(t)$ , minimum separation radius  $r_o$ , and detection radius  $\delta_r$ .

with given initial state  $x_o(0) = x_{o,0}$  and continuously differentiable function vector  $\chi$ . Note that the model (10) is not known to the mobile system, but the latter can only measure the current state  $x_o(t)$  at time  $t$  if  $\|x(t) - x_o(t)\| \leq \delta_r$  applies.

*Assumption 2:* It is assumed that  $u(t)$  with  $\|u(t)\| \leq u_{max}$  exists with  $(\dot{x}(t) - \dot{x}_o(t))^T \cdot \dot{x}_o(t) > 0$  for any point of time  $t \geq 0$ .

This assumption enables that the mobile system is able to evade the obstacle.

#### D. Problem Statement

Given a mobile system (1) and a reference trajectory  $x_d(t)$  satisfying Assumption 1 as well as an obstacle with position  $x_o(t)$  moving according to (10) and satisfying Assumption 2, the objective is to design a control law:

$$u(t) = \kappa(x(t), x_d(t), x_o(t)), \quad (11)$$

such that:

- (i) the performance requirement (5) holds for all  $t \geq 0$ ,
- (ii) the safety constraint (9) is satisfied for all  $t \geq 0$ , and
- (iii) all closed-loop signals remain bounded.

### III. CONTROL DESIGN

The deviation part of the control input introduced by (3) is first decomposed into:

$$u_{\delta,n}(t) = u_t(t) + u_c(t) \quad (12)$$

with  $u_t(t) \in \mathbb{R}^n$  denoting the component for reference tracking, and  $u_c(t) \in \mathbb{R}^n$  the component for collision avoidance. The complete input applied to (1) under consideration of saturation is:

$$u(t) = \begin{cases} \bar{u}(t) + u_{\delta,n}(t) & \text{if } \|\bar{u}(t) + u_{\delta,n}(t)\| \leq u_{max} \\ u_{max} \frac{\bar{u}(t) + u_{\delta,n}(t)}{\|\bar{u}(t) + u_{\delta,n}(t)\|} & \text{otherwise} \end{cases} \quad (13)$$

which preserves the direction of a solution without saturation while enforcing the input constraint. The saturation mismatch is defined as:

$$\tilde{u}_\delta(t) = u_\delta(t) - u_{\delta,n}(t), \quad u_\delta(t) := u(t) - \bar{u}(t), \quad (14)$$

and vanishes whenever no saturation occurs.

#### A. Tracking Component

Each normalized error  $\xi_i(t)$  is passed through the smooth and strictly monotone transformation function, as in [13]:

$$T(\xi_i) = \frac{1}{2} \ln \left( \frac{1 + \xi_i}{1 - \xi_i} \right), \quad (15)$$

which satisfies  $T(\xi_i) \rightarrow \pm\infty$  as  $\xi_i \rightarrow \pm 1$ . The tracking component is defined component-wise with a gain  $k_i \in \mathbb{R}$  by:

$$u_{t,i}(t) = -k_i T(\xi_i(t)), \quad i \in \{1, \dots, n\}, \quad (16)$$

which penalizes any motion that would violate (5).

*Remark 1:* For  $u_{t,i}(t)$  to have a corrective effect, it is required that  $[B]_i k_i > 0$ , where  $[B]_i$  denotes the sum of elements of the  $i$ -th row of matrix  $B$ .

#### B. Collision Avoidance Component

Once the obstacle enters the detection region, a repulsive input is generated. The avoidance term is defined to:

$$u_c(t) = \phi(t) \frac{x(t) - x_o(t)}{\|x(t) - x_o(t)\|} \quad (17)$$

with the distance-dependent scalar gain:

$$\phi(t) = \begin{cases} \frac{1}{d(x(t), x_o(t))} - \frac{1}{\delta_r - r_o} & \text{if } \|x(t) - x_o(t)\| < \delta_r \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

The gain  $\phi(t)$  vanishes on the detection boundary and grows large as  $d \rightarrow 0$ , ensuring a strongly repulsive action as the distance approaches  $r_o$ .

#### C. Adaptive Performance Function

The adaptive performance function is chosen by:

$$\dot{\rho}(t) = -\lambda(\rho(t) - \rho_\infty) - k_r \min(0, m(t)) \quad (19)$$

with the gain  $k_{r,i} > 0$  and the mismatch indicator:

$$m(t) = e(t)^\top u_c(t) - \|\tilde{u}_\delta(t)\|. \quad (20)$$

The first term in (19) drives  $\rho(t)$  exponentially towards its steady-state value  $\rho_\infty$ . The second term is active only if  $m(t) < 0$ . The latter condition holds true if either the avoidance action opposes the tracking objective, i.e.,  $e(t)^\top u_c(t) < 0$ , or if the actuator saturates, i.e.,  $\|\tilde{u}_\delta(t)\| > 0$ . The case  $\min(0, m(t)) < 0$  induces a positive contribution to  $\dot{\rho}(t)$ , thus possibly widening the admissible error envelope constituted by the performance function  $\rho(t)$ . In the case  $m(t) \geq 0$ , the second term in (19) vanishes, and the performance function recovers its nominal exponential convergence to  $\rho_\infty$ .

The overall control structure comprising the components for tracking, collision avoidance, input saturation, and the adaptive performance function is summarized in Fig. 2.

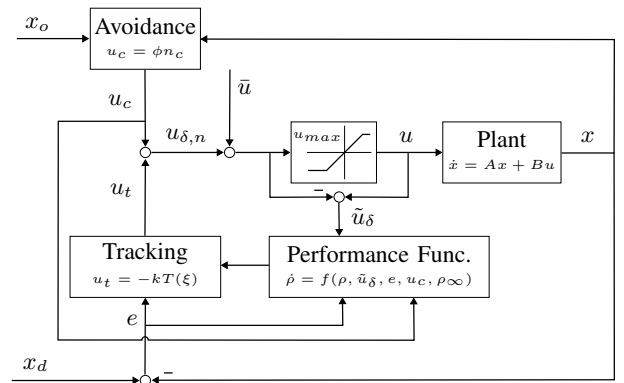


Fig. 2. Block diagram of the proposed control structure; the time argument is omitted from all signals for space reasons.

#### IV. FEASIBILITY OF COLLISION AVOIDANCE

The collision avoidance term  $u_c(t)$  generates a repulsive action, but does by itself not guarantee that the mobile system can physically bypass the obstacle under the input constraint (13). Since the direction of  $u_c(t)$  is fixed by construction in (17), this section determines the scalar gain  $\phi^*(t)$  required for safe passage and derives a corresponding condition on  $u_{\max}$ .

##### A. Speed Decomposition

The closed-loop velocity of the mobile system follows from (1), (3) and (12) to:

$$\dot{x}(t) = \underbrace{Ax(t) + B\bar{u}(t) + Bu_t(t)}_{=: v_f(t)} + \phi(t) Bn_c(t). \quad (21)$$

In here,  $v_f(t) \in \mathbb{R}^n$  collects all contributions determined by the current state and the tracking law, and:

$$n_c(t) = \frac{x(t) - x_o(t)}{\|x(t) - x_o(t)\|} \quad (22)$$

is the unit repulsion direction fixed by (17). Only the gain  $\phi(t)$  remains to be chosen in order to redirect  $\dot{x}(t)$  away from the obstacle.

##### B. Tangent Condition and Required Gain

The minimum requirement for safe passage is that the total velocity  $\dot{x}(t)$  points at least tangentially to the safety circle of radius  $r_o$  centred at  $x_o(t)$ . A point  $x_p \in \mathbb{R}^n$  on this circle satisfies  $\|x_p - x_o(t)\| = r_o$  and the condition for tangential direction is:

$$(x_p - x(t))^\top (x_p - x_o(t)) = 0. \quad (23)$$

According to the Pythagorean theorem, the distance from the mobile system to any such tangent point is:

$$\ell(t) = \sqrt{\|x(t) - x_o(t)\|^2 - r_o^2}. \quad (24)$$

The unit direction towards a tangent point is denoted by  $n_p(t) = (x_p - x(t))/\ell(t)$ . Among all tangent points satisfying (23), the one closest in angle to  $v_f(t)$  is selected, as it requires the smallest corrective gain  $\phi^*$ . The geometric construction for a two-dimensional case is illustrated in Fig. 3.

The requirement that  $\dot{x}(t)$  needs to be aligned to  $n_p(t)$  yields:

$$v_f(t) + \phi^*(t) Bn_c(t) = \alpha(t) n_p(t), \quad (25)$$

in which  $\alpha(t) = n_p^\top(t) v_f(t) > 0$  is the projection of the determined velocity onto the tangent direction. Projecting both sides of (25) onto  $Bn_c(t)$  leads to the solution for  $\phi^*(t)$ :

$$\phi^*(t) = \frac{\alpha(t) n_c^\top(t) B^\top n_p(t) - n_c^\top(t) B^\top v_f(t)}{n_c^\top(t) B^\top B n_c(t)}. \quad (26)$$

This assignment is well-defined whenever  $n_c^\top(t) B^\top B n_c(t) \neq 0$ , what is in turn guaranteed by regularity of  $B$  and  $\|n_c\| = 1$ .

Now it becomes apparent that  $u_{\max}$  is large enough to allow circumvention of the obstacle (thus contributing to satisfying Assumption 2), if the following condition holds:

$$\|\bar{u}(t) + u_t(t) + \phi^*(t) n_c(t)\| \leq u_{\max}, \quad (27)$$

with  $\phi^*(t)$  as defined in (26). This guarantees the existence of an admissible total input that steers the mobile system tangentially past the obstacle without violating the input constraint (13).

*Remark 2:* The condition (27) depends on the system matrices  $A$ ,  $B$ , the tracking gains  $k_i$ , and the obstacle geometry  $r_o$ ,  $\delta_r$ , and it can be verified offline at the time of controller design for the worst-case relative configuration of the mobile system and the obstacle.

#### V. STABILITY ANALYSIS

The performance function  $\rho(t) > 0$  governs the admissible tracking error. The performance requirement (5) reads:

$$-\rho(t) < e_i(t) < \rho(t), \quad i \in \{1, \dots, n\}, \quad \forall t \geq 0, \quad (28)$$

with normalized error component  $\xi_i(t) = e_i(t)/\rho(t) \in (-1, 1)$ . The adaptive performance function obeys (19) with the mismatch indicator (20) and the gains  $\lambda$ ,  $k_r > 0$ . The constrained input signal is defined as in (13).

*Theorem 1:* Let the Assumptions 1 and 2 as well as condition (27) hold, and let  $\rho_0 > |e_i(0)|$  for all  $i \in \{1, \dots, n\}$ . Furthermore, assume that a finite time  $T_f \geq 0$  exists with  $\|x(t) - x_o(t)\| \geq \delta_r$  for all  $t \geq T_f$ . With the control law (13) and the adaptive performance function (19), the following holds for all  $t \geq 0$ :

- (i)  $\xi_i(t) \in (-1, 1)$ , i.e.,  $|e_i(t)| < \rho(t)$  for all  $i \in \{1, \dots, n\}$ ,
- (ii)  $\rho(t) \rightarrow \rho_\infty$  as  $t \rightarrow \infty$ , and
- (iii) all closed-loop signals remain bounded.

*Proof:* Property (i): Since  $\rho_0 > |e_i(0)|$  for all  $i$ , each  $\xi_i(0) \in (-1, 1)$ . For each component  $i$ , forward invariance of the set  $\Omega = \{e_i : |e_i| < \rho(t)\}$  is established by showing that the condition

$$\dot{e}_i(t) < \dot{\rho}(t) \quad (29)$$

holds on the upper boundary  $e_i = \rho$ , and by symmetry also on the lower boundary  $e_i = -\rho$ .

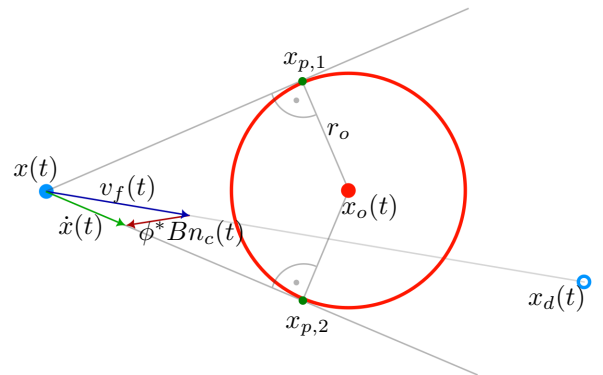


Fig. 3. Tangent condition: optimal tangent point  $x_{p,2}$ , determined velocity  $v_f(t)$ , required correction  $\phi^* B n_c(t)$ , and resulting velocity  $\dot{x}(t)$ .

For  $\xi_i \rightarrow 1$ , the transformed normalized error converges according to  $T(\xi_i) \rightarrow +\infty$ . The  $i$ -th component of the error dynamics (4) leads to:

$$\dot{e}_i = [A]_i e - [B]_i k_i T(\xi_i) + [B]_i u_c + [B]_i \tilde{u}_\delta. \quad (30)$$

Since  $[B]_i k_i > 0$  and without input constraints, the term  $-[B]_i k_i T(\xi_i)$  diverges to  $-\infty$  as  $\xi_i \rightarrow 1$ , while all remaining terms, including  $\dot{\rho}(t)$ , remain bounded. Consequently,

$$\dot{e}_i \rightarrow -\infty \quad \text{as } \xi_i \rightarrow 1. \quad (31)$$

It remains to verify that (29) holds when saturation occurs,  $\|\tilde{u}_\delta\| > 0$ , and when avoidance opposes tracking,  $e^\top u_c < 0$ . In either case, it is  $m(t) < 0$  and the performance function increases. As the term  $-[B]_i k_i T(\xi_i)$  diverges to  $-\infty$  and saturation cuts the nominal input,  $\|\tilde{u}_\delta\| \rightarrow \infty$ . Similarly, in the case of collision avoidance for  $d \rightarrow 0$ ,  $\|u_c\| \rightarrow \infty$  follows, such that  $m(t) \rightarrow -\infty$  and  $\dot{\rho}(t) \rightarrow +\infty$ . Therefore, condition (29) is satisfied even under input saturation and critical obstacle proximity; thus,  $\xi_i(t) \in (-1, 1)$  holds for all  $i$  and all  $t \geq 0$ .

**Property (ii):** The condition (27) and the assumption that the mobile system leaves the detection region at the finite time  $T_f$  implies that  $u_c(t) = 0$  for all  $t \geq T_f$ . Since  $e(t)$  and  $x_d(t)$  are bounded, it holds that  $u_{\delta,n}(t) \rightarrow 0$  for  $e(t) \rightarrow 0$ , i.e., saturation no longer occurs after any finite time  $T_s \geq T_f$ . For  $t \geq T_s$ ,  $m(t) = 0$  and the adaptive law reduces to  $\dot{\rho}(t) = -\lambda(\rho(t) - \rho_\infty)$ , from which exponential convergence  $\rho(t) \rightarrow \rho_\infty$  follows.

**Property (iii):** From property (i) follows that  $|e_i(t)| < \rho(t)$  for all  $i$  and  $t \geq 0$ . Since the correction term (20) only increases  $\rho$  when active, and by property (ii) it vanishes after  $T_s$ ,  $\rho(t)$  is bounded above on  $[0, T_s]$  and converges monotonically to  $\rho_\infty$  thereafter. Boundedness of  $e(t)$  follows componentwise from  $|e_i(t)| < \rho(t)$ , boundedness of  $x(t)$  from  $x(t) = e(t) + x_d(t)$ , and boundedness of  $u(t)$  from the saturation law (13). ■

## VI. SIMULATION RESULTS

In this section, a simulation study in a 3D-space illustrates the behavior of the proposed control framework. The mobile system evolves according to (1) with chosen system matrices

$$A = \begin{bmatrix} -2.1 & 0.4 & -0.3 \\ -0.5 & -1.8 & 0.6 \\ 0.2 & -0.3 & -1.5 \end{bmatrix}, \quad B = \begin{bmatrix} 1.2 & 0.3 & -0.2 \\ 0.4 & 1.5 & 0.3 \\ -0.1 & 0.2 & 1.1 \end{bmatrix},$$

which satisfy Assumption 1. With use of a parameter with  $T = 30$  s, the desired trajectory is specified to:

$$x_d(t) = \left[ 4 \cdot \left( 1 - \cos \frac{\pi t}{T} \right) \quad 0.6 \sin^2 \frac{\pi t}{T} \quad 0.25 \sin \frac{2\pi t}{T} \right]^\top.$$

The steady-state input  $\bar{u}(t)$  satisfies  $\|\bar{u}(t)\| \leq u_{\max} = 15$  for all  $t \in [0, T]$ , such that condition (27) holds. The mobile system starts at  $x_0 = x_d(0) + [0, -0.4, 0.2]^\top$ .

The moving obstacle approaches the mocking system head-on along the  $x_1$ -axis starting at  $x_o(0) = [7.9, 0.45, 0.15]^\top$  with constant speed. The detection radius is  $\delta_r = 2.0$  and the safety radius  $r_o = 0.35$ . This configuration makes an avoidance maneuver necessary and forces

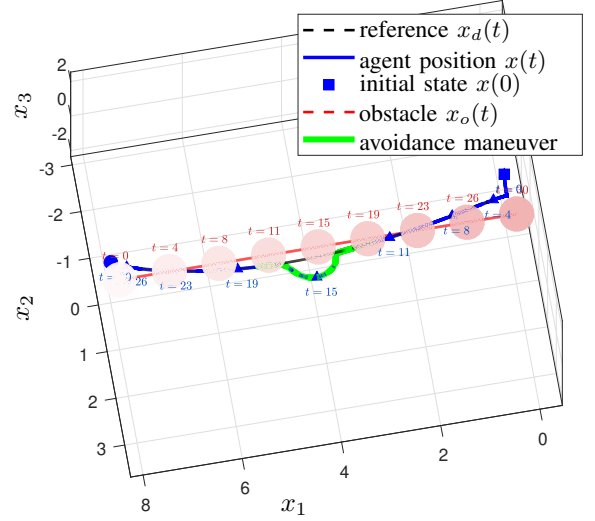


Fig. 4. Three-dimensional trajectories: mobile system  $x(t)$  (blue), reference  $x_d(t)$  (black, dashed), obstacle  $x_o(t)$  (red, dashed), safety spheres of radius  $r_o$  at selected instants (shaded, opacity increasing with time), and active avoidance phase (green).

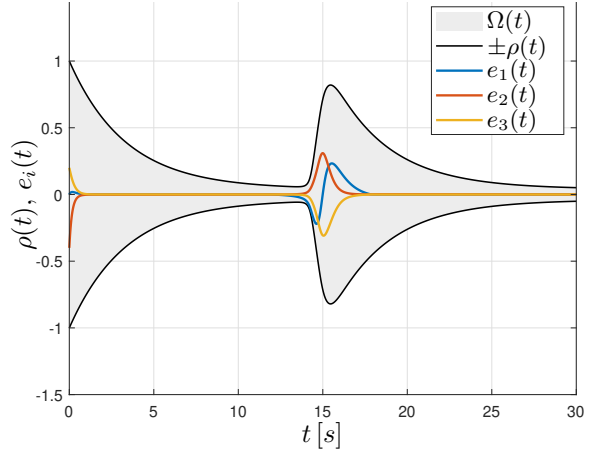


Fig. 5. Tracking errors  $e_i(t)$  (coloured) and adaptive performance bound  $\pm\rho(t)$  (black); the shaded region indicates the invariant set  $\Omega(t)$ .

the mobile system to deviate from  $x_d(t)$ . The performance function is initialised at  $\rho_0 = 1.0 > |e_i(0)|$  for all  $i$ , with steady-state value  $\rho_\infty = 0.04$  and convergence rate  $\lambda = 0.30$ . The adaptive gain is  $k_r = 1.0$  and the tracking gains are  $k_i = 3$  for all  $i$ .

Figure 4 shows the trajectories of the mobile system, the desired path, and the obstacle in  $\mathbb{R}^3$ , together with snapshots of the safety sphere at selected instants of time. The green segment marks the active avoidance phase during which the obstacle lies within the detection radius. The mobile system tracks the desired trajectory while being deflected around the obstacle, whose safety radius is never violated.

Figure 5 displays the tracking errors  $e_i(t)$  together with the adaptive performance bound  $\pm\rho(t)$ . The shaded region corresponds to the invariant set  $\Omega(t)$ . For all three components and at all times, the errors remain strictly within  $\Omega(t)$ , confirming property (i) of Theorem 1. A temporary expansion of the performance function is visible during

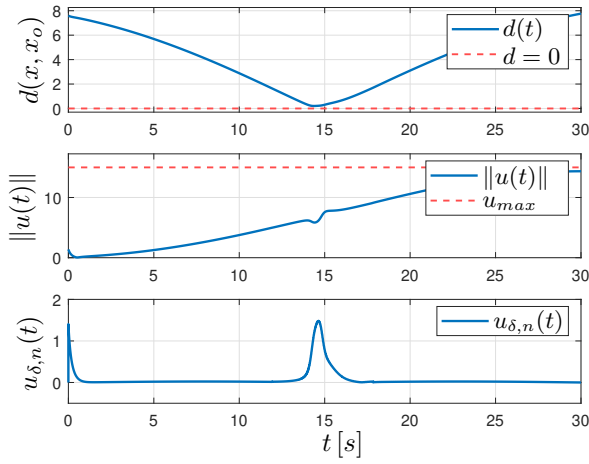


Fig. 6. Top: distance  $d(x(t), x_o(t))$ ; the constraint  $d > 0$  (red, dashed) is never violated ( $d_{min} = 0.21$ ). Centre: control input  $\|u(t)\|$  and bound  $u_{max}$  (red, dashed). Bottom: nominal deviation input  $\|u_{\delta,n}(t)\|$ .

the avoidance maneuver, reflecting the adaptive widening induced by the mismatch indicator (20). Once the obstacle leaves the detection region,  $\rho(t)$  resumes its exponential convergence toward  $\rho_\infty$ , consistent with property (ii).

Figure 6 confirms the three remaining properties: The upper part shows the distance  $d(x(t), x_o(t))$ , which remains strictly positive throughout, verifying the safety constraint (9). The input constraint (13) is always satisfied, as can be seen in the middle part by the visualization of the norm of the control input  $\|u(t)\|$  next to  $u_{max}$ . Finally, the nominal deviation norm  $\|u_{\delta,n}(t)\|$  shown in the lower part returns to zero as the mobile system converges to the reference trajectory, in agreement with the delta-system argument of Section II. The simulations demonstrate that the proposed adaptive performance control law achieves collision-free trajectory tracking. The performance function expands when needed and recovers its nominal convergence once the obstacle interaction and input saturation vanish, in full agreement with the theoretical analysis of Section V.

## VII. CONCLUSIONS

This paper presented a prescribed performance control framework for trajectory tracking in linear time-invariant systems subject to input constraints and a moving obstacle. The approach integrates three core elements: (i) an adaptive performance function that expands the error envelope upon input saturation or collision avoidance actions and recovers nominal convergence once both effects subside; (2) a repulsive collision avoidance term whose gain grows as the distance approaches the minimal admissible separation radius; and (3) a formulation of an error system which establishes well-posedness of the adaptive law near the reference. A geometric feasibility condition based on a tangent criterion characterizes the minimum control range sufficient for safe passage past the obstacle and is verifiable offline at the time of controller design. A stability analysis proved forward invariance of the prescribed performance function and boundedness of all closed-loop signals, and simulation

results in a three-dimensional space confirmed collision-free tracking and consistent transient performance.

Compared to MPC-based approaches, the proposed method does not rely on optimization, i.e., the online effort is restricted to the evaluation of the developed control laws, and are thus much smaller. Compared to approaches based on CBF, the framework provides explicit transient performance guarantees through the time-varying error envelope.

Several directions are conceivable for future research. The present framework assumes a single moving obstacle; an extension to multiple obstacles requires a relevance-based selection mechanism, analogous to the prediction of closest approach criterion employed in [15], to avoid overly conservative super-position of repulsive actions. A second and obvious extension concerns the deployment of the proposed method for nonlinear dynamics.

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