Observer Design for Conformable Fractional Positive Continuous-time Systems

D. Krokavec

***Abstract*— The Lyapunov method is applied to the synthesis of the state observer of semilinear positive systems with conformable fractional derivative of the state vector and vector nonlinearity satisfying the local Lipschitz condition. The design conditions are presented in the form of a set of linear matrix inequalities that reflect the parametric constraints defined by the structure of Metzler matrices and the limiting consequences of diagonal stabilization in positive system synthesis tasks. It is shown that when solving this problem through the feasibility of the proposed set of linear matrix inequalities, the observer error is positive and stable. Finally, the correctness of the obtained theoretical results is verified by a numerical example.**

I. Introduction

The class of existing positive models has proven useful in modeling many systems with non-negative states and outputs, [1], [2], despite limitations requiring innovative theoretical approaches to solving the positivity problem, [3], [4]. Since the standard forms of describing the dynamics of these systems are related to the system matrices of the Metzler structure, the theory of Metzler matrices, [5], [6], forms a platform for their analysis. Within these constraints, most general linear system design techniques cannot be applied to the design of linear systems with dynamics defined by the Metzler matrix structure. Under these conditions, the Lyapunov method appears suitable for the analysis and synthesis of positive systems, as it allows one to easily include in the solution the constraints resulting from the positivity of the system, [7], [8].

To simplify some interpretations regarding fractional derivatives, a conformable fractional derivative is proposed in [9] which appears to meet the characteristics of the standard derivative. As a result, this concept has been incorporated into many studies, where, for example, [10] proposes a generalization of conformable fractional derivative. Scientists have recently focused more on conformable fractional derivatives in applications in stability analysis of conformable fractional systems, [11], non-fragile H_∞ observers for conformable fractional systems, [12], sensor fault reconstruction, [13] and consensus in linear conformable fractional multiagent systems [14]. To the author's knowledge, there are very few publications dealing with the design of observers for positive systems with conformable fractional derivative, [15], because to investigate the existence of positive solutions it is desirable to take into account a suitable representation of the parametric constraints, [16]. Application of the conformable fractional derivatives in physics are presented in [17], conformal prediction in biological systems is analyzed in [18].

To achieve a further extension of the assignment allowing to take into account the nonlinear function in the conformable fractional positive model of the system, its additive representation is assumed, known as Lipschitz-type nonlinearity [19]. Such a concept of observer design with respect to the existing nonlinearity allows the linear part of the description of the system dynamics to be extended by a nonlinear Lipschitz segment. This idea does not require significant modifications to the solution for the case of Metzler observer dynamics of a system with conformable fractional structure when it comes to representing this influence on the observer design strategy, which effectively represents the effect of the positivity of nonlinearity.

Based on this implementation concept, diagonal positive definite matrix variables and related diagonal matrix parametric constraints are introduced to achieve the design objectives. The design assumes the existence of a Lipschitz nonlinear function, which conditions the solution on a defined value of the Lipschitz constant in relation to a conformable fractional derivation in the positive model of the system.

The paper is structured as follows. Basic properties related to conformable fractional derivatives are presented in Section II. Section III presents basic procedures for observer design for a class of semilinear systems with conformable fractional derivatives and completes the conditions for the solvability of the observer design problem. The methodology for parameterizing the structure of Metzler matrices, the LMI for their representation, and the consequences of diagonal stabilization in the observer design task for semilinear conformable fractional positive systems is presented in Section IV. The results of the numerical design are presented in Section V, basic issues and some remarks regarding the synthesis of the state observer for this type of systems are discussed in Section VI.

Throughout the article, $\boldsymbol{x}^{\mathrm{T}}$, $\boldsymbol{X}^{\mathrm{T}}$ denote the transpose of the vector $\boldsymbol{x}$, and the matrix $\boldsymbol{X}$, diag$\,[\,\cdot\,]$ denotes a (block) diagonal matrix. For a square symmetric matrix $\boldsymbol{X} \prec 0$ means that $\boldsymbol{X}$ is a negative definite matrix, $\boldsymbol{I}_n$ denotes an identity matrix of order n, $\boldsymbol{X}^{-1}$, $\rho(\boldsymbol{X})$ denote the inverse and spectrum of eigenvalues of a square matrix $\boldsymbol{X}$, is $\boldsymbol{l}$ is an n-dimensional vector of the form $\boldsymbol{l} = [1 \cdots 1]^{\mathrm{T}}$. The indicator $\mathbb{R}$ ($\mathbb{R}_+$) denotes the set of (nonnegative) real numbers, $\mathbb{R}^{n\times n}$ ($\mathbb{R}_+^{n\times n}$) denotes the set of $n \times n$ (nonnegative) real matrices and $\mathbb{R}_{-+}^{n\times n}$ denotes the set of $n \times n$ matrices with strictly Metzler structure.

This work was not supported by any organization.

The author is with the Institute of Artificial Intelligence, Faculty of Electrical Engineering and Informatics, Technical University of Košice, 042 00 Košice, Slovakia dusan.krokavec@tuke.sk

II. Conformable Fractional Derivatives

In the following text, the given real function $\psi(t,\alpha)\geq 1$, $\alpha\in(0,1]$ is understood as a fractional conformable function, defined with the relation to the Gâteaux differential of $f(t+\varepsilon\psi(t,\alpha))$ at t in the direction $\psi()$ [20].

Definition 1: Let $\psi(t,\alpha)=1$, $\alpha\in(0,1]$ be a fractional conformable function then the general conformable fractional derivative at the point $t\geq 0$ of a given function $f(t):(0,\infty)\rightarrow\mathbb{R}$ is defined as, [10]

$$D^{\alpha}_{\psi}\big(f\big)(t)=\lim_{\epsilon\rightarrow 0+}\frac{f(t+\varepsilon\psi(t,\alpha))-f(t)}{\varepsilon} \tag{1}$$

if the limit exists.

Definition 2: The conformable fractional derivative of order $\alpha\in(0,1]$ at the point $t\geq 0$ of a given function $f(t):(0,\infty)\rightarrow\mathbb{R}$ is given as, [9],

$$D^{\alpha}\big(f\big)(t)=\lim_{\epsilon\rightarrow 0+}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon} \tag{2}$$

assuming that the right-hand side of this expression exists as a finite number.

Remark 1: It is evident that if $\psi(t,\alpha)=t^{1-\alpha}$ then $D^{\alpha}_{\psi}\big(\big)$ coincides with (2). Considering $\psi(t,\alpha)=te^{\epsilon}e^{t^{-\alpha}}-t$, [21] then,

$$D^{\alpha}\big(f\big)(t)=\lim_{\epsilon\rightarrow 0+}\frac{f(te^{\epsilon}e^{t^{-\alpha}})-f(t)}{\varepsilon} \tag{3}$$

Since according to [21] $\psi(t,\alpha)=\varepsilon t^{1-\alpha}+o(\varepsilon^2)$, this conformable fractional derivative has the same form as (2) if $o(\varepsilon^2)$ is omitted and these definitions can be considered as the same type of definition.

If $0<\alpha\leq 1$ and the functions f, g are α differentiable at the point $t>0$, then the conformable fractional derivative (2) satisfies for all scalars $a,b\in\mathbb{R}$ and f,g in the domain of D^{α} the following properties, [22],

$$\begin{aligned}D^{\alpha}\big(af+bg\big)&=aD^{\alpha}\big(f\big)+bD^{\alpha}\big(g\big)\\ D^{\alpha}\big(fg\big)&=fD^{\alpha}\big(g\big)+gD^{\alpha}\big(f\big)\end{aligned} \tag{4}$$

For $\alpha=1$ the conformable fractional derivative coincides with the classical first derivative $df(t)/dt$, [23], that is each function $f(t):(0,\infty)\rightarrow\mathbb{R}$ has a conformable fractional derivative if and only if it has a classical first derivative.

Definition 3: Given a function $f(t):(0,\infty)\rightarrow\mathbb{R}$ satisfying (2), then the conformable fractional Laplace transform of the $f(t)$ at a point $t>0$ of order α, $0<\alpha\leq 1$, is, [24],

$$F^{\alpha}(s)=\int_0^{\infty}e^{-s\frac{t^{\alpha}}{\alpha}}f(t)t^{\alpha-1}\mathrm{d}t \tag{5}$$

provided the integral exists.

Given a conformable fractional linear system, [25],

$$D^{\alpha}\big(\boldsymbol{q}\big)(t)=\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{u}(t) \tag{6}$$

where $\boldsymbol{q}(0)=\boldsymbol{q}_0\in\mathbb{R}^n$ is an initial state, $D^{\alpha}\big(\big)$ is the conformable fractional derivative operator (2), $\boldsymbol{q}(t)\in\mathbb{R}^n$, $\boldsymbol{u}(t)\in\mathbb{R}^r$, $\boldsymbol{A}\in\mathbb{R}^{n\times n}$, $\boldsymbol{B}\in\mathbb{R}^{n\times r}$, $t\in(0,\tau]$ then, applying the inverse conformable fractional Laplace transform, one can get the solution of system (6) in this way, [26],

$$\boldsymbol{q}_u(t)=e^{\boldsymbol{A}\frac{1}{t^{\alpha}}\alpha}\boldsymbol{q}_0+\int_0^t e^{\boldsymbol{A}\frac{1}{\alpha}(t^{\alpha}-s^{\alpha})}\boldsymbol{B}\boldsymbol{u}(s)s^{\alpha-1}\mathrm{d}s \tag{7}$$

III. Semilinear Conformable Fractional Structures

The system (5) in the semilinear conformable fractional structure is given by the relations

$$\begin{aligned}D^{\alpha}\big(\boldsymbol{q}\big)(t)&=\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{f}(\boldsymbol{q}(t))\\ \boldsymbol{y}(t)&=\boldsymbol{C}\boldsymbol{q}(t)\end{aligned} \tag{8}$$

where $D^{\alpha}\big(\big)$ is the conformable fractional derivative operator, $\boldsymbol{q}(t)\in\mathbb{R}^n$, $\boldsymbol{u}(t)\in\mathbb{R}^r$, $\boldsymbol{A}\in\mathbb{R}^{n\times n}$, $\boldsymbol{B}\in\mathbb{R}^{n\times r}$, $\boldsymbol{C}\in\mathbb{R}^{m\times n}$ and $\boldsymbol{f}(\boldsymbol{q}(t))\in\mathbb{R}^n$ is bounded and Lipschitz continuous.

The system

$$\begin{aligned}D^{\alpha}\big(\boldsymbol{q}_e\big)(t)&=\boldsymbol{A}\boldsymbol{q}_e(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{f}(\boldsymbol{q}_e(t))+\\ &\quad+\boldsymbol{J}(\boldsymbol{y}(t)-\boldsymbol{y}_e(t))\\ \boldsymbol{y}_e(t)&=\boldsymbol{C}\boldsymbol{q}_e(t)\end{aligned} \tag{9}$$

is a conformable fractional observer of the system (8), where $\boldsymbol{q}_e(t)\in\mathbb{R}^n$, $\boldsymbol{y}_e(t)\in\mathbb{R}^m$ and $\boldsymbol{f}(\boldsymbol{q}_e(t))\in\mathbb{R}^n$ are the observed vectors, .

Assumption 1: [27], [28] Let $\boldsymbol{h}(\boldsymbol{s}):\mathbb{X}\rightarrow\mathbb{R}^n$ be a vector function defined on $\mathbb{X}\subset\mathbb{R}^n$ and $\overline{\boldsymbol{s}}\in\mathbb{X}$. The function $\boldsymbol{h}(\boldsymbol{s})$ is said to satisfy locally Lipschitz condition at $\overline{\boldsymbol{s}}$ if there exist a neighborhood $\mathcal{S}$ of $\overline{\boldsymbol{s}}$ and $\eta>0$ such that

$$\|\boldsymbol{h}(\boldsymbol{x})-\boldsymbol{h}(\boldsymbol{y})\|\leq\eta\|\boldsymbol{x}-\boldsymbol{y}\|\ \text{for all}\ \boldsymbol{x},\boldsymbol{y}\in\mathcal{S} \tag{10}$$

In this sense it is assumed that there exists a known positive scalar $\eta\in\mathbb{R}_+$ for the known nonlinear terms $\boldsymbol{f}(\boldsymbol{q}(t))\in\mathbb{R}^n$, $\boldsymbol{f}(\boldsymbol{q}_e(t))\in\mathbb{R}^n$ to satisfy the Lipschitz condition

$$\|\boldsymbol{f}(\boldsymbol{q}(t))-\boldsymbol{f}(\boldsymbol{q}_e(t))\|\leq\eta\|\boldsymbol{q}(t)-\boldsymbol{q}_e(t)\| \tag{11}$$

Defining the vector errors

$$\begin{aligned}\boldsymbol{e}_q(t)&=\boldsymbol{q}(t)-\boldsymbol{q}_e(t)\\ \boldsymbol{e}_y(t)&=\boldsymbol{y}(t)-\boldsymbol{y}_e(t)=\boldsymbol{C}\boldsymbol{e}_q(t)\end{aligned} \tag{12}$$

and using (8) and (9) it yields

$$\begin{aligned}D^{\alpha}\big(\boldsymbol{e}_q\big)(t)=&D^{\alpha}\big(\boldsymbol{q}\big)(t)-D^{\alpha}\big(\boldsymbol{q}_e\big)(t)\\ =&\boldsymbol{A}\boldsymbol{q}_e(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{f}(\boldsymbol{q}_e(t))-\\ &-\boldsymbol{A}\boldsymbol{q}_e(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{f}(\boldsymbol{q}_e(t))+\\ &+\boldsymbol{J}(\boldsymbol{y}(t)-\boldsymbol{y}_e(t))\\ =&(\boldsymbol{A}-\boldsymbol{J}\boldsymbol{C})\boldsymbol{e}_q(t)+\boldsymbol{f}(\boldsymbol{q}(t))-\boldsymbol{f}(\boldsymbol{q}_e(t))\end{aligned} \tag{13}$$

which implies

$$D^{\alpha}\big(\boldsymbol{e}_q\big)(t)=\boldsymbol{A}_e\boldsymbol{e}_q(t)+\boldsymbol{f}(\boldsymbol{q}(t))-\boldsymbol{f}(\boldsymbol{q}_e(t)) \tag{14}$$

where

$$\boldsymbol{A}_e=\boldsymbol{A}-\boldsymbol{J}\boldsymbol{C} \tag{15}$$

Lemma 1: [29] (Petersen's Lemma) Let $\boldsymbol{X}\in\mathbb{R}^{n\times p}$, $\boldsymbol{Y}\in\mathbb{R}^{p\times n}$, $\boldsymbol{Z}=\boldsymbol{Z}^{\mathrm{T}}\in\mathbb{R}^{n\times n}$ then

$$\boldsymbol{Z}+\boldsymbol{X}\boldsymbol{Y}+\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}}\prec 0 \tag{16}$$

if and only if there exist real $\lambda>0$ such that

$$\boldsymbol{Z}+\lambda\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}+\lambda^{-1}\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}\prec 0 \tag{17}$$

Theorem 1: If for the observer dynamics (15) and for a known positive scalar $\eta \in \mathbb{R}_+$ from (11) there exists a symmetric positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{S} \in \mathbb{R}^{n\times m}$ such that the following set of LMIs

$$\boldsymbol{P} = \boldsymbol{P}^T \succ 0 \tag{18}$$

$$\begin{bmatrix} \boldsymbol{PA} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} - \boldsymbol{SC} - \boldsymbol{C}^{\mathrm{T}}\boldsymbol{S}^{\mathrm{T}} + \boldsymbol{I}_n & \eta\boldsymbol{P} \\ \eta\boldsymbol{P} & -\boldsymbol{I}_n \end{bmatrix} \prec 0 \tag{19}$$

is feasible, then $\boldsymbol{e}_q(t) \to \boldsymbol{0}$ if $t \to \infty$ and the observer gain matrix can be obtained as

$$\boldsymbol{J} = \boldsymbol{P}^{-1}\boldsymbol{S} \tag{20}$$

Proof: Considering the Lyapunov function

$$v(\boldsymbol{e}_q(t)) = \boldsymbol{e}_q^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_q(t) > 0 \tag{21}$$

where $\boldsymbol{P} \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, then the conformable fractional derivative of $V(\boldsymbol{e}_q(t))$ is, when using Lemma 1,

$$\begin{aligned} &D^{\alpha}\big(v(\boldsymbol{e}_q(t))\big) \\ &= D^{\alpha}\big(\boldsymbol{e}^{\mathrm{T}}{}_q(t)\big)\boldsymbol{P}\boldsymbol{e}_q(t) + \boldsymbol{e}_q^{\mathrm{T}}(t)\boldsymbol{P}D^{\alpha}\big(\boldsymbol{e}_q(t)\big) \\ &= (\boldsymbol{A}_e\boldsymbol{e}_q(t) + \boldsymbol{f}(\boldsymbol{q}(t)) - \boldsymbol{f}(\boldsymbol{q}_e(t)))^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}_q(t) + \\ &+ \boldsymbol{e}_q^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}_e\boldsymbol{e}_q(t) + \boldsymbol{f}(\boldsymbol{q}(t)) - \boldsymbol{f}(\boldsymbol{q}_e(t))) = \\ &= \boldsymbol{e}_q^{\mathrm{T}}(t)(\boldsymbol{P}\boldsymbol{A}_e + \boldsymbol{A}_e^{\mathrm{T}}\boldsymbol{P})\boldsymbol{e}_q(t) + \\ &+ \boldsymbol{e}_q^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{f}(\boldsymbol{q}(t)) - \boldsymbol{f}(\boldsymbol{q}_e(t))) + \\ &+ (\boldsymbol{f}(\boldsymbol{q}(t)) - \boldsymbol{f}(\boldsymbol{q}_e(t)))^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}_q(t) \\ &\le \boldsymbol{e}_q^{\mathrm{T}}(t)(\boldsymbol{P}\boldsymbol{A}_e + \boldsymbol{A}_e^{\mathrm{T}}\boldsymbol{P})\boldsymbol{e}_q(t) + \\ &+ \lambda^{-1}\boldsymbol{e}_q^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{P}\boldsymbol{e}_q(t) + \\ &+ \lambda(\boldsymbol{f}(\boldsymbol{q}(t)) - \boldsymbol{f}(\boldsymbol{q}_e(t)))^{\mathrm{T}}(\boldsymbol{f}(\boldsymbol{q}(t)) - \boldsymbol{f}(\boldsymbol{q}_e(t))) \\ &< 0 \end{aligned} \tag{22}$$

and, with respect to (11),

$$\begin{aligned} &D^{\alpha}\big(v(\boldsymbol{e}_q(t))\big) \\ &\le \boldsymbol{e}_q^{\mathrm{T}}(t)(\boldsymbol{P}\boldsymbol{A}_e + \boldsymbol{A}_e^{\mathrm{T}}\boldsymbol{P})\boldsymbol{e}_q(t) + \\ &+ \lambda^{-1}\boldsymbol{e}_q^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{P}\boldsymbol{e}_q(t) + \lambda\eta^2\boldsymbol{e}_q^{\mathrm{T}}(t)\boldsymbol{e}_q(t) \\ &< 0 \end{aligned} \tag{23}$$

Setting $\lambda\eta^2 = 1$ then $\lambda^{-1} = \eta^2$ and (23) gives

$$\begin{aligned} &D^{\alpha}\big(v(\boldsymbol{e}_q(t))\big) \\ &\le \boldsymbol{e}_q^{\mathrm{T}}(t)(\boldsymbol{P}\boldsymbol{A}_e + \boldsymbol{A}_e^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{I}_n + \eta^2\boldsymbol{P}^2)\boldsymbol{e}_q(t) \\ &< 0 \end{aligned} \tag{24}$$

and (24) implies, using only one tuning scalar variable $\eta > 0$,

$$\boldsymbol{P}\boldsymbol{A}_e + \boldsymbol{A}_e^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{I}_n + \eta^2\boldsymbol{P}^2 \prec 0 \tag{25}$$

Applying the properties of Schur's complement to (25) thus gives

$$\begin{bmatrix} \boldsymbol{P}(\boldsymbol{A} - \boldsymbol{JC}) + (\boldsymbol{A} - \boldsymbol{JC})^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{I}_n & \eta\boldsymbol{P} \\ \eta\boldsymbol{P} & -\boldsymbol{I}_n \end{bmatrix} \prec 0 \tag{26}$$

and with

$$\boldsymbol{S} = \boldsymbol{PJ} \tag{27}$$

(26) implies (19) and (27) gives (20). This concludes the proof. ■

IV. Semilinear Conformable Fractional Positive Structures

The relations (8) evolves the semilinear conformable fractional positive structure if $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is strictly Metzler, $\boldsymbol{B} \in \mathbb{R}_+^{n\times r}$, $\boldsymbol{C} \in \mathbb{R}_+^{m\times n}$ are nonnegative, $\boldsymbol{q}(t) \in \mathbb{R}_+^n$, $\boldsymbol{y}(t) \in \mathbb{R}_+^m$, $\boldsymbol{f}(\boldsymbol{q}(t)) \in \mathbb{R}_+^n$ are positive and $\boldsymbol{u}(t) \in \mathbb{R}^r$.

Definition 4: [30] A square matrix $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is strictly Metzler if all its diagonal elements are negative and all its off-diagonal elements are positive.

Proposition 1: [31] An autonomous conformable fractional positive system

$$D^{\alpha}\big(\boldsymbol{q}\big)(t) = \boldsymbol{A}\boldsymbol{q}(t) \tag{28}$$

where $\boldsymbol{q}(t) \in \mathbb{R}_+^n$ and $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is strictly Metzler and Hurwitz, is positive and asymptotically stable if there exist a diagonal positive definite matrix $\boldsymbol{W} \in \mathbb{R}^{n\times n}$ such that

$$\boldsymbol{W}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{W} \prec 0, \quad \boldsymbol{W} \succ 0 \tag{29}$$

The positivity of (28) means that $\boldsymbol{q}(0) \ge 0$ implies that $\boldsymbol{q}(t) \ge 0$ for all $t > 0$, the diagonally $\boldsymbol{W}$ forces diagonal stabilizability, [32].

Definition 5: [33] A matrix $\boldsymbol{L} \in \mathbb{R}^{n\times n}$ is a permutation matrix if exactly one item in each column and row is equal to 1 and all other elements are equal to 0.

If the circulant structure of $\boldsymbol{L} \in \mathbb{R}^{n\times n}$ is

$$\boldsymbol{L}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{n-1} \\ 1 & \boldsymbol{0}^{\mathrm{T}} \end{bmatrix}, \quad \boldsymbol{L}^{\mathrm{T}}\boldsymbol{L} = \boldsymbol{I}_n \tag{30}$$

the following proposition yields.

Proposition 2: The defined relations from Definition 4 to the strictly positive Metzler matrix $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ form n^2 parametric constraints

$$\boldsymbol{A} = \{a_{hl}\}_{h,l=1}^{n}, \ a_{ll} < 0, \ a_{hl,h\neq l} > 0 \ \forall \ hl \in \langle 1, n\rangle \tag{31}$$

and if the strictly Metzler matrix $\boldsymbol{A}$ is represented in this rhombic form constructed by circular shifts of its columns

$$\boldsymbol{A}_{\Theta} = \begin{bmatrix} -a_{11} & & & & \\ a_{21} & -a_{22} & & & \\ a_{31} & a_{32} & -a_{33} & & \\ \vdots & \vdots & \vdots & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & -a_{nn} \\ & a_{12} & a_{13} & \cdots & a_{1n} \\ & & a_{23} & \cdots & a_{2n} \\ & & & \ddots & \vdots \\ & & & & a_{n-1,n} \end{bmatrix} \tag{32}$$

then (32) imply the diagonal matrices $\boldsymbol{A}_{\Theta}(\nu+h,\nu) \in \mathbb{R}_+^{n\times n}$

$$\boldsymbol{A}_{\Theta}(\nu{+}h,\nu) = \mathrm{diag}\left[a_{1+h,1}\cdots a_{n,n-h}\, a_{1,n-h+1}\cdots a_{h,n}\right] \tag{33}$$

which redefine (31) by the following LMI set

$$\begin{aligned} \boldsymbol{A}_{\Theta}(\nu,\nu) \prec 0, \ h = 0 \\ \boldsymbol{A}_{\Theta}(\nu+h,\nu) \succ 0, \ h = 1,\ldots,n-1 \end{aligned} \tag{34}$$

Thus, respecting the diagonal matrices $\boldsymbol{A}_{\Theta}(\nu+h,\nu)$ and using powers of the matrix $\boldsymbol{L}$ for $h = l,\cdots,n-1$, the strictly Metzler matrix $\boldsymbol{A}$ is parameterized as, [34]

$$\boldsymbol{A} = \sum_{h=0}^{n-1}\boldsymbol{L}^h\boldsymbol{A}_{\Theta}(\nu+h,\nu) \tag{35}$$

Since the task is a solution for the estimator dynamics, it is necessary to generalize the principle of diagonal parameterization for the matrix $\boldsymbol{A}_e \in \mathbb{R}_{-+}^{n\times n}$ using the principle presented in [35].

Lemma 2: Let $\boldsymbol{A} \in \mathbb{R}_{-+}^{n\times n}$ be a strictly Metzler matrix, $\boldsymbol{C} \in \mathbb{R}_{+}^{m\times n}$ be nonnegative and $\boldsymbol{J} \in \mathbb{R}_{+}^{n\times m}$ be strictly positive. Using the set (34) of constraint diagonal matrices $\boldsymbol{A}_\Theta(\nu+h,\nu) \in \mathbb{R}_{+}^{n\times n}$, $h=0,\dots,n-1$, and constructing diagonal matrices for $i=1,\dots,m$, $\boldsymbol{C}_i \in \mathbb{R}_{+}^{n\times n}$, $\boldsymbol{J}_i \in \mathbb{R}_{+}^{n\times n}$, $\boldsymbol{J}_{ih} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_i\boldsymbol{L}^h \in \mathbb{R}_{+}^{n\times n}$ in relation to the following row and column representations

$$\boldsymbol{C} = \begin{bmatrix}\boldsymbol{c}_1^{\mathrm{T}}\\ \vdots\\ \boldsymbol{c}_m^{\mathrm{T}}\end{bmatrix}, \quad \boldsymbol{J} = \begin{bmatrix}\boldsymbol{j}_i \cdots \boldsymbol{j}_m\end{bmatrix} \tag{36}$$

such that

$$\begin{aligned}\boldsymbol{J}_i &= \operatorname{diag}\left[\boldsymbol{j}_i\right] = \operatorname{diag}\left[j_{1i}\cdots j_{ni}\right]\\ \boldsymbol{C}_i &= \operatorname{diag}\left[\boldsymbol{c}_i^{\mathrm{T}}\right] = \operatorname{diag}\left[c_{i1}\cdots c_{in}\right]\end{aligned} \tag{37}$$

then the matrix $\boldsymbol{A}_e \in \mathbb{M}_{-+}^{n\times n}$ defining the observer dynamics is parameterized as

$$\boldsymbol{A}_e = \sum_{h=0}^{n-1} \boldsymbol{L}^h \Big(\boldsymbol{A}_\Theta(\nu+h,\nu) - \sum_{i=1}^{m} \boldsymbol{J}_{ih}\boldsymbol{C}_i\Big) \tag{38}$$

Note that (38) also introduces Metzler parametric constraints for the observer dynamics defined by the matrix $\boldsymbol{A}_e$.

Considering the latter formulations, the following design conditions are obtained.

Theorem 2: If for the dynamics of the observer (15), where $\boldsymbol{A} \in \mathbb{R}_{-+}^{n\times n}$ is strictly a Metzler matrix, $\boldsymbol{C} \in \mathbb{R}_{+}^{m\times n}$ is a nonnegative matrix, $\boldsymbol{J} \in \mathbb{R}_{+}^{n\times m}$ is a strictly positive matrix, and for a known positive scalar $\eta \in \mathbb{R}_+$ compactly defined in (11) there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{S}_i \in \mathbb{R}^{n\times n}$ such that for $i=1,\dots,m$ and all $h \in [1, n-1]$ the following set of LMIs is feasible

$$\boldsymbol{P} \succ 0, \quad \boldsymbol{S}_i \succ 0 \tag{39}$$

$$\boldsymbol{P}\boldsymbol{A}(\nu,\nu) - \sum_{i=1}^{m} \boldsymbol{S}_i\boldsymbol{C}_i \prec 0 \tag{40}$$

$$\boldsymbol{P}\boldsymbol{L}^h\boldsymbol{A}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{i=1}^{m} \boldsymbol{S}_i\boldsymbol{L}^h\boldsymbol{C}_i\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{41}$$

$$\begin{bmatrix}\boldsymbol{P}\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{I}_n-\sum_{i=1}^{m}(\boldsymbol{S}_i\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_i+\boldsymbol{C}_i\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{S}_i) & * \\ \eta\boldsymbol{P} & -\boldsymbol{I}_n\end{bmatrix} \prec 0 \tag{42}$$

then $\boldsymbol{e}_q(t) \to \boldsymbol{0}$ if $t \to \infty$ and the observer gain matrix can be obtained as

$$\boldsymbol{J}_i = \boldsymbol{P}^{-1}\boldsymbol{S}_i, \ \boldsymbol{j}_i = \boldsymbol{J}_i\boldsymbol{l}, \ \boldsymbol{J} = \begin{bmatrix}\boldsymbol{j}_1 \cdots \boldsymbol{j}_m\end{bmatrix} \tag{43}$$

Here $*$ denotes a symmetric item in a symmetric matrix and the n-dimensional vector $\boldsymbol{l} \in \mathbb{R}^n$ is defined as

$$\boldsymbol{l}^{\mathrm{T}} = \begin{bmatrix}1\,1\cdots 1\end{bmatrix} \tag{44}$$

Proof: Starting from (27), then obviously one can get for $i=1,\dots,m$

$$\boldsymbol{S}_i = \boldsymbol{P}\boldsymbol{J}_i \tag{45}$$

and with $\boldsymbol{J}_{ih} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_i\boldsymbol{L}^h$, (45) and (38) then (26) can be rewritten as

$$\begin{bmatrix}\boldsymbol{P}\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{I}_n-\sum_{i=1}^{m}(\boldsymbol{S}_i\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_i+\boldsymbol{C}_i\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{S}_i & \eta\boldsymbol{P} \\ \eta\boldsymbol{P} & -\boldsymbol{I}_n\end{bmatrix} \prec 0 \tag{46}$$

which implies (42).

By premultiplying the left side by $\boldsymbol{P}$ and then multiplying the right side by $\boldsymbol{L}^{h\mathrm{T}}$, we can then get from (38) for any $h \in [1, n-1]$

$$\boldsymbol{P}\boldsymbol{L}^h\boldsymbol{A}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{i=1}^{m} \boldsymbol{P}\boldsymbol{J}_i\boldsymbol{L}^h\boldsymbol{C}_i\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{47}$$

which implies with respect to (45) the inequality (41).

If we set $h=0$ and multiply the left side of the inequality (38) by the matrix $\boldsymbol{P}$, we get

$$\boldsymbol{P}\boldsymbol{A}(\nu,\nu) - \sum_{i=1}^{m} \boldsymbol{P}\boldsymbol{J}_i\boldsymbol{C}_i \prec 0 \tag{48}$$

which implies with respect to (45) the inequality (40). This concludes the proof. ■

Remark 2: The LMI (40) reflects the structural constraints for the elements on the main diagonal of a strictly Metzler matrix $\boldsymbol{A}_e \in \mathbb{M}_{-+}^{n\times n}$, the LMI set (41) reflects the structural constraints for the off-diagonal elements of a strictly Metzler matrix $\boldsymbol{A}_e \in \mathbb{M}_{-+}^{n\times n}$ and (42) guarantees that $\boldsymbol{A}_e \in \mathbb{M}_{-+}^{n\times n}$ is Hurwitz.

V. Illustrative Example

The semilinear conformable fractional structure (8) is constructed on the matrix parameters

$$\boldsymbol{A} = \begin{bmatrix}-4 & 1 & 5\\ 1 & -5 & 1\\ 1 & 2 & -3\end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix}0\\1\\0\end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix}1\,0\,0\end{bmatrix}$$

$$\rho(\boldsymbol{A}) = \left\{-1.5451 \ \ -6.3610 \pm 0.3352\mathrm{i}\right\}$$

$$\boldsymbol{f}(\boldsymbol{q}(t)) = \begin{bmatrix}q_2(t)q_3(t) & 0 & 0\end{bmatrix}^{\mathrm{T}}$$

where $\|q_2(t)\| \le 0.3$, $\|q_3(t)\| \le 0.28$ while the state vector $\boldsymbol{q}(t)$ is nonnegative.

The Jacobian matrix is given by, [36],

$$\boldsymbol{J}_f = \left[\frac{\partial \boldsymbol{f}(\boldsymbol{q}(t))}{\partial \boldsymbol{q}(t)}\right] = \begin{bmatrix}0 & q_3 & q_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \ \boldsymbol{J}_{fm} = \begin{bmatrix}0 & 0.28 & 0.3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

and, with relation to (10), a Lipschitz constant can be taken as [37]

$$\left\|\frac{\partial \boldsymbol{f}(\boldsymbol{q}(t))}{\partial \boldsymbol{q}(t)}\right\|_2 \le \|\boldsymbol{J}_{fm}\|_2 = 0.4104, \ \|\boldsymbol{J}_{fm}\|_\infty = \eta = 0.58$$

To adapt for diagonal stability, one can obtain auxiliary parameters

$$\boldsymbol{L}=\begin{bmatrix}\boldsymbol{0}^{\mathrm{T}} & 1\\ \boldsymbol{I}_2 & \boldsymbol{0}\end{bmatrix},\ \boldsymbol{l}^{\mathrm{T}}=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$$

$$\boldsymbol{A}_{\Theta}(\nu,\nu)=\operatorname{diag}\begin{bmatrix}-4 & -5 & -3\end{bmatrix}$$

$$\boldsymbol{A}_{\Theta}(\nu+1,\nu)=\operatorname{diag}\begin{bmatrix}1 & 2 & 5\end{bmatrix},\ \boldsymbol{A}_{\Theta}(\nu+2,\nu)=\operatorname{diag}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

$$\boldsymbol{C}_1=\operatorname{diag}\begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

It can be verified that according to (35)

$$\begin{aligned}\boldsymbol{A}&=\boldsymbol{L}^0\boldsymbol{A}_{\Theta}(\nu,\nu)+\boldsymbol{L}^1\boldsymbol{A}_{\Theta}(\nu+1,\nu)+\boldsymbol{L}^2\boldsymbol{A}_{\Theta}(\nu+2,\nu)\\&=\begin{bmatrix}-4 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & -3\end{bmatrix}+\begin{bmatrix}0 & 0 & 5\\ 1 & 0 & 0\\ 0 & 2 & 0\end{bmatrix}+\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}\\&=\begin{bmatrix}-4 & 1 & 5\\ 1 & -5 & 1\\ 1 & 2 & -3\end{bmatrix}\end{aligned}$$

The set of design LMIs (39)-(42) is feasible in general and solving them in MATLAB environment using the standard SeDuMi package, [38], then for fixed $\eta=0.58$ there we have

$$\boldsymbol{P}=\operatorname{diag}\begin{bmatrix}0.2126 & 0.4754 & 0.6469\end{bmatrix}\succ 0$$

$$\boldsymbol{S}_1=\operatorname{diag}\begin{bmatrix}0.8321 & 0.2778 & 0.3813\end{bmatrix}\succ 0$$

As it is prescribed by (43), synthesizing $\boldsymbol{J}$ from these data then

$$\boldsymbol{J}=\begin{bmatrix}3.9138\\ 0.5844\\ 0.5895\end{bmatrix}>0$$

and (15) implies strictly Metzler and Hurwitz matrix $\boldsymbol{A}_e$, where

$$\boldsymbol{A}_e=\begin{bmatrix}-7.9138 & 1 & 5\\ 0.4156 & -5 & 1\\ 0.4105 & 2 & -3\end{bmatrix},\ \rho(\boldsymbol{A}_e)=\begin{Bmatrix}-1.7855\\ -5.9302\\ -8.1981\end{Bmatrix}$$

It is clear that the observer dynamics are set up such that the vector error of the state estimate converges to zero exponentially with time and it is obvious that for such a spectrum of eigenvalues, numerical simulation can be omitted. The potential problem of modifying the eigenvalues of the observer dynamics can be addressed by adding a quadratic constraint to the Lyapunov function, [39], [40].

This numerical example shows that the design problem can be reduced to an equivalent LMI set with a convex solution. Moreover, since $\boldsymbol{A}_e$ is a strictly Metzler structure, (9) implies a positive estimated vector $\boldsymbol{q}_e(t)$ for a given conformable fractional positive structure. It is evident that the desired parametric properties are achieved using the proposed constraint representation.

VI. Conclusions

This paper presents a methodology for solving the observer design problem for semilinear conformable fractional positive systems. The proposed set of LMIs reflects the Lipschitz condition on the nonlinearity of the system, the constraints arising from the Metzler matrix structure of the system dynamics, and the diagonal stabilizability consequences in the observer design task. The focus on structures with conformable fractional derivatives adds a certain degree of simplicity and intuitiveness to the presented methodology and its range of applications, making it a valuable tool in modeling and analysis of various engineering fields. It is evident that the synthesized observer for conformable fractional positive structures converges positively for $t>0$ if the matrix inequality constraints are satisfied.

The applicability of the proposed method is of course bound to the class of positive systems, where the matrix $\boldsymbol{A}$ must be strictly Metzler and the Lyapunov matrix $\boldsymbol{P}$ must be a diagonal positive definite matrix, while the advantages of conformal fractional differentiation may be limited for positive systems with complex memory. From this point of view an interesting practical class of positive systems based on dynamics in the form of ostensible Metzler matrices is represented by models for aircrafts, [41]. Since these tools offer effective research potential for demonstrating valid models for the analysis and synthesis of ostensible Metzler systems, these systems represent a potential area of practical application.

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