Non-Singular Terminal Super-Twisting Control of 2-DOF Robotic Manipulator using Higher-Order Sliding Mode Observer

Thiyagarajan Anushalalitha, Rahul Kumar Sharma, and K. Sankar

Abstract—**In this paper, tracking problem is considered for a class of non-linear systems. Super-Twisting Control law has been designed for the system using only position information available by measurement. The technique uses a non-singular terminal sliding surface with fast convergence properties ensuring robustness in the presence of parametric uncertainties and external disturbances. The velocity information required in the control design is obtained from higher-order sliding mode observer which uses only position information for its estimation in finite time. The sliding surface and the tracking errors converge to zero in finite time. The designed control law is illustrated by considering an example of 2-Degrees-of-Freedom (DOF) robotic manipulator. The simulation results show accuracy and robustness of the designed technique.**

Index Terms—**Super-Twisting Control, Non-Singular Terminal Sliding Mode Control, Higher-Order Sliding Mode Observer, 2-DOF Robotic Manipulator.**

I. Introduction

Trajectory tracking is one of the most crucial and widely considered problems in designing non-linear systems [1]. Generally, the output variables of such systems are required to follow a time-varying trajectory to achieve the desired motion [2]. The resulting motion is required to be smooth for the trajectory tracking with the desired transient behaviour [3]. However, their performance is affected due to uncertain parameters and disturbances which may be present due to payload variations and friction [4]. The effects of parametric uncertainties and disturbances should be minimized for accurate and robust performance [2] [5]. Also, the tracking of the desired position should be achieved in finite time [6] [7]. For achieving the required objectives, the designed control law should be robust. This work considers the tracking problem for a class of non-linear systems for which Super-Twisting Control (STC) law is designed with non-singular terminal sliding surface which requires only position information [8] [9] [10]. The required velocity information is estimated in finite time by using Higher-Order-Sliding Mode (HOSM) Observer [6] [11] [12].

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Various robust control design techniques are used for non-linear systems to mitigate the effects of uncertainties and disturbances [13] - [15]. Sliding Mode Control (SMC) is a robust approach which deals with matched disturbances [16] - [19]. However, it produces undesired chattering effect which adversely affects the performance. It occurs because the control law contains signum function which is discontinuous. There are various techniques to minimize the effect of chattering. The saturation function is employed in place of signum function to avoid switching which is responsible for the chattering effect. HOSM control approaches are also designed to deal with chattering effect [6]. Super-Twisting Control (STC) is second-order SMC technique which produces continuous control input [14] [15] [20]. With this control law, the sliding surface with its derivative go to zero in finite time. However, the states go to zero asymptotically. It is important to explore some appoaches such that the states converge in finite time. The design of sliding surface has an important role which determines the dynamic behaviour as well as the convergence of the states of the system. So, it is important to explore some of the techniques of designing the sliding surface for the system.

The sliding surface may be selected in various ways to achieve the required performance [16]. Linear sliding surface is the basic design with which the states go to zero asymptotically. For achieving finite-time convergence of states to zero, various types of sliding surfaces have been designed using non-linear functions. Such sliding surfaces have improved transient response characteristics and convergence begaviour. These functions may be improved to include additional terms which result in faster convergence of the states. With conventional terminal sliding surface, the states go to zero in finite time [8]. However, it results in singularity issue with slow convergence of the states. This problem is overcome by using the non-singular terminal sliding surface avoiding singularity with the states going to zero in finite time [9] [10]. For achieving faster convergence of the states, a fast non-singular terminal sliding surface based approach is used in this work. Using this technique, the desired performance is achieved for the tracking problem of the system.

Generally, the information of all the states is required for designing the control law for the tracking problem of the system. However, all the states may not always be available by measurement through sensors. Some state of the system may not be measured due to inaccesibility, Also, individual sensors may be required to measure each of the states. This increases the overall cost as the order of the system increases. Also, the measurements of the states through sensors introduces noise in the system for which signal conditioning circuits are required to be designed. Generraly, only output information is available by measurement in actual systems. Using the output information, the remaining states may be estimated by designing observers and differentiators [21] [22]. There are various model based observer design techniques. The basic design is the Luenberger observer which has a linear structure with asymptotic convergence of the estimation errors. There are many non-linear observer design techniques applicable for various class of non-linear systems. Sliding Mode Observers are robust non-linear observers which may be used to obtain the desired behaviour of the estimation errors. HOSM observer is widely used for states and disturbance estimation for its compensation in the control law. It is important to mention that STC used with Super-Twisting Observer generates discontinuous control law which is not suitable for implementation in experimental setups [15]. In the same work, it is concluded that STC uses with HOSM observer produces continuous control law. In this work, STC is designed for trajectory tracking problem of 2-DOF robotic manipulator based on [15]. It is assumed that only position output is known for design. The estimated states are provided in finite time by the HOSM observer. The obtained information of the states are used in the sliding surface required in the control law. Using this approach, high accuracy and robustness is achieved in the performance of the robotic system. With the designed control law, the states go to zero in finite time.

There are many control approaches for tracking problem to achieve acuuracy and robustness with the desired behaviour. SMC based technique is used for the robotic system in [23]. It uses saturation function in place of the discontinuous signum function to overcome the chattering issue providing the desired response. A regressor based SMC approach with exponential convergence of the tracking error dynamics is designed and implemented on experimental setup in [24]. An adaptive STC based technique is used for 2-DOF robotic manipulator in [25]. A neural network based SMC is designed in [26]. In this scheme, the maximum value of the disturbances is obtained by using neural network and is used for disturbance compensation to achieve the required performance. A continuous fast non-singular terminal SMC is used with finite-time estimation of uncertainty using an observer in [27]. A terminal SMC is formulated using adaptive disturbance observer to obtain the disturbances to achieve zero tracking error of the system in finite time [28]. A non-singular fast terminal SMC using HOSM observer is used to achieve fast tracking of position error in [29]. STC is used with HOSM observer estimating the uncertainties resulting in accurate position tracking in [30].

The paper continues as follows: Section II presents the equations related to the motion of the robotic manipulator with parametric uncertainty. Section III describes conventional Non-Singular Terminal Super-Twisting Control law. Section IV discusses Super-Twisting Control using fast non-singular terminal SMC with velocity information obtained from HOSM Observer. Section V gives the results of the simulation and Section VI concludes.

II. Modeling of 2-DOF Robotic Manipulator

The dynamic model of 2-DOF robotic manipulator is [1] [4],

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) = \tau_i + \tau_d \tag{1}$$

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad \tau_i = \begin{bmatrix} \tau_{i1} \\ \tau_{i2} \end{bmatrix} \tag{2}$$

Here, $q \in \mathbb{R}^2$ is the joint angular position of the manipulator. The two joint variables are q_1 and q_2, respectively. $\dot{q} \in \mathbb{R}^2$ is the velocity vector containing the two joints velocities. Similarly, $\ddot{q} \in \mathbb{R}^2$ represents acceleration of the two joints. $\tau_i = [\tau_{i1}, \tau_{i2}]^\top \in \mathbb{R}^2$ denotes joint torque. $\tau_d \in \mathbb{R}^2$ contains the disturbances and uncertainties as the robot interacts with the environment. $M_0(q) \in \mathbb{R}^{2\times 2}$ denotes the nominal inertia matrix which is positive definite and symmetric. $C_0(q,\dot{q})\dot{q} \in \mathbb{R}^2$ contains nominal Coriolis and centrifugal forces and $G_0(q) \in \mathbb{R}^2$ is nominal gravitational vector. The matrices $M_0(q), C_0(q,\dot{q}), G_0(q)$ are given by,

$$M_0(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix} \tag{3}$$

$$\begin{aligned} M_{11} &= J_1 + J_2 + m_1 r^2 + m_2(l_1^2 + r_2^2) + 2\cos(q_2) m_2 l_1 r_2 \\ M_{12} &= M_{21} = J_2 + m_2 r_2^2 + m_2 l_1 r_2 \cos(q_2) \\ M_{22} &= J_2 + m_2 r_2^2 \end{aligned} \tag{4}$$

$$\begin{aligned} C_0(q,\dot{q}) &= \begin{bmatrix} -m_2 l_1 r_2 \sin(q_2)\dot{q}_2 & -m_2 l_1 r_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ m_2 l_1 r_2 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \\ C_0(q,\dot{q})\dot{q} &= \begin{bmatrix} -m_2 l_1 r_2 \sin q_2 \dot{q}_2 (2\dot{q}_1 + \dot{q}_2) \\ -m_2 l_1 r_2 \sin q_2 \dot{q}_1^2 \end{bmatrix} \\ G_0(q) &= \begin{bmatrix} (m_1 r_1 + m_2 l_1) g \cos(q_1) + m_2 r_2 g \cos(q_1 + q_2) \\ m_2 r_2 g \cos(q_1 + q_2) \end{bmatrix} \end{aligned} \tag{5}$$

Considering parametric uncertainties,

$$\begin{aligned} (M_0(q) + \Delta M(q))\ddot{q} + &(C_0(q,\dot{q}) + \Delta C(q,\dot{q}))\dot{q} \\ &+ G_0(q) + \Delta G(q) = \tau_i + \tau_d \end{aligned} \tag{6}$$

where, $\Delta M(q), \Delta C(q,\dot{q}), \Delta G(q)$ represent the parameter perturbations. Generraly, these perturbations are considered in the parameters for a realistic representation of the system.
Rearranging,

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) = \tau_i + \tau_{dl} \tag{7}$$

where, $\tau_{dl} = \tau_d - \Delta M(q)\ddot{q} - \Delta C(q,\dot{q})\dot{q} - \Delta G(q)$ is the lumped disturbance.
The model is,

$$\ddot{q} = M_0^{-1}(q)(\tau_i - C_0(q,\dot{q})\dot{q} - G_0(q) + \tau_{dl}) \tag{8}$$

III. Non-Singular Terminal Super-Twisting Control Design

Consider the class of non-linear systems,

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + g(x)\nu + d(t,x)\end{aligned} \tag{9}$$

where, $f(x) : \mathbb{R}^2 \to \mathbb{R}^2$ is the non-linear system function, $g(x) : \mathbb{R}^2 \to \mathbb{R}^2$ is the input non-linear function. $\nu \in \mathbb{R}$ is nominal input. $d(t,x)$ is bounded disturbance. $x = [x_1, x_2]^\top \in \mathbb{R}^2$ contains the states. Consider that x is completely known. Using the method of Feedback Linearization [1],

$$\nu = g^{-1}(x)(u - f(x)) \tag{10}$$

provided $g^{-1}(x)$ exists. Here, u is the actual control input. Substituting the expression of ν given by Eq. (10) in the system described by Eq. (9), the dynamics reduces to,

$$\dot{x}_1 = x_2 \tag{11a}$$

$$\dot{x}_2 = u + d \tag{11b}$$

Selecting the non-singular terminal sliding surface as [8],

$$s = x_1 + \frac{1}{\beta} x_2^{p/q} \tag{12}$$

where, β is a positive constant and p, q are positive odd integers such that $1 < p/q < 2$. Differentiating Eq. (12),

$$\dot{s} = \dot{x}_1 + \frac{1}{\beta}\frac{p}{q} x_2^{p/q-1}\dot{x}_2 \tag{13}$$

Here, the sliding surface derivative, $\dot{s}$ does not contain any term which may result is the problem of singularity. It has an advantage over the conventional terminal SMC which suffers from the issue of singularity. The reduced-order system becomes,

$$\begin{aligned}\dot{x}_1 &= \beta(s - x_1)^{q/p} \\ \dot{s} &= \dot{x}_1 + \frac{1}{\beta}\frac{p}{q} x_2^{p/q-1}\dot{x}_2\end{aligned} \tag{14}$$

Using (11a) in (14), the equation becomes,

$$\dot{s} = x_2 + \frac{1}{\beta}\frac{p}{q} x_2^{p/q-1}\dot{x}_2 \tag{15}$$

Using Eq. (11b) in Eq. (15), the dynamics of s becomes,

$$\dot{s} = x_2 + \frac{1}{\beta}\frac{p}{q} x_2^{p/q-1}(u + d) \tag{16}$$

It can be expressed as,

$$\dot{s} = x_2 + \frac{1}{\beta}\frac{p}{q} x_2^{p/q-1}u + d' \tag{17}$$

where, $d' = \frac{1}{\beta}\frac{p}{q}x_2^{p/q-1}d$. The control law is,

$$u = -\beta\frac{q}{p}x_2^{1-p/q}\big(1 + k_1|s|^{1/2}\operatorname{sign}(s) + k_2\int_o^t \operatorname{sign}(s)d\tau\big) \tag{18}$$

Substituting the control law in Eq. (17),

$$\dot{s} = -k_1|s|^{1/2}\operatorname{sign}(s) - k_2\int_o^t \operatorname{sign}(s)d\tau + d' \tag{19}$$

It may be expressed as,

$$\begin{aligned}\dot{s} &= -k_1|s|^{1/2}\operatorname{sign}(s) + v \\ \dot{v} &= -k_2\operatorname{sign}(s) + \dot{d}'\end{aligned} \tag{20}$$

So, s becomes zero in finite time. The reduced-order dynamics is,

$$\begin{aligned}\dot{x}_1 &= -\beta x_1^{q/p} \\ x_2 &= -\beta x_1^{q/p}\end{aligned} \tag{21}$$

So, x_1 and x_2 become zero in finite time [7]. The equations are,

$$\begin{aligned}\dot{q}_1 &= q_2 \\ \dot{q}_2 &= M_0^{-1}(q)(\tau_i - C_0(q,\dot{q})\dot{q} - G_0(q) + \tau_{dl})\end{aligned} \tag{22}$$

Defining the trajectory tracking errors as,

$$e_{q_1} = q_1 - q_d, \quad e_{q_2} = q_2 - \dot{q}_d$$

The error dynamics becomes,

$$\dot{e}_{q_1} = e_{q_2} \tag{23a}$$

$$\dot{e}_{q_2} = M_0^{-1}(q)(\tau_i - C_0(q,\dot{q})\dot{q} - G_0(q) + \tau_{dl}) - \ddot{q}_d \tag{23b}$$

Designing the fast non-singular terminal sliding surface as [8],

$$\hat{s} = e_{q_1} + \frac{1}{\beta}\hat{e}_{q_2}^{p/q}\operatorname{sign}(\hat{e}_{q_2}) \tag{24}$$

where, $\hat{e}_{q_2} = \hat{q}_2 - \dot{q}_d$ depends on the estimated state provided by the observer. So, Eq. (23a) becomes,

$$\begin{aligned}\dot{e}_{q_1} &= q_2 - \dot{q}_d \\ &= e_2 + \hat{q}_2 - \dot{q}_d \\ &= e_2 + \hat{e}_{q_2} \\ &= e_2 + \beta^{q/p}|\hat{s} - e_{q_1}|^{q/p}\operatorname{sign}(\hat{s} - e_{q_1})\end{aligned} \tag{25}$$

Differntiating Eq. (24),

$$\begin{aligned}\dot{\hat{s}} &= \dot{e}_{q_1} + \frac{1}{\beta}\frac{p}{q}|\hat{e}_{q_2}|^{p/q-1}\dot{\hat{e}}_{q_2} \\ &= e_{q_2} + \frac{1}{\beta}\frac{p}{q}|\hat{q}_2 - \dot{q}_d|^{p/q-1}(\dot{\hat{q}}_2 - \ddot{q}_d) \\ &= q_2 - \dot{q}_d + \frac{1}{\beta}\frac{p}{q}|\hat{q}_2 - \dot{q}_d|^{p/q-1}(\dot{\hat{q}}_2 - \ddot{q}_d) \\ &= e_2 + \hat{q}_2 - \dot{q}_d + \frac{1}{\beta}\frac{p}{q}|\hat{q}_2 - \dot{q}_d|^{p/q-1}(\dot{\hat{q}}_2 - \ddot{q}_d)\end{aligned} \tag{26}$$

The reduced-order system dynamics becomes,

$$\begin{aligned}\dot{e}_{q_1} &= e_2 + \beta^{q/p}|\hat{s} - e_{q_1}|^{q/p}\operatorname{sign}(\hat{s} - e_{q_1}) + e_2 \\ \dot{\hat{s}} &= e_2 + \hat{q}_2 - \dot{q}_d + \frac{1}{\beta}\frac{p}{q}|\hat{q}_2 - \dot{q}_d|^{p/q-1}(\dot{\hat{q}}_2 - \ddot{q}_d)\end{aligned} \tag{27}$$

IV. Non-Singular Terminal Super-Twisting Control using Higher-Order Sliding Mode Observer

HOSM observer is used to obtain the unknown velocity $q_2 = \dot{q}$. The estimated velocity information, $\hat{q}_2$ is used by the designed control law for trajectory tracking. With Super-Twisting Observer, the resulting control law becomes discontinuous as discussed in [15]. So, HOSM Observer is designed in this section which achieves continuous input based on [15]. Using HOSM observer, the information of $\hat{x}_2$ is achieved in finite time and is provided to STC based control law. Using Eq. (22), the observer dynamics for the nominal model is designed as [6],

$$\begin{aligned}\dot{\hat{q}}_1 &= \hat{q}_2 + \alpha_1|q_1-\hat{q}_1|^{\frac{2}{3}}\operatorname{sign}(q_1-\hat{q}_1)\\ \dot{\hat{q}}_2 &= \hat{q}_3 + \alpha_2|q_1-\hat{q}_1|^{\frac{1}{3}}\operatorname{sign}(q_1-\hat{q}_1)\\ &\qquad + M_0^{-1}(q)(\tau_i - C_0(q,\dot{q})\dot{q} - G_0(q))\\ \dot{\hat{q}}_3 &= \alpha_3\operatorname{sign}(q_1-\hat{q}_1)\end{aligned} \tag{28}$$

Defining the error variables as,

$$\begin{aligned}e_1 &:= q_1 - \hat{q}_1\\ e_2 &:= q_2 - \hat{q}_2\end{aligned} \tag{29}$$

The error dynamics takes the form,

$$\begin{aligned}\dot{e}_1 &= -\alpha_1|e_1|^{\frac{2}{3}}\operatorname{sign}(e_1) + e_2\\ \dot{e}_2 &= -\alpha_2|e_1|^{\frac{1}{3}}\operatorname{sign}(e_1) - \hat{q}_3 + d\\ \dot{\hat{q}}_3 &= \alpha_3\operatorname{sign}(e_1)\end{aligned} \tag{30}$$

Defining new variable, $e_3 := -\hat{q}_3 + d$, the error dynamics is,

$$\begin{aligned}\dot{e}_1 &= -\alpha_1|e_1|^{\frac{2}{3}}\operatorname{sign}(e_1) + e_2\\ \dot{e}_2 &= -\alpha_2|e_1|^{\frac{1}{3}}\operatorname{sign}(e_1) + e_3\\ \dot{e}_3 &= -\alpha_3\operatorname{sign}(e_1) + \dot{d}\end{aligned} \tag{31}$$

The error variables, e_1, e_2, e_3 become zero in finite time [6]. Rewriting the reduced-order dynamics of 2-DOF robotic manipulator,

$$\begin{aligned}\dot{e}_{q_1} &= e_2 + \beta^{q/p}|\hat{s} - e_{q_1}|^{q/p}\operatorname{sign}(\hat{s} - e_{q_1})\\ \dot{\hat{s}} &= e_2 + \hat{q}_2 - \dot{q}_d + \frac{1}{\beta}\frac{p}{q}|\hat{q}_2 - \dot{q}_d|^{p/q-1}(\dot{\hat{q}}_2 - \ddot{q}_d)\end{aligned} \tag{32}$$

Using the error dynamics, $\dot{\hat{q}}_2$ from Eq. (30) in the reduced-order system dynamics (32),

$$\begin{aligned}\dot{e}_{q_1} &= e_2 + \beta^{q/p}|\hat{s} - e_{q_1}|^{q/p}\operatorname{sign}(\hat{s} - e_{q_1})\\ \dot{\hat{s}} &= e_2 + \hat{q}_2 - \dot{q}_d + \frac{1}{\beta}\frac{p}{q}|\hat{q}_2 - \dot{q}_d|^{p/q-1}\big(\hat{q}_3 + \alpha_2|e_1|^{\frac{1}{3}}\operatorname{sign}(e_1)\\ &\quad + M_0^{-1}(q)(\tau_i - C_0(q,\dot{q})\dot{q} - G_0(q)) - \ddot{q}_d\big)\end{aligned} \tag{33}$$

Substituting the expression of $\hat{q}_3$ from (30),

$$\begin{aligned}\dot{e}_{q_1} &= e_2 + \beta^{q/p}|\hat{s} - e_{q_1}|^{q/p}\operatorname{sign}(\hat{s} - e_{q_1})\\ \dot{\hat{s}} &= e_2 + \hat{q}_2 - \dot{q}_d + \frac{1}{\beta}\frac{p}{q}|\hat{q}_2 - \dot{q}_d|^{p/q-1}\\ &\quad\Bigg[\alpha_2|e_1|^{\frac{1}{3}}\operatorname{sign}(e_1) + \int_0^t \alpha_3\operatorname{sign}(e_1)d\tau\\ &\quad + M_0^{-1}(q)(\tau_i - C_0(q,\dot{q})\dot{q} - G_0(q)) - \ddot{q}_d\Bigg]\end{aligned} \tag{34}$$

Designing the control input as [15],

$$\begin{aligned}\tau_i &= C_0(q,\dot{q})\dot{q} + G_0(q) + M_0(q)\\ &\quad\Bigg[-\alpha_2|e_1|^{\frac{1}{3}}\operatorname{sign}(e_1) - \int_0^t \alpha_3\operatorname{sign}(e_1)d\tau + \ddot{q}_d\\ &\quad - \beta\frac{q}{p}|\hat{q}_2 - \dot{q}_d|^{1-p/q}\big(-k_1|\hat{s}|^{\frac{1}{2}}\operatorname{sign}(\hat{s})\\ &\quad - \int_0^t k_2\operatorname{sign}(\hat{s})d\tau + \hat{q}_2 - \dot{q}_d\big)\Bigg]\end{aligned} \tag{35}$$

Using the control input (35) in Eq. (34),

$$\begin{aligned}\dot{e}_{q_1} &= e_2 + \beta^{q/p}|\hat{s} - e_{q_1}|^{q/p}\operatorname{sign}(\hat{s} - e_{q_1})\\ \dot{\hat{s}} &= e_2 - k_1|\hat{s}|^{\frac{1}{2}}\operatorname{sign}(\hat{s}) - \int_0^t k_2\operatorname{sign}(\hat{s})d\tau\end{aligned} \tag{36}$$

It may be expressed as,

$$\begin{aligned}\dot{e}_{q_1} &= e_2 + \beta^{q/p}|\hat{s} - e_{q_1}|^{q/p}\operatorname{sign}(\hat{s} - e_{q_1})\\ \dot{\hat{s}} &= e_2 - k_1|\hat{s}|^{\frac{1}{2}}\operatorname{sign}(\hat{s}) + v\\ \dot{v} &= -k_2\operatorname{sign}(\hat{s})\end{aligned} \tag{37}$$

which has the structure of STC and guarantees the convergence of $\hat{s}$ and $\dot{\hat{s}}$ to zero in finite time [15]. As observer is designed faster, $e_1 = e_2 = 0$ occurs before $\hat{s} = \dot{\hat{s}} = 0$. Then, the tracking error dynamics reduces to,

$$\dot{e}_{q_1} = -\beta^{q/p}|e_{q_1}|^{q/p}\operatorname{sign}(e_{q_1}) \tag{38}$$

$$e_{q_2} = -\beta^{q/p}|e_{q_1}|^{q/p}\operatorname{sign}(e_{q_1}) \tag{39}$$

So, the errors, e_{q_1}, e_{q_2} become zero in finite time [7]. The non-linear sliding surface is designed to overcome significant limitations in classical SMC and for achieving faster and stronger convergence properties.

V. Simulation Results

The designed STC control law with the HOSM Observer is simulated by taking the dynamic model of 2-DOF robotic manipulator. The system parameters are $m_1 = m_2 = 1$ kg, $r_1 = r_2 = 0.5$ m, $J_1 = J_2 = 5$ kgm^2, $g = 9.8$ m/s^2. The gain values of the non-singular terminal SMC are $p = 5$, $q = 3$, $\beta = 2.5$. For the HOSM observer, $\alpha_1 = 1.45 \times 10^{1/3}$, $\alpha_2 = 16 \times 10^{1/2}$, $\alpha_3 = 2 \times 10$. The comparisons of the actual and desired link positions and velocities are shown from Figs. 1 to 4. Fig. 5 shows the sliding surface. The tracking errors of the link positions and veclocities are given in Fig. 6. and Fig. 7. The control input is given in Fig. 8. For HOSM Observer, the errors in the estimated position and velocity of the links are shown in Fig. 9. and Fig. 10.

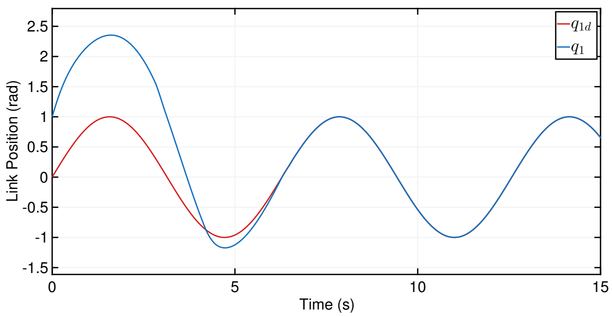

Fig. 1. Link Position

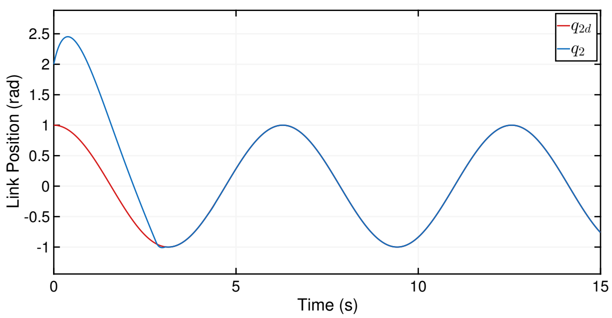

Fig. 2. Link Position

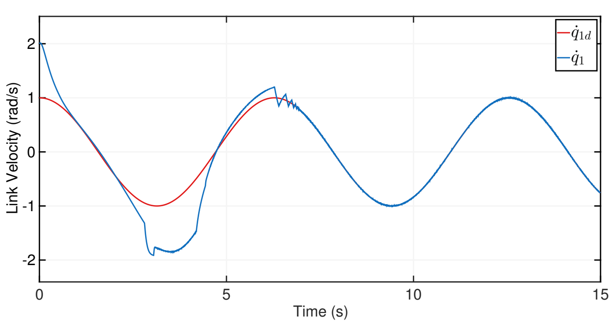

Fig. 3. Link Velocity

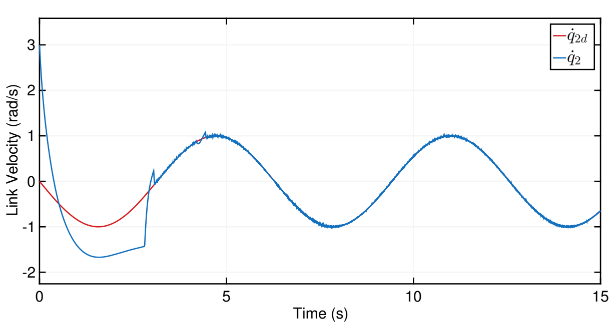

Fig. 4. Link Velocity

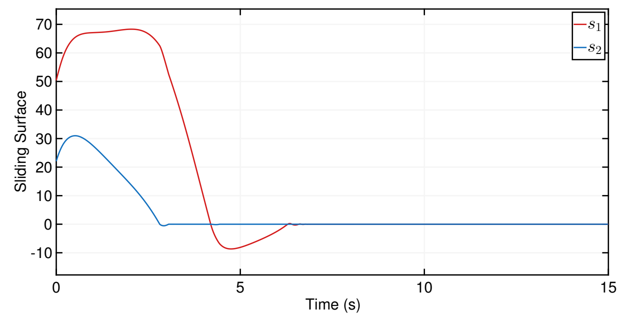

Fig. 5. Sliding Surface

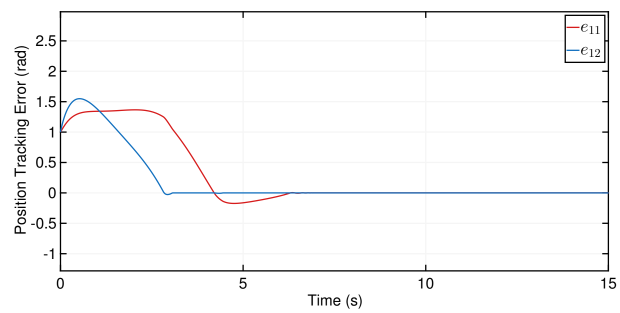

Fig. 6. Link Position Tracking Error

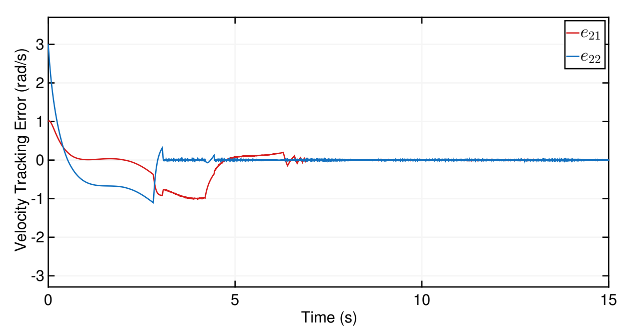

Fig. 7. Link Velocity Tracking Error

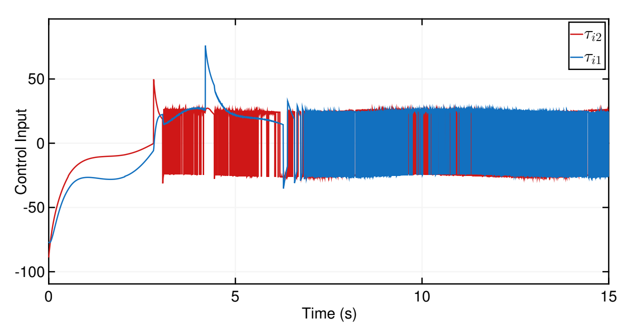

Fig. 8. Control Input

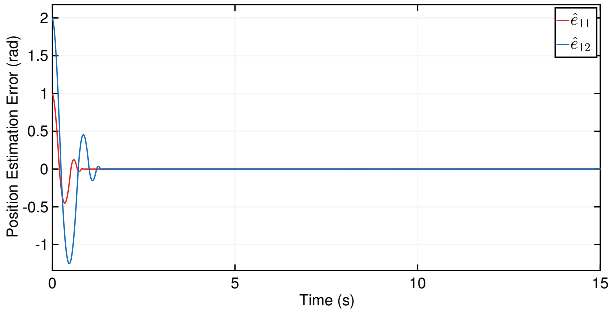

Fig. 9. Link Position Estimation Error

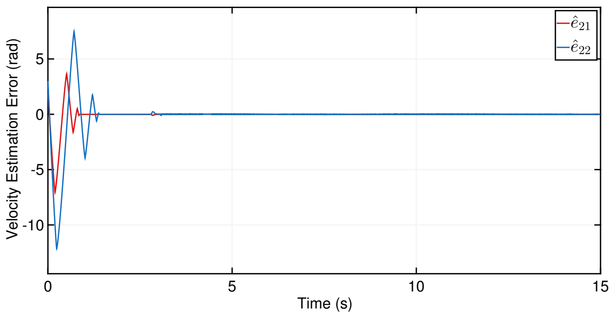

Fig. 10. Link Velocity Estimation Error

VI. Conclusion

In this work, Non-Singular Terminal Super-Twisting Control law is designed for the tracking problem of 2-DOF robotic manipulator. The link velocities are estimated using Higher-Order Sliding Mode Observer. The estimated velocity is used in the control law. The fast non-singular terminal sliding surface involving fractional terms allows the states of the system to converge to zero in finite time. The simulation results show improved tracking performance with high accuracy and robustness as compared to the conventional techniques.

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