A Case Study of the Heterogeneous Dial-A-Ride Problem in the Airport

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Abstract—**The Dial-a-Ride Problem (DARP) from or to airports is a variant of the vehicle routing problem. It should fulfill the transportation needs of people traveling between predetermined pick-up and drop-off points. This paper presents a brief review of the literature on the DARP and proposes a comparison of two approaches. In the first approach, the transportation for airport and hotel arrivals and departures involves separate vehicles. In the second approach, operations are insured simultaneously by the same vehicles. This research proposes a mathematical models using the CPLEX solver. The proposed models aim to find the optimal solution for each approach to help the company decide what types of vehicles to use and which route to follow according to its needs. Computational results based on real data and benchmark instances show the effectiveness of the proposed approaches.**

I. Introduction

Today, the use of transportation services has increased mainly in urban areas due to the expansion of cities. Consequently, it causes increased traffic congestion, limited parking spaces, and other environmental problems. One way to help tackle the matter is through the implementation of shared travel modes. It consists of moving a fleet of vehicles from a depot to satisfy the transport demands. Several people would request a vehicle to pick them up from a certain location and deliver them to another at a specific time based on airplane departure and arrival. This issue is a complex combinatorial optimization problem in cases of large instances.

The present paper explores the Dial-a-Ride Problem (DARP) in the context of airport shuttles, highlighting the difficulties involved in picking up and dropping off travelers between their accommodation and the airport. This service is more economical than what classic taxis offer. DARP is a variant of the Vehicle Routing Problem (VRP). In short, it is an optimization problem that should satisfy transport demands at a lower cost. In practice, several constraints should be considered, such as arriving to the airport 2 to 3 hours before the departure of the plane, the limited vehicle capacity, shortening the wait time of customers, etc. All these constraints make the problem more complex. The VRP is shown to be NP-hard in its basic version [1].

In this study, a static version of the DARP is considered, which means that all the requests are known in advance, while the dynamic version refers to the possibility of emerging new requests while the vehicle is touring. The airport shuttle is the combination of the three problems: DARP, which generalizes the Pick-up and Delivery problem (PDP), which is itself an extension of the VRP. The rest of the paper is organized as follows: Section 2 presents a brief literature review of the DARP. Section 3 defines three mathematical formulations of the problem. Section 4 reports numerical experiments. Finally, Section 5 presents the concluding remarks obtained from this work.

II. The State of the Art

Krumke et al. [2] classify the DARP as a variant of VRP, which generalizes the PDP (Pick-up and Delivery problem) where the transport of goods is replaced by the transport of persons.

The PDP is a particular vehicle routing problem that involves pickup and delivery (VRPPD). The VRP with simultaneous pickup and delivery (VRPSPD) received the most attention in research because it pertains to various sectors of the transport industry. The author provides an example of the simultaneous recovery of empty bottles and the delivery of soft drinks.

In the VRPSPD some clients have a pickup demand, some have a delivery demand, and at least one client has a simultaneous pickup and delivery demand. Louati et al. [3] resolves the VRPPD based on a real-life case study. The prototype developed is to find the best path for simultaneously delivering the company's products and collecting the empty boxes. The author also developed three different Mixed Integer Linear Programming (MILP) models designed for a mega poultry company. These models are a VRP with simultaneous pickup and delivery, a VRP with either simultaneous and/or separated pickup and delivery, and a VRP with simultaneous and flexible delivery and pickup.

Specific case studies by Mekonenn and Balasundaram [4] include the distribution of beverages and the collection of recyclable containers. The author used Clark-Wright's saving algorithm to solve the mathematical model based on real data from the Ethiopian MOHA juice factory.

The problem in [5] is formulated as MILP. This mathematical approach allows for the precise representation of the routing problem, including constraints and objectives related to logistics operations, specifically in rural areas of China. The authors proposed, a branch-and-cut algorithm is proposed for solving the literature on DARP has evolved to encompass various models applied in various use cases. The introduction of mixed-integer programming formulations and branch-and-cut algorithms marked a significant advancement in solving the DARP, allowing for more complex problem instances to be addressed.

The integration of time windows into the DARP has been a focal point in several research. Røpke et. al. [6] introduced new formulations for the pickup and delivery problem with time windows, which closely relate to the DARP. Their work emphasizes the necessity of managing the elapsed time between pick-ups and deliveries, a critical aspect of user satisfaction.

Parragh et al [7] also published a study of an H-DARP problem of patient transport using heterogeneous vehicles. The mathematical model proposed has been modified to take into

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account the heterogeneity of passengers and vehicles. The author adapted the heuristic "neighborhood variable" proposed in by Parragh [8] to solve this problem. The results obtained were compared with Cordeau and Laporte [9].

Rist et. al [10] contribute to the field by proposing a new mixed-integer programming formulation for the DARP, coupled with a branch and cut algorithm. Their formulation incorporates new valid inequalities, which are essential for improving the solution space and computational efficiency. This innovative approach underscores the importance of algorithmic advancements in solving complex routing problems.

Further expanding on the DARP, Rist et. al [10] also explore a Column Generation and Combinatorial Benders Decomposition Algorithm specifically for the Selective Dial-A-Ride Problem. This method showcases the versatility of DARP formulations and the potential for tailored solutions based on specific operational constraints.

The work of Gaul et al. [11] contributes to the optimization of ride pooling services by providing innovative formulations" tight (MILP), which lead to improved computational efficiency for the Dial-a-Ride Problem.

In the literature review, the majority of authors do not attempt to compare approaches. They rather concentrate on a given problem, add constraints and characteristics to it, and then propose solutions based on exact or heuristic methodologies. In addition, in order to solve the problem-based usage cases, most authors use an exact method to improve the computational efficiency of the Dial-a-Ride problem and related problems. This paper Evaluate integrated versus sequential pick-up and delivery models.

III. MATHEMATICAL FORMULATIONS OF THE DIFFERENT PROBLEMS

There are several ways to model the same problem, which essentially depend on the decision variables. The models proposed in this study are inspired by Louati et al. [3]. Three mathematical formulations are proposed to solve the following problems:

1. The pick-up model refers to the process of picking up travelers from their hotels and taking them to the airport.
2. The delivery model refers to the process of picking up clients at the airport and delivering them to their hotels.
3. The pick-up and delivery model describes simultaneously picking up clients to take them to the airport and delivering them to their hotels.

A. *Pick-up model*

The H-DRAP is modeled on a fully directed graph G = (V, A), where V = {1,..,N} designates the set of stations (hotel, tourist destination ...) and A the set of arcs.

For every station, an arc (i,j) connects them, however, an airport's arrival or departure is never connected. Considered is a non-negative travel time t_ij. Each client makes an advance pickup request. At the same hotel, multiple customers may be picked up at once. A diverse fleet, including taxis, shuttles (i.e., big taxis), and minibuses, has its origins at the airport (stations 1 and 2 for arrivals and departures).

Parameter:

m: Number of available vehicles,
v : The number of vehicles used,
n: Number of Stations (hotels, residences, and airports),
t_{ij}: Travel time from station i to station j,
s : Service time at station i,
p : Number of passengers picked up at station i.
CAP_k : Capacity of vehicle k,
Dplane : the flight departure time,

Decision Variables:

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle k travels from station i to station j} \\ 0 & \text{otherwise} \end{cases}$$

y_{ijk}: The number of passengers that vehicle k transported from i to j.
z_{ik} : The passengers to be picked up in station i by vehicle k
A_{ik} : Arrival time of vehicle at station i

The linear model is as follows:

$$Min \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} t_{ij} * X_{ijk} \tag{3.1}$$

Subject to:

$$\sum_{k=1}^{m}\sum_{i=1}^{n} X_{ijk} = 1 . \quad \forall i \neq j \ ; \ \forall \ j = 3 \dots n . \tag{3.2}$$

$$\sum_{k=1}^{m}\sum_{j=1}^{n} X_{ijk} = 1 . \quad \forall i \neq j \ ; \ \forall \ i = 3 \dots n . \tag{3.3}$$

$$\sum_{j=3}^{n} X_{1jk} \leq 1 , \quad \forall \ k = 1 \dots m . \tag{3.4}$$

$$\sum_{i=3}^{n} X_{i2k} \leq 1 , \quad \forall \ k = 1 \dots m . \tag{3.5}$$

$$\sum_{j=3}^{n}\sum_{k=1}^{m} X_{1jk} v . \tag{3.6}$$

$$\sum_{j=2}^{n}\sum_{k=1}^{m} X_{j2k} = v \quad . \tag{3.7}$$

$$\sum_{j>1}^{n} X_{ijk} - \sum_{j\neq 2}^{n} X_{jik} = 0 , \quad \forall \ i = 3 \dots n, k = 1 \dots m . \tag{3.8}$$

$$y_{ijk} \leq M * X_{ijk} \begin{cases} X_{jik} = 1 & \rightarrow y_{ijK} \leq M \\ X_{jik} = 0 & \rightarrow y_{ijK} \leq 0 \end{cases}, \quad \forall \ i,j = 1 \dots n, k = 1 \dots m . \tag{3.9}$$

$$Y_{ijk} \leq CAP_k, \quad \forall j = 3, \dots n . \tag{3.10}$$

$$\sum_{i\neq 2}^{n} y_{ijK} + z_{jk} = \sum_{i=2}^{n} y_{jik}, \quad j = 3, \dots n, \quad k = 1 \dots m. \tag{3.11}$$

$$\sum_{K=1}^{m} \sum_{i=3}^{n} y_{1ik} = 0 \tag{3.12}$$

$$\sum_{K=1}^{m} z_{ik} = p_i, \tag{3.13}$$

$$A_{1k} = 0; \tag{3.14}$$

$$A_{jk} \geq A_{ik} + S_i + T_{ij} - M * (1 - X_{ijk}), \tag{3.15}$$

$$A_{2k} \geq A_{jk} + S_j + T_{j2} - M * (1 - X_{j2k}), \quad A_2 \leq Dplane \quad \forall j = 3, \dots n, \quad k = 1 \dots m, \quad i = 1..n. \tag{3.16}$$

$$X_{ij} \in \{0,1\}; \quad 1 \leq i, j \leq n; \tag{3.17}$$

$$1 \leq k \leq m; \quad y_{ijk,,}, z_{ik} \geq 0$$

« *The choice of M>>0:* $M = \sum_{i,j=1}^{n} T_{ij} + \sigma$ »

The objective function (3.1) aims to reduce the total route duration. Constraints (3.2) and (3.3) ensure that each station is visited exactly once by a single vehicle (Taxi: Urban On-Demand Taxi, Shuttle: Intercity Shared Taxi, or Minibus: Shared Minibus). Constraints (3.4) and (3.5) make sure that each vehicle only departs from and returns to the airport once. Constraints (3.6) and (3.7) denote the number of vehicles used. The continuity of the routing is guaranteed by constraint (3.8): each vehicle entering a station must also leave it. Constraint (3.9) ensures that customers are transported from i to j only if the vehicle travels from i to j. Constraint (3.10) fixes the vehicle capacity.

The collection constraint (3.11) updates the number of customers in vehicle k. Constraint (3.12) ensures that the vehicles leave empty from the depot. Constraint (3.13) guarantees that all passengers are picked up. The time constraint (3.14) assumes that the tour starts at time 0. The constraint (3.15) determines the arrival time at station j when the vehicle visits station j after station i. The constraint (3.16) assumes that the arrival date at the airport A_2 needs to be earlier than the required arrival date. Finally, constraint (3.17) represents the type of the decision variables.

It is worth mentioning that the above program is run several times. At each run, the number of vehicles v is increased manually (constraint (3.6)). This operation is used to determine the ideal number of vehicles. Such an operation reduces the search space since we are using an equality constraint rather than an inequality constraint.

B. Delivery model

The "Delivery" Model is nearly the same as the previous one. The difference appears in constraints (3.11), (3.12), and (3.13), and some of the parameters and decision variables.

The parameters of the problem:

d_i: Number of clients dropped off at i.

Decision Variables:

$\boldsymbol{f_{ik}}$: The passengers dropped off at station i by the vehicle k

The delivery constraint:

$$\sum_{i\neq 2}^{n} y_{ijK} - f_{jk} = \sum_{i=2}^{n} y_{jik} \quad j = 3, \dots n, k = 1 \dots m. \tag{3.18}$$

$$\sum_{K=1}^{m} \sum_{i=3}^{n} y_{1ik} = d_1 \tag{3.19}$$

$$\sum_{K=1}^{m} f_{ik} = d_i, \quad \forall\ i = 2 \dots n \tag{3.20}$$

$$f_{ik} \geq 0 \tag{3.21}$$

The delivery constraint (3.18) defines the number of customers in the vehicle. Constraint (3.19) presents the number of persons picked up at the airport. Constraint (3.20) denotes the number of passengers delivered in station i by vehicle k, and (3.21) represents the decision variable.

C. Pick-up and delivery model

The "pick-up and delivery" model is nearly identical to the previous models. It represents, in some ways, the integration of the two previous models.

The collection contraints ((3.11), (3.12) and (3.13)) and the delivery contraints ((3.18), (3.19) and (3.20)) are modeled as follows:

$$\sum_{i\neq 2}^{n} y_{ijK} + z_{jk} - f_{jk} = \sum_{i=2}^{n} y_{jik} \quad \forall j = 3, \dots n, k = 1 \dots m. \tag{3.22}$$

$$\sum_{K=1}^{m} \sum_{i=3}^{n} y_{1ik} = d_1 \tag{3.23}$$

$$\sum_{K=1}^{m} \sum_{i=3}^{n} y_{i2k} = p_2 \tag{3.24}$$

$$\sum_{K=1}^{m} z_{ik} = p_i, \quad \forall\ i = 2 \dots n \tag{3.25}$$

$$\sum_{K=1}^{m} f_{ik} = d_i, \quad \forall\ i = 3 \dots n \tag{3.26}$$

$$\sum_{i\neq 2}^{N} y_{ijK} \geq f_{jk} \quad \forall\ i = 2 \dots n, k = 1 \dots m. \quad (3.27)$$

The Pick-up and delivery constraint (3.22) defines the number of customers in the vehicle. Constraint (3.23) determines the people picked up at the airport. Constraint (3.24) refers to the number of people picked up from all of the stations to reach the airport. While constraint (3.25) stands for the number of passengers to pick-up in station i by vehicle k. Constraint (3.26) represents the number of passengers to be delivered in station i by vehicle k. Last but not least, constraint (3.**27**) shows that the number of customers transported is higher than delivered for each station.

IV. COMPUTATIONAL EXPERIMENTS

A. *Actual data*

The first dataset is based on real-world data. This concerns hotels in the popular tourist destinations of Sousse and Monastir, Tunisia, as well as the Monastir airport.

The travel times (minutes) are determined using Google Maps API, and the tests are carried out with 50 and 100 customers for either the pick-up or the delivery models and with 100 and 200 customers for the mixed model. The number of hotels is fixed at 10 Hotels, and it offers 3 "taxis", 3 "Shuttle", and 2 "minibuses" (Table I). Mathematical Programming was implemented in the CPLEX 12.6 solver. It was run on an Intel Core i3 CPU 2.39 GHz, with 4 GO RAM.

TABLE I. DATA USED FOR RESOLVING MODELS

Models	Nombre Person P	Nombre Vehicle V	Capacity Vehicle
Pick-up	50/100	3 Taxis (T), 3 Shuttles (S) 2 Minibus (M)	Taxi= 4, Shuttle=8 Minibus= 40 places.
Delivery			
Pick-up and Delivery	100/200		

B. *Pick-upproblem results with actual data*

Initially, the number of travelers is set to 50, then to 100. A loop is used to increase the number of vehicles until convergence. Table II presents the results obtained by the "pick-up" model.

TABLE II. RESULTS OF THE PICK-UP MODEL

P \ V	50			P \ V	100		
	TT[b]	CPU[a]	Veh		TT	CPU	Veh
2	85	3,60	2M	5	155	3,46	3S,2M
3	91	3,25	1T,1S,1M	6	197	3.4	4S,2M
4	99	3,60	1T,2S,1M				

a. Run time in seconds, b Run time in minutes, T Taxi, S shuttles, M minibus

To pick up 50 people with 2 vehicles, the travel time is equal to 85 minutes; while for 3 and 4 vehicles, the total time increases to 91 and 99 minutes, respectively. With three vehicles, the characteristics of the fleet also changed, since a "minibus" was replaced by a taxi and a shuttle. The 4-vehicle option is worse than the 3-vehicle one. The decision maker could choose between the two vehicles or the three vehicles option, depending on the fleet readiness. To collect 100 people, the most appropriate number and type of vehicles would be 2 minibuses and 3 shuttles.

C. *Delivery problem results with actual data*

First, the tests are carried out with 50 people, then with 100 people. At each run, the number of vehicles increased until the change in result was no longer interesting.

Table III presents the results obtained by the "delivery" model.

TABLE III. RESULTS OF THE DELIVERY MODEL

P \ V	50			P \ V	100		
	TT[b]	CPU[a]	Veh		TT	CPU	Veh
2	91	4,48	2M	5	149	3.2	3S,2M
3	93	3,35	3S	6	198	3,8	4S,2M
4	97	2,7	1T,3M				

a. Run time in seconds, b Run time in minutes, T Taxi, S shuttles, M minibus

To drop off 50 people using two vehicles, the travel time is approximately 91 minutes. With 3 or 4 vehicles, the total time increases to 93 and 97 minutes, respectively, but the composition of the fleet is not the same. For this reason, it is recommended to use 2 minibuses or 3 shuttles, depending on the fleet availability. In each situation, the most appropriate number and type of vehicles that would be used to deliver either 100 should be 2 minibuses and 3 shuttles.

D. *Pick-up and delivery problem results with actual data*

First, 100 individuals are employed for the testing, and then 200. Table IV presents the results obtained by the "pick-up and delivery" model.

To transport 100 people with 2 vehicles, the travel time is equal to 85 minutes, while for 3 and 4 vehicles, the total time increases to 91 and 99 minutes, respectively.

TABLE IV. RESULTS OF THE PICK-UP AND DELIVERY MODEL

P \ V	100			P \ V	200		
	TT[b]	CPU[a]	Veh		TT	CPU	Veh
2	85	6,7	2M	5	141	9,7	3S,2M
3	91	8,3	1T,2M	6	155	5,2	4S,2M
4	99	6,8	2T,1S,1 M				

a. Run time in seconds, b Run time in minutes, T Taxi, S shuttles, M minibus

With 4 vehicles, a "minibus" is replaced by 2 taxis and a shuttle. The 3-vehicle option is worse than the 4-vehicle one. For this reason, it is recommended to choose between the two vehicles or the four vehicles option, depending on the fleet availability. In each situation, the most appropriate number and type of vehicles that would be used to pick up and deliver 200 should be 2 minibuses and 3 shuttles.

E. Results Comparison

The following table V summarizes the results of the three models.

TABLE V. THE RESULTS OF THE THREE MODELS

Models	P	Veh	TT
Pick-up	50	2M	85
	100	3S, 2M	155
Delivery	50	2M	91
	100	3B, 2M	149
Pick-up and Delivery	100	2M	85
	200	3B, 2M	141

The route duration to pick up only 50 people is equal to 85 minutes. It also requires 91 minutes to deliver 50 people. The sum of time in both situations is 170 minutes, which is more than a mixed tour (i.e. pickup and delivery) of 100 people simultaneously (85 minutes). It takes 4 minibuses to pick up and deliver 100 people separately. However, it requires only 2 minibuses in case of a pickup-up and delivery service. Similarly, 4 minibuses were used to pick up and deliver 200 people separately. In contrast, only 2 minibuses were used to pick up and deliver passengers during the same travel.

Analysis of results with benchmark data

With the purpose of testing these models, two benchmark instances are considered in a small-sized problem situation (Table VI). Both instances are analyzed by Augerat [12] and deal with the type of CVRP. They are also presented in the form of a file which includes: the demand of each node, the vehicle's capacity, the number of vehicles used, and the distance between the nodes.

TABLE VI. RESULTS WITH BENCHMARK DATA

Modeles	Instances	TT	CPU
Pick-up	P-n19-k2 **Optimal value: 212**	212	55.1
	P-n22-k2 **Optimal value: 216**	216	10
Delivery	P-n19-k2	212	32,5
	P-n22-k2	216	10,47

V. CONCLUSIONS

This study describes a heterogeneous dial-a-ride problem and proposes three models related to the real airport case that help the company in determining which vehicle types to use and which route to follow based on its demands.

Computational tests demonstrate that the problem can be resolved by utilizing a pickup-delivery service, which has been proved to be more effective.

In the future, we are looking to combine our proposed model with artificial intelligence algorithms like deep learning and neural networks to improve the quality of the solution and provide a more sustainable method.

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