Stochastic Bi-objective Optimization Model for a Single-Period Sustainable Supplier Selection and Ordering Problem*

Rima Cheaytou[1], Sadeque Hamdan[2], and Ali Cheaitou[3,4]

Abstract— **In the current global context with increasing competition and pressure for more social and, in particular, environmental sustainability, selecting the best suppliers becomes critical for the success of any organization. This paper, therefore, studies the sustainable supplier selection and order allocation problem of a retailer facing stochastic demand for a short life-cycle product. Moreover, the suppliers are considered to be capacitated and heterogeneous and evaluated based on economic, environmental, and social criteria. The problem is modeled using a bi-objective optimization framework. As inputs to this model, Analytic Hierarchy Process is used in order to determine the importance weights of the sustainability criteria and the end customer satisfaction. Moreover, Fuzzy Technique for Order of Preference by Similarity to Ideal Solution is then used in order to assess the suppliers based on pre-determined environmental and social sustainability criteria. Furthermore, profit-based and sustainability-based multi-supplier newsvendor subproblems are developed. These subproblems are solved numerically to characterize the optimal policy, and then through a scalarization approach, the bi-objective model is also solved numerically. In addition, a numerical case study is solved to analyze the optimal solution.**

Keywords: Sustainability; stochastic; inventory; perishable products; bi-objective optimization.

I. INTRODUCTION

Short life-cycle products face stochastic demand difficult to predict and a short selling season. Many services and products can be categorized in this category such as fashion products and perishable food produce. For such kinds of products, one of the most used ordering and inventory models, in the literature, is the "*Newsvendor*" model [1].

Moreover, the growing awareness about sustainability and particularly social and environmental impacts of businesses has increased the pressure on business organizations. Even international organizations such as the International Organization for Standardization (ISO) have published standards on the social responsibility and sustainable procurement of companies such as ISO 20400 [2], and the United Nations' Sustainability Development Goals have become necessary to be considered when companies design their supply chains [3].

At the same time, with globalization, it has become far easier for companies looking to reduce their procurement costs but also concerned by the social and environmental sustainability value of the products that they purchase to compare many suppliers and to choose the best. It appears, therefore, that the problem of supplier selection and order allocation for short life-cycle products with random end customer demand is very important to many industries especially when considering multiple suppliers in a global context with different costs and different environmental and social sustainability performances. In this context, researchers have been considering sustainability (environmental and social) aspects in supplier selection and order allocation models [4] and some in inventory control models [5]. However, to the best of the authors' knowledge, none of the existing literature works had addressed the capacitated multiple-supplier newsvendor model with environmental, economic and social sustainability criteria.

We develop a multiple-supplier framework for a newsvendor-type problem in which we take into account environmental and social aspects in addition to the economic performance. The framework allows retailers to select the best supplier(s), from which the orders of a short life-cycle product can be placed, and to determine their corresponding optimal quantities. It comprises a bi-objective optimization model in which the first objective maximizes the retailer's profit and the second objective maximizes the sustainability value of the purchased products. The sustainability value is determined through a multi-criteria decision making (MCDM) approach. First, Analytic Hierarchy Process (AHP) is used to determine the importance weight of environmental and social sustainability for the company compared to the end customer satisfaction in terms of product availability, based on a set of criteria. Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is then used to assess the performance of the available suppliers based on a set of environmental (green) and social sustainability criteria. To solve the proposed bi-objective model, the profit and the sustainability based subproblems are first solved separately, then the bi-objective model is solved, which allows to characterize the optimal solution of the two single-objective subproblems and for the bi-objective problem. Moreover, some managerial insights are obtained based on a numerical study.

*This work was not supported by any organization

[1]Rima Cheaytou is with Mathematics Division, American University in Dubai, Sheikh Zayed Road, P.O. Box 28282 Dubai, United Arab Emirates, rcheaytou@aud.edu

[2]Sadeque Hamdan is with Bangor Business School, Bangor University, Bangor, Gwynedd, LL57 2DG, United Kingdom, s.hamdan@bangor.ac.uk

[3]Ali Cheaitou is with the Department of Industrial Engineering and Engineering Management, University of Sharjah, 27272 Shajah, United Arab Emirates, acheaitou@sharjah.ac.ae

[4]Ali Cheaitou is with the Sustainable Systems, Technologies & Infrastructure Research Center (SSTIRC), Research Institute of Sciences and Engineering, University of Sharjah, 27272 Sharjah United Arab Emirates, acheaitou@sharjah.ac.ae

II. MODEL

The notations of the model are given as follows:

- n: Total number of available suppliers.
- c_i: Cost per unit owed to supplier i, $i = 1, ..., n$.
- p: Unit selling price charged by the retailer to the end customers.
- s: Unit salvage value.
- π: Unit penalty shortage cost.
- D: Stochastic demand to be fulfilled by the retailer.
- $f(.)$: Probability Density Function (PDF) of the demand.
- Q_i: Quantity ordered from supplier i.
- $\sum_{i=1}^{n} Q_i$: Total quantity ordered from all suppliers.
- $\mathbf{Q} \equiv \{Q_1, ..., Q_n\}$: Vector of quantities ordered from the suppliers.
- K_i: Supply capacity of supplier i.
- w^{GS}: Total importance weight of the green and social aspects obtained from AHP.
- ϖ^c: Importance weight given by the company to the negative impact of a shortage on the company's image, obtained using AHP.
- w^c: Importance weight given by the company to the positive impact of a satisfied customer on the company's image, obtained using AHP.
- w_i: Performance score of supplier i with respect to the sustainability aspects (including green and social factors) obtained using fuzzy TOPSIS.
- w_i^{GS}: Relative performance score of supplier i with respect to the and social criteria obtained by multiplying w_i by w^{GS}.
- $\Gamma(\mathbf{Q})$: Retailer's profit objective function as function of $\mathbf{Q}$.
- $W_{ST}(\mathbf{Q})$: Retailer's sustainability objective function as function of $\mathbf{Q}$.
- Z($\mathbf{Q}$): Retailer's scalarized bi-objective function as function of $\mathbf{Q}$.

We refer the reader to [6] for the fuzzy TOPSIS procedure and to [7] for the AHP procedure.

We denote by Q_i^* the optimal value of Q_i and $\mathbf{Q}^*=\{Q_i^*\}$ the corresponding vector of decision variables. As it is known in the newsvendor problem, the retailer has to make decisions before the beginning of the selling season which is the case in this profit model. In addition, the retailer may place orders from n independent suppliers. The retailer's first objective is to maximize the expected profit for the selling season denoted $\Gamma(\mathbf{Q})$ with

$$\begin{aligned}
\Gamma(\mathbf{Q}) = \quad & p \int_0^{\sum_{i=1}^n Q_i} D\, f(D) dD \qquad (1)\\
+ \quad & p \sum_{i=1}^{n} Q_i \int_{\sum_{i=1}^n Q_i}^{\infty} f(D) dD \\
+ \quad & s \int_0^{\sum_{i=1}^n Q_i} \left(\sum_{i=1}^{n} Q_i - D \right) f(D) dD \\
- \quad & \pi \int_{\sum_{i=1}^n Q_i}^{\infty} \left(D - \sum_{i=1}^{n} Q_i \right) f(D) dD \\
- \quad & \sum_{i=1}^{n} c_i\, Q_i
\end{aligned}$$

The retailer's profit maximization problem is then defined as

$$\max_{\mathbf{Q}} \quad \Gamma(\mathbf{Q}) \tag{2}$$

subject to

$$0 \leq Q_i \leq K_i, \forall i = 1, ..., n. \tag{3}$$

The sustainability problem aims at calculating the quantity that maximizes the green value W_{ST} where

$$\begin{aligned}
W_{ST}(\mathbf{Q}) \quad = \quad & \sum_{i=1}^{n} w_i^{GS} \times Q_i \qquad (4)\\
+ \quad & w^c \int_0^{\sum_{i=1}^n Q_i} D\, f(D) dD \\
+ \quad & w^c \int_{\sum_{i=1}^n Q_i}^{\infty} (\sum_{i=1}^{n} Q_i) f(D) dD \\
- \quad & w^{GS} \int_0^{\sum_{i=1}^n Q_i} \left(\sum_{i=1}^{n} Q_i - D \right) f(D) dD \\
- \quad & \varpi^c \int_{\sum_{i=1}^n Q_i}^{\infty} \left(D - \sum_{i=1}^{n} Q_i \right) f(D) dD.
\end{aligned}$$

Equation (4) represents the weighted total sustainability value for ordering $\mathbf{Q}$ from the selected suppliers. The first term represents the weighted environmental and social sustainability, the second and third terms represent the weighted positive economic sustainability related to the end customer satisfaction (product availability), the fourth term represents the weighted negative sustainability related to the salvaged products at the end of the selling season that may include returning them (emissions because of transportation) or disposing of them, and the fifth term represents the weighted negative economic sustainability related to the end customer dissatisfaction (product unavailability).

The retailer's optimization problem is defined as follows

$$\max_{\mathbf{Q}} \ W_{ST}(\mathbf{Q}) \tag{5}$$

subject to (3).

The profit objective function maximizes the expected profit (Γ) defined in Equation (1). The sustainability objective function aims at maximizing the sustainability value (W_{ST}) defined in Equation (4). Thus, the bi-objective model is defined as follows:

$$\max_{\mathbf{Q}} \ \Gamma(\mathbf{Q}), \tag{6}$$

$$\max_{\mathbf{Q}} \ W_{ST}(\mathbf{Q}) \tag{7}$$

subject to (3).

Such bi-objective problems can be solved using different techniques including the scalarization techniques such as the Weighted Comprehensive Criterion Method denoted by (WCCM) [8] which has been used in this work. We do not detail here the steps of the WCCM method and we only present the final normalized objective function which will be minimized subject to (3), which infers

$$\begin{aligned} \min_{\mathbf{Q}} Z(\mathbf{Q}) &= 100 \times (\omega_1(\Gamma^* - \Gamma(\mathbf{Q}))/\Gamma^* \\ &+ \omega_2(W_{ST}^* - W_{ST}(\mathbf{Q}))/W_{ST}^*) \\ &= 100 \times (-\omega_1 * \Gamma(\mathbf{Q})/\Gamma^* \\ &- \omega_2 * W_{ST}(\mathbf{Q})/W_{ST}^* + \omega_1 + \omega_2), \quad (8) \end{aligned}$$

subject to (3).

where ω_1 and ω_2 are the objective function relative weights. They are set by the organization in a way that follows its strategy and so that $\omega_1 + \omega_2 = 1$. In addition, Γ and W_{ST} can be substituted in (8) by their respective expressions (see Eqs. (1) and (4)).

III. Numerical study

The model described in Section II has been implemented in Wolfram Mathematica 8.0. The implementation follows the solution approach described earlier, and starts by solving the profit subproblem, then the sustainability subproblem and finally the bi-objective subproblem.

A numerical example, which relies on generated and not a real set of data, is used, in this section, to shed light on the implementation aspects of the proposed approach.

A. Input data

The numerical example considers ten suppliers with different capacity levels, perceived green and social sustainability levels, and unit prices. Table I provides their main characteristics.

TABLE I

Input data of the numerical example

Supplier i	K_i	c_i	Green and social performance level
1	300	30	Low
2	300	30	Very high
3	300	31	Very high
4	300	32	Very high
5	200	16	Low
6	200	18	Medium
7	200	19	Medium
8	400	32	Medium
9	200	18	Medium
10	200	22	Medium

It is worth noting that Table I only provides indicative green and social performance levels, which are used to classify the suppliers on a high level using a linguistic scale. The measured and detailed performance levels have been determined using fuzzy TOPSIS, using a scenario with 3 decision makers and ten criteria, and the results are shown in Table II. Table II shows that the best three suppliers in descending order in terms of green and social performance are 2, 4 and 3 respectively.

Moreover, the other parameters used are as follows: $p = 75$, $s = 10$, $\pi = 20$. For the demand, we assumed a normal distribution with an average value of $\mu = 1000$ and a standard deviation of $\sigma = 300$. In order to determine the three parameters w^{GS}, ϖ^c, and w^c, AHP is used. A scenario with three decision makers and eight criteria is assumed which led to the following results: $w^{GS} = 0.408$, $\varpi^c = 0.344$, and $w^c = 0.248$.

TABLE II

Relative performance weights of the suppliers

Supplier i	w_i	w_i^{GS}
1	0.25	0.10
2	0.53	0.22
3	0.51	0.21
4	0.52	0.21
5	0.24	0.10
6	0.35	0.14
7	0.35	0.15
8	0.36	0.15
9	0.35	0.14
10	0.36	0.15

TABLE III

Optimal solution of the profit and sustainability subproblems

Supplier i	Q_i of the profit subproblem	Q_i of the sustainability subproblem
1	0	0
2	216.4	300
3	0	300
4	0	300
5	200	0
6	200	0
7	200	0
8	0	217.8
9	200	0
10	200	0

TABLE IV

Pareto optimal solutions

ω_1	ω_2	$\Gamma(\mathbf{Q})$ bi-objective	$W_{ST}(\mathbf{Q})$ bi-objective	Z (%)
0.1	0.9	38535.7	220.99	2.25
0.2	0.8	38717.9	220.84	4.27
0.3	0.7	38974.3	220.46	6.23
0.4	0.6	45509.4	205.65	6.79
0.5	0.5	48415	195.86	
0.6	0.4	48446.4	195.68	4.79
0.7	0.3	48479.4	195.40	3.64
0.8	0.2	48512.4	194.94	2.46
0.9	0.1	48541.3	194.17	1.25

These input values have been used to solve the two subproblems and the bi-objective problem as described earlier. The results of the single objective subproblems are reported in Table III and the Pareto solutions of the bi-objective problem are reported in Table IV. As it can be seen from Table IV, the bi-objective solution is closer to the profit solution for high ω_1 values, closer to the sustainability solution for high ω_2 values which is as expected. Moreover, improving the sustainability value from the worst solution of 194.17 to the best solution of 220.99, which corresponds to an improvement of 13.81%, happens at the expense of a decrease in the profit value from 48541.3 to 38535.7 and a relative decrease of 20.61%.

IV. CONCLUSION

This study highlights the growing necessity of integrating environmental and social sustainability into business decision-making, particularly in supplier selection within supply chains. It proposes a bi-objective newsvendor model that jointly optimizes profit and sustainability, moving beyond traditional cost- and service-based criteria. The model is solved numerically and provides the optimal policies with flexibility to balance economic performance and sustainability goals. Numerical results show that sustainability often comes at a cost, which remains a barrier to adoption; however, an appropriate weighting of sustainability objectives can lead to effective trade-offs without significantly harming profitability. The findings also suggest that public policies, incentives, and regulatory frameworks can support firms in adopting sustainable practices. While the model is limited to a single-product, single-period setting without emergency replenishment, future research could extend it to more complex, dynamic, and uncertain supply chain environments.

REFERENCES

[1] Y. Qin Y, R. Wang, A.J. Vakharia, Y. Chen, and M.M.H. Seref, he newsvendor problem: Review and directions for future research, uropean Journal of Operational Research, vol. 213, pp. 361-374, 2011.

[2] ISO 20400 - sustainable procurement, International Organization for Standardization, https://www.iso.org/publication/PUB100410.html, Accessed on 05/10/2019.

[3] Y.J. Cai and T.M. Choi, united nations sustainable development goals perspective for sustainable textile and apparel supply chain management. Transportation Research Part E: Logistics and Transportation Review, vol. 141, 102010, 2020.

[4] L. Liu, M. Zhang, and W. Ye, he adoption of sustainable practices: A suppliers perspective. Journal of Environmental Management, vol. 232, pp. 692701, 2019.

[5] Y. Bouchery, A. Ghaffari, Z. Jemai, Y. Dallery, ncluding sustainability criteria into inventory models. European Journal of Operational Research, vol. 222, pp. 229240, 2012.

[6] S. Hamdan and A. Cheaitou, upplier selection and order allocation with green criteria: An MCDM and multi-objective optimization approach, Computers & Operations Research, vol. 81, pp. 282-304, 2017.

[7] T.L. Saaty, ecision making with the analytic hierarchy process, International Journal of Services Sciences, vol. 1, pp. 83-98, 2008.

[8] A. Kamali, S. Fatemi Ghomi and F. Jolai, multi-objective quantity discount and joint optimization model for coordination of a single-buyer multi-vendor supply chain. Computers & Mathematics with Applications, vol. 62, pp. 32523269, 2011.