The Cost of Quality in Circular Economy: Balancing AI Investment and Profitability in CRD Waste Management

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***Abstract*— The management of Construction, Renovation, and Demolition (CRD) waste faces challenges due to low material sortability and limited effectiveness of manual sorting, which lead to impurity rates and financial losses. This study proposes the integration of Industry 4.0 technologies, AI-driven computer vision and robotic sorting into a Reverse Logistics Network Design (RLND) under uncertainty. We develop a two-stage stochastic Mixed-Integer Quadratically Constrained Program (MIQCP) to model the nonlinear relationship between investment and sorting efficiency, using a cost of quality function to reflect diminishing returns. To address the non-convexity of the model, a McCormick linearization approach is employed with Gurobi optimizer. Results from a case study in Quebec demonstrate that AI-enabled sorting improves effective recycling rates from a baseline of 0.47–0.58 to 0.72–0.85. However, this transition results in an 11.1% reduction in short-term profitability. These results illustrate the cost of quality required to guarantee the high-purity recycled materials necessary for a sustainable circular economy.**

***Index Terms*— Reverse logistics; Circular economy; Cost of quality, Stochastic optimization; MIQCP; Construction waste; McCormick linearization.**

I. INTRODUCTION

The shift toward a circular economy has made reverse logistics (RL) a strategic priority, particularly for the Construction, Renovation, and Demolition (CRD) sector. Generating up to 100 billion tons of waste globally each year, the CRD industry struggles with low recovery rates, with about 35% of this waste sent directly to landfills [1]. In Quebec alone, nearly half of the 3.51 million tons of CRD waste generated in 2021 was disposed of without sorting [2]. While Reverse Logistics Network Design (RLND) aims to optimize material recovery [3], final sorting at Collection Centers (CCs) relies heavily on inefficient manual processes, resulting in high impurity rates [4].

Despite advancements in automated sorting technologies capable of exceeding 90% classification accuracy [5], [6], their economic integration into strategic network planning remains underdeveloped. Crucially, most existing stochastic RLND models [7], [8] treat effective recycling rates as fixed, exogenous parameters. This simplification neglects the economic reality that achieving the high material purity required for secondary markets demands deliberate capital investment and operational trade-offs.

To bridge this gap, we extend a Two-Stage Stochastic Mixed-Integer Quadratically Constrained Program (MIQCP) that explicitly endogenizes the quality of sorted waste by introducing a Cost of Quality (COQ) function.

The study addresses the following research questions:

- How does investment in advanced sorting technology affect recycling performance in a stochastic CRD reverse logistics network?
- What is the trade-off between improving recycling quality and its impact on profitability?

II. LITERATURE REVIEW

Reverse logistics (RL) in the construction sector is critical for mitigating environmental impacts, yet its implementation is often limited by the complexities of waste sorting [9], [10]. Effective recycling depends on the purity of recovered materials. However, traditional sorting methods, such as manual labor and basic mechanical separation, suffer from low consistency, high labor intensity, and limited

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scalability [11]. While recent advancements in AI-enabled sorting and computer vision can achieve classification accuracies exceeding 90% [5], [6], their high capital costs and uncertain economic returns often deter adoption.

Despite these technological capabilities, the economic modeling of such investments within RLND remains underdeveloped. Most existing RLND models treat recycling rates as fixed parameters, neglecting the fact that higher material purity requires deliberate investment. [12] introduced the Cost of Quality into supply chain design, but their framework is deterministic and does not address stochasticity conditions.

In the context of CRD waste management purity levels directly dictate profitability [5]. Recycling performance operates under significant uncertainty due to fluctuating supply volumes, variable material composition, and dynamic supply locations. [8], [7] addressed these challenges through a two-stage stochastic RLND framework incorporating mobile Source Separation Centers. Nevertheless, their network remains constrained by the fixed efficiency limits of manual sorting at downstream CCs.

To address the lack of decision-support models jointly analyzing technology, quality, and uncertainty, this study directly extends the stochastic formulation of [7]. By incorporating a COQ function to capture diminishing returns to technological investment, we quantify the explicit economic trade-offs between short-term profitability and material purity in RLND.

III. Optimization Model Development

A. Proposed MIQCP Extension

TABLE I

Indices and Sets

Symbol	Description
$m \in M$	CRD material type
$s \in S$	CRD waste source
$o \in O$	Source Separation Center
$f \in F$	Collection Center
$c \in C$	Secondary market
$t \in T$	Planning period
$u \in U$	Scenarios

To address the fixed-efficiency limitation of manual sorting at CCs, we extend the base stochastic MILP of [7] by treating material quality as an endogenous decision variable, rather than reproducing its full formulation, we detail only the proposed investment dependent quality extension, which adds a continuous facility investment variable and an effective recycling-rate function with diminishing returns. The base MILP is transformed into a MIQCP. All standard flow conservation and capacity constraints remain unchanged. Tables I, II, III present all the used sets, parameters and variables in the new model respectively.

TABLE II

Parameters

Symbol	Description	Unit
$I^{\max}$	Maximum investment allowed per collection center	\$/facility
α_m	Linear efficiency coefficient	efficiency/\$
β_m	Diminishing-return coefficient	efficiency/\$^{2}
r^0_{mtu}	Baseline recycling efficiency	fraction
$r^{\min}$	Minimum required recycling efficiency	fraction
U^X_f	Upper bound on total input flow received by facility f	tons
L^X_f	Lower bound on total input flow equal to 0	tons
L^E_{mtu}	Lower bound on recycling efficiency, equal to r^0_{mtu}	fraction
U^E	Upper bound on recycling efficiency, fixed to 1	fraction

TABLE III

Decision Variables

Symbol	Description	Domain/Unit
I_f	Technological investment allocated to collection center f	$[0, I^{\max}]$, \$
E_{mftu}	Effective recycling efficiency after investment	$[r^0_{mtu}, 1]$
Z_{mftu}	McCormick variable for $X^{\text{in}}_{mftu}E_{mftu}$	tons
X_{msftu}	Flow from source s to collection center f	tons
X_{moftu}	Flow from source separation center o to collection center f	tons
X_{mfctu}	Recycled flow from collection center f to secondary market c	tons
β_f	1 if facility f is open/active, 0 otherwise	binary

New Constraints:

1) Investment is allowed only in active collection centers (where β_f is the existing binary facility-status variable):

$$I_f \leq I_{max} \cdot \beta_f \quad \forall f \in F \tag{1}$$

2) Inspired by [12], The linkage between the

technological investment decision (I_f) and the effective recycling efficiency (E_{mftu}) is explicitly modeled in Equation 2. The recycling efficiency obtained after investment is defined as a quadratic function where initial investments yield rapid quality gains (α_m), while excessive investment leads to technological saturation (β_m):

$$E_{mftu}(I_f) = r^0_{mtu} + \alpha_m I_f - \beta_m I_f^2 \quad (2)$$

Subject to the strict physical limit:

$$r^0_{mtu} \leq E_{mftu} \leq 1 \quad (3)$$

3) The total input flow of material m entering collection center f during period t under scenario u is defined as: $X_{total} = \sum_{s\in S} X_{msftu} + \sum_{o\in O} X_{moftu}$.
4) The static recycling-capacity constraint of the base model is then replaced by the following dynamic constraint. This ensures the output of recycled material does not exceed the effective sorting rate multiplied by the total input flow:

$$\sum_{c\in C} X_{mfctu} \leq \left(\sum_{s\in S} X_{msftu} + \sum_{o\in O} X_{moftu}\right) \cdot \left(r^0_{mtu} + \alpha_m I_f - \beta_m I_f^2\right) \quad (4)$$

for all $m \in M,\ f \in F,\ t \in T,\ u \in U$.
5) Finally, the objective function is updated to maximize the expected profit of the original network $[Z]$ minus the total technological investment costs:

$$\max \text{profit} = [Z] - \sum_{f\in F} I_f \quad (5)$$

IV. Resolution Method

This model was solved using Gurobi solver 12.0.3 integrated in python 3.9 on an i5-11 GEN CPU with 8GB RAM computer. The solver termination limit is set for 60 seconds. In order to model and solve the model in gurobi, we had to linearize the constraint 4 using McCormick Envelopes

A. McCormick Envelopes

McCormick envelopes were chosen instead of solving the model directly as a non-convex MIQCP or using more detailed piecewise linear approximations, because they transform the bounded bilinear terms into a simpler linear form that can be handled efficiently by Gurobi. This makes them a practical choice that keeps a good balance between model accuracy and solution time [13].

To linearize the following product $Z_{mftu} = (\sum_s X_{msftu} + \sum_{o\in O} X_{moftu}) \cdot E_{mftu}$. Becomes $Z_{mftu} = X_{total} \cdot E_{mftu}$

Lower Bounds

for all $m \in M,\ f \in F,\ t \in T,\ u \in U$

$$Z_{mftu} \geq L^E_{mtu} X_{total} \quad (6)$$

$$Z_{mftu} \geq U^X_f E_{mftu} + X_{total} - U^X_f \quad (7)$$

Upper Bounds

for all $m \in M,\ f \in F,\ t \in T,\ u \in U$

$$Z_{mftu} \leq U^X_f E_{mftu} + L^E_{mtu} X_{total} - U^X_f L^E_{mtu} \quad (8)$$

$$Z_{mftu} \leq X_{total} \quad (9)$$

Final Output Constraint 4 becomes for all $m \in M,\ f \in F,\ t \in T,\ u \in U$

$$\sum_{c\in C} X_{mfctu} \leq Z_{mftu} \quad (10)$$

B. Implementation details & Data

The bilinear term $X_{total}.E_{mftu}$ was linearized using McCormick envelopes. The lower bound of X_{total} is 0, while its upper bound U^X_f is obtained from the maximum input capacity of collection center f in the original RLND instance. Therefore, $0 \leq X_{total} \leq U^X_f$. The recycling-efficiency bounds are defined as $L^E_{mtu} = r^0_{mtu}$ and $U^E = 1$, where r^0_{mtu} is the baseline recycling rate and 1 represents the physical upper limit of 100% recycling efficiency.

TABLE IV

COMPUTATIONAL RESULTS COMPARISON

Metric	Basic Model	Investment Model	Variation
Objective (Profit)	\$12,271,125	\$10,906,720.05	-11.1%
CPU Time	0.10 s	0.36 s	+0.26 s
Model Type	MILP	MIQCP (Linearized)	-
Constraints	Linear	McCormick Envelopes	-

The baseline operational and stochastic data for the Quebec case study are drawn from the base model and [8].

Investment parameters were calibrated to capture diminishing returns of facility upgrades. I_{max} is capped at \$300,000, representing a realistic upper bound for deploying an AI-driven recycling robot [14]. The COQ function is governed by α_m, calibrated to yield a 10–20% efficiency improvement per \$100,000 invested [6], [15], and β_m, which models technological saturation near the physical accuracy limit. For the single material type tested, $\alpha_m = 2.0 \times 10^{-6}$ and $\beta_m = 3.3 \times 10^{-12}$.

The tested network instance consists of 10 sources (S), 8 existing and 5 potential CCs (F), 7 SSCs (O), evaluated over 5 planning periods (T) under 2 stochastic scenarios (U)

C. Experiment and Results

The model was solved with one instance, for comparison, we run two variations of the stochastic network design problem.

- Basic Model: The standard Two-Stage Stochastic MILP with fixed recycling parameters.
- Investment Model: The proposed model with continuous investment variable, linearized via McCormick Envelopes.

The McCormick linearization increased computational time from 0.10s to 0.36s, remaining negligible for the tested instance ($\leq$ 1 second). The solver chose to invest in existing Collection Centers to improve recycling rates rather than opening new CCs or Source Separation Centers.

Table IV shows that the investment model yields a profit decrease of approximately \$1.37 million. This difference represents the Cost of Quality (COQ).

While the basic model appears more profitable, it overestimates revenue by ignoring capital costs required to guarantee material quality. As shown in Figure 1, the investment model increases effective recycling rates from 0.475–0.575 to 0.72–0.85 (avg. gain of 0.25), with a quadratic relationship where \$100k investment yields 20% efficiency improvement. The resulting 11% profit gap represents the Cost of Quality, unrecovered within the five planning periods.

D. Sensitivity analysis

To assess model performance and technology impact, we conducted a sensitivity analysis on the two key parameters α_m and β_m, varying each in $[-20\%, -10\%, 0\%, +10\%, +20\%]$.

- **Variation of** α_m

The parameter α_m represents the initial rate of quality improvement per dollar invested. As shown in Figure 2 and Table V, When the technology is 20% less efficient, the binding $r^{min} = 0.60$ constraint forces the solver to over-invest across all facilities, making costs disproportionate to quality gains and reducing profit to \$1.65M, an 85% drop. The -10% case remains close to the base case profit. By contrast, higher technological efficiency (α_m = +20%) increases profit by 3.8% to 11.3\$ million and allows the solver to achieve the optimal effective recycling rate, around 0.77–0.79, with less investment. Overall, the results confirm the trade-off between investment cost and quality.

- **Variation of** β_m

The sensitivity analysis on the diminishing return coefficient β_m evaluates the network's resilience to technological saturation. A higher β_m indicates that the technology reaches its performance limit more rapidly as investment increases, and unlike α, it acts strictly as a penalty.

As shown in Figure 2 and Table VI, in the +20% scenario, total profit drops by 6.6% from 10.9\$ million to 10.18\$ million, while the recycling rate declines from 0.7844 to 0.7697. When β increases by 10%, the model is forced to raise investment in f2, f4, f8 from \$211k to \$253k, while the recycling

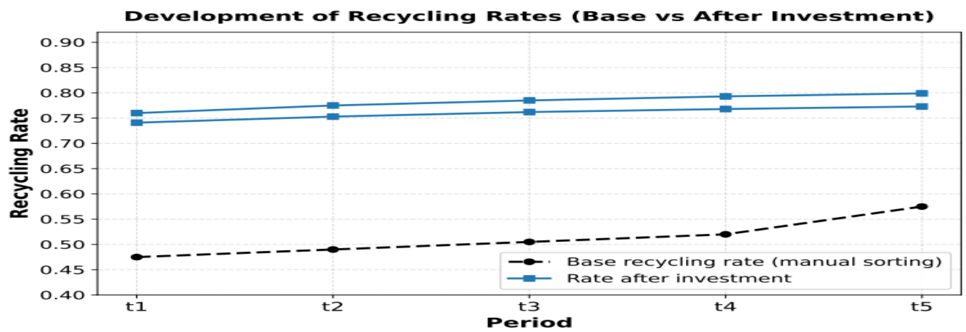

Fig. 1. Comparison between baseline manual sorting and technology-enhanced recycling rates

TABLE V
SENSITIVITY ANALYSIS OF LINEAR EFFICIENCY COEFFICIENT (α_m)

Variation	Profit ($)	Avg. Rate	f1	f2, f4, f8	f3, f7	f5
-20%	1,648,246	0.7075	$158k / 0.694	$237k / 0.718	$221k / 0.716	$221k / 0.716
-10%	9,605,255	0.7631	$194k / 0.749	$268k / 0.769	$237k / 0.765	$205k / 0.754
0% (Base)	**10,906,720**	**0.7844**	**$174k / 0.772**	**$211k / 0.799**	**$176k / 0.774**	**$176k / 0.774**
+10%	11,168,930	0.7869	$145k / 0.774	$167k / 0.799	$145k / 0.774	$158k / 0.789
+20%	11,322,993	0.7879	$126k / 0.774	$143k / 0.799	$126k / 0.774	$141k / 0.797

Note: Values for facilities are reported as investment / effective recycling rate.

TABLE VI
SENSITIVITY ANALYSIS OF DIMINISHING RETURN COEFFICIENT (β_m)

Variation	Profit ($)	Avg. Rate	f1	f2, f4, f8	f3, f7	f5
-20%	11,076,535	0.7869	$158k / 0.774	$181k / 0.799	$158k / 0.774	$171k / 0.789
-10%	11,005,823	0.7869	$166k / 0.774	$193k / 0.799	$166k / 0.774	$182k / 0.789
0% (Base)	**10,906,720**	**0.7844**	**$174k / 0.772**	**$211k / 0.799**	**$176k / 0.774**	**$176k / 0.774**
+10%	10,706,359	0.7822	$177k / 0.764	$253k / 0.798	$192k / 0.774	$192k / 0.774
+20%	10,186,916	0.7697	$174k / 0.752	$237k / 0.776	$221k / 0.773	$197k / 0.764

Note: Values for facilities are reported as investment / effective recycling rate.

rate still slightly decreases from 0.799 to 0.798. Overall, a higher β imposes a ceiling on quality and lowers profit ($720k loss).

V. CONCLUSION AND LIMITATIONS

This study extends stochastic RLND in the CRD industry by endogenizing sorting quality through an investment dependent function with diminishing returns. The resulting two-stage stochastic MIQCP was linearized with McCormick envelopes and solved efficiently. Results from the Quebec case study show that advanced sorting improves effective recycling rates from 0.47–0.58 to 0.72–0.85, but reduces short-term profitability by 11.1%, thereby quantifying the Cost of Quality. The findings highlight Collection Centers as the main quality bottleneck and show the value of treating sorting quality as an endogenous decision variable.This study is a proof of concept limited to a single network instance, two scenarios, and one material type, with investment-efficiency parameters derived from literature rather than empirical facility data. Additionally, the model assumes fixed secondary market prices and does not account for environmental objectives or technology maintenance costs. Future work should test larger and

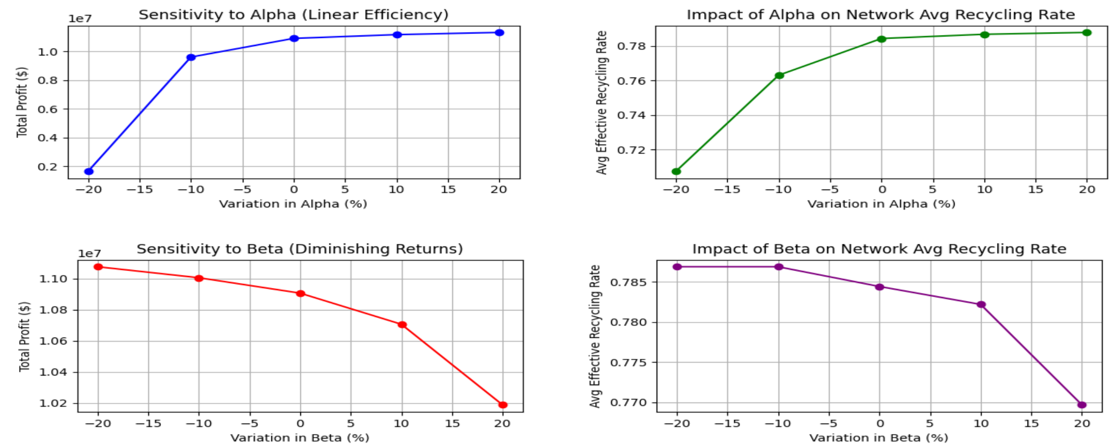

Fig. 2. Sensitivity analysis results: profit, and average recycling rate for α_m (top) and β_m (bottom).

more diverse instances, incorporate heterogeneous waste streams, dynamic recycled-material pricing, and broader policy scenarios.

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