Evaluating SOCP for AC Optimal Power Flow in Stressed Power System Conditions

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***Abstract*—The Optimal Power Flow is essential for ensuring the reliable and cost-effective operation in modern power systems. However, solving the AC Optimal Power Flow problem is difficult because it is highly non-convex and computationally demanding, particularly when the power system operates under stressed conditions. Convex relaxation methods, such as Second-Order Cone Programming, have been widely used to obtain faster solutions, although their accuracy can be reduced in meshed transmission networks. This paper evaluates the performance of the strengthened SOCP-III relaxation for solving AC-OPF problems under stressed loading conditions. The study is carried out using the IEEE 30, 57 and 118 bus test systems respectively. Where the system load is gradually increased to represent different levels of operating stress. The obtained solutions are compared with the results of conventional AC-OPF to assess the effectiveness of the approach. The evaluation focuses on generation cost and computational efficiency. In addition, the approach preserves acceptable voltage profiles even at higher loading levels. These findings demonstrate that SOCP-III provides a good balance between computational speed and solution accuracy . The method is using increasing load conditions (100%–125%) especially suitable for real-time and large-scale power system optimization.**

***Index Terms*—AC Optimal Power Flow (AC-OPF), Second-Order Cone Programming (SOCP), SOCP-III Relaxation, Stressed Operating Conditions, IEEE Bus Systems, Generator Constraint Violations.**

I. Introduction

Modern power systems are becoming increasingly complex due to the growth in electricity demand, integration of renewable energy sources, and expansion of transmission networks. With the increasing complexity of modern power systems, efficient operational planning has become increasingly important for maintaining both reliability and cost-effective operation. Among the various optimization techniques used in power system studies, OPF plays a significant role in determining the best operating condition of the network. The main objective of OPF is to achieve an economical and secure system operation while ensuring that all technical and operational constraints are satisfied. These constraints typically include power balance requirements, allowable voltage ranges, and transmission line flow limits, which are necessary for maintaining stable and secure power system performance. Among the different OPF formulations, the AC-OPF model provides the most accurate representation of power system behavior because it incorporates the nonlinear AC power flow equations.

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Despite its accuracy, the AC-OPF problem is hard to solve due to its nonlinear and non-convex nature of it. These features make the optimization problem computationally expensive especially for large-scale networks or applications that require fast decision-making. With the increasing scale and complexity of power systems, the efficient solution of AC-OPF has become an important research challenge.

In recent years several convex relaxation techniques have been proposed to address these difficulties. One popular approach is the Semidefinite Programming (SDP) relaxation,

which has strong theoretical guarantees and is often able to produce highly accurate solutions. The SDP formulations, however, usually need more computational time and memory, which may restrict their practical use in real-time or large-scale power system applications. Relative to SDP, SOCP can be solved much faster with still reasonable solution quality. However, tight solutions for meshed transmission networks may not always be obtained using standard SOCP relaxations, which can lead to significant optimality gaps compared to the true AC-OPF solution.

To enhance performance of SOCP-based approaches, recent works have proposed strengthened formulations that tighten the relaxation without significantly increasing computational .

cost . One such approach is the SOCP-III relaxation, which generalizes the basic SOCP model by adding extra linear tightening constraints obtained using convex envelope techniques.The AC-OPF problem was formulated based on the SOCP-III relaxation and solved using the SDPT3 solver as the backend numerical engine. Both CVX and Mosek tools were used for optimization in MATLAB and were called in turn.

This paper assesses the performance of the SOCP-III relaxation to solve AC-OPF problems under stressed operating conditions. The test network considered is the IEEE 30, 57 and 118 bus systems respectively. Different levels of loading are considered to represent different operating scenarios. The aim of this work is to study the accuracy, feasibility and computational efficiency of the SOCP-III formulation when the system is operated close to the limits.

II. Convex Relaxation Using SOCP

A. Maintaining Convex Approximation of SOCP

The SOCP relaxation approach converts the highly nonlinear and non-convex AC-OPF problem into a convex optimization model, making it easier to solve with modern optimization techniques. This reformulation significantly reduces computational complexity and enables faster and more efficient analysis, especially for large-scale power systems, while still providing solutions that closely approximate the original AC-OPF problem. To obtain a computationally efficient formulation, by defining auxiliary variables and replacing nonlinear equalities with conic inequalities. It requires much less computation time and memory than SDP, making it suitable for large systems and real-time operation. Although the solution may be slightly approximate, it provides a good balance between accuracy and computational efficiency for practical large-system applications.

SOCP Relaxation:

$$\min_{P,Q,V} \quad \sum_i f_i(P_i) \tag{1}$$

$$\text{s.t.} \quad P_{ij} + jQ_{ij} = (W_{ii} - W_{ij})y_{ij}^* \tag{2}$$

$$P_i + jQ_i = \sum_j (P_{ij} + jQ_{ij}) \tag{3}$$

$$\underline{P_i} \leq P_i \leq \overline{P_i} \tag{4}$$

$$\underline{Q_i} \leq Q_i \leq \overline{Q_i} \tag{5}$$

$$\underline{V_i}^2 \leq W_{ii} \leq \overline{V_i}^2 \tag{6}$$

$$P_{ij}^2 + Q_{ij}^2 \leq \overline{S}_{ij}^2 \tag{7}$$

$$\left\| \begin{bmatrix} 2W_{ij} \\ W_{ii} - W_{jj} \end{bmatrix} \right\| \leq W_{ii} + W_{jj}, \quad \forall ij \tag{8}$$

These equations transform the nonlinear AC power flow optimization into a convex SOCP problem by introducing lifted voltage variables, enforcing physical operating limits, and replacing nonconvex voltage product relationships with SOC constraints [1].

Standard form SOCP:

$$\min_x \quad f^* x \tag{9}$$

$$\text{s.t.} \quad \|A_i x + b_i\| \leq c_i^* x + d_i \tag{10}$$

These equations refer from [1] in this x is the Decision variable vector, A, b,c,d and i are constraint coefficient matrix, constant vector, constraint coefficient vector, constant scalar and constraint index respectively. Basic SOCP relaxation is computationally efficient but produces loose solutions with significant optimality gaps in meshed transmission networks, making it inadequate for high-accuracy in large systems. To enhanced SOCP relaxations that achieve near-SDP accuracy with significantly lower computation time for large-scale AC-OPF problems. Four methods are used to strengthen SOCP [2]. So that it captures voltage angle physics and produces near-SDP accuracy while remaining computationally fast for large systems.

B. Cycle-Based Strengthening using McCormick Relaxation

This method improves the standard SOCP by enforcing network cycle (angle consistency) constraints, which are ignored in the basic SOCP. This method strengthens SOCP by enforcing voltage angle consistency over network cycles. The nonlinear cycle equations are decomposed into bilinear constraints and relaxed using McCormick envelopes, resulting in a tighter and more accurate convex formulation. Cycle angle condition is considered in a power network, the sum of voltage angle differences across any closed loop must ensures physical feasibility of power flow. Convert angle constraints into algebraic form [2].

The angle relations are rewritten using SOCP variables:

$$c_{ij} = V_i V_j \cos(\theta_i - \theta_j), \qquad s_{ij} = V_i V_j \sin(\theta_i - \theta_j)$$

variable c, s, i and j denotes the Cosine component between buses, Sine component between buses and i,j are buses respectively. But McCormick-based cycle method provides a static outer approximation of bilinear constraints, which may lead to a looser feasible region. Bilinear terms like xy are replaced by McCormick envelopes, which are only outer approximations of the true nonlinear relation [2]. This creates a larger feasible region than the actual physical solution set.

C. Arctangent Convex Envelope Based Strengthening

This method improves the standard SOCP formulation by incorporating the voltage angle relationship in a convex form. In the AC power flow model, the angle difference between two buses is given by

$$\theta_i - \theta_j = \tan^{-1}\left(\frac{s_{ij}}{c_{ij}}\right)$$

where c_{ij} and s_{ij} represent the cosine and sine components of the voltage product [2].

Since the arctangent function is nonlinear and nonconvex, it cannot be directly included in the SOCP model. To address this issue, the method constructs a convex polyhedral envelope that approximates the arctangent relationship within known bounds of c_{ij} and s_{ij} . These envelopes are formed using a set of linear inequalities that tightly bound the feasible angle region.

But the feasible region becomes larger than the actual physical region, leading to approximation error. The arctangent envelope method relies on a fixed polyhedral approximation of the nonlinear angle relation, which results in a relaxed feasible region [2].

D. SDP-Cut Based SOCP Strengthening

This method enhances the classical SOCP relaxation by iteratively adding valid linear inequalities derived from semidefinite programming (SDP) constraints. Initially, the standard SOCP problem is solved to obtain a candidate solution. If this solution violates the positive semidefinite properties that would be satisfied in the SDP relaxation, additional linear constraints (cuts) are generated to eliminate the infeasible region.

The improved SOCP model becomes

$$X_{ii} X_{jj} \geq |X_{ij}|^2$$

$$v_k^T X v_k \geq 0, \qquad k = 1, 2, \ldots, K$$

where each v_k generates a new cut.

The generated cuts are meant to separate the current SOCP solution from the feasible region defined by the SDP formulation while keeping the convex structure of the problem. These additional inequalities are incorporated into the SOCP model, after which the optimization problem is solved again. The procedure continues iteratively until the violations are sufficiently reduced or eliminated. By gradually tightening the feasible region through these cuts, the approach is able to obtain solutions that are close to the accuracy of SDP while retaining the computational advantages of SOCP.

However, the method requires solving the SOCP problem repeatedly, since new cuts are introduced during each iteration. As a result, the total computational effort can become higher than that of a single SOCP formulation.Sections II.B–II.D review three categories of SOCP strengthening strategies. The formulation evaluated in this work referred to as SOCP-III which follows the approach of [3], which avoids arctangent and iterative SDP-cut methods. Instead, it introduces auxiliary rectangular voltage variables and replaces nonlinear voltage products with equivalent quadratic conic constraints, as detailed in the below subsection.

E. SOCP-III Formulation for AC-OPF

SOCP-III, as proposed in [3], is the specific formulation implemented and evaluated in this paper. Unlike the McCormick-based (Section II.B) and arctangent-based (Section II.C) methods, SOCP-III does not rely on outer polyhedral approximations of angle constraints. Unlike the SDP-cut approach (Section II.D), it does not require iterative re-solving. Instead, it tightens the basic SOCP relaxation by introducing three auxiliary variables per branch c_{ii}, c_{ij}, and s_{ij} and enforcing the following additional constraints that exactly link these variables to the rectangular voltage components e_i, f_i. These variables allow the nonlinear power flow equations to be expressed in a form that can be handled using conic optimization techniques. The additional variables are defined as [3]

$$c_{ii} = e_i^2 + f_i^2$$

$$c_{ij} = e_i e_j + f_i f_j$$

$$s_{ij} = e_i f_j - e_j f_i$$

where e_i and f_i denote the real and imaginary parts of the equations at bus i.

The SOCP-III formulation implemented the basic SOCP model (Equations 1–8) with four additional sets of constraints: (i) auxiliary rectangular voltage variable definitions for c_{ii}, c_{ij}, and s_{ij}; (ii) the conic exactness constraint

$$c_{ij}^2 + s_{ij}^2 \leq c_{ii}\, c_{jj};$$

(iii) consistency constraints linking the auxiliary variables to the lifted UV matrix; and (iv) linear angle difference bounds on s_{ij} and c_{ij}. These additions tighten the relaxation without requiring iterative solving or nonlinear arctangent constraints, preserving the computational advantages of SOCP. The formulation of the SOCP-III is explain in the flowchart of figure.1 with its conditions and constraints used are given from equations 1-8. While the SOCP-III formulation was proposed in [3], its behavior under progressively stressed loading conditions has not been systematically characterized in the existing literature. The quantitative evaluation of the feasibility boundary of SOCP-III under load stress levels up to 125% of nominal and identification of the loading threshold beyond which SOCP-III transitions from optimal to inaccurate to infeasible.

TABLE I
IEEE 30-Bus Comparison

Method	Generation Cost ($/hr)	Optimality Gap
Basic SOCP	804.5	1.8%
McCormick Strengthening	802.9	2%
Arctangent Envelope	801.8	2.03%
SDP-Cut Strengthening	800.9	0.2%
SOCP-III	801.1	0.01%

III. Stressed Operating Conditions

Stressed operating conditions represent network states where the power system operates close to or beyond its operational limits due to high load demand, large renewable generation penetration, or network congestion. Under such conditions, voltage magnitudes may violate permissible limits and transmission lines may approach their thermal capacity. these scenarios are specially designed to evaluate the robustness and efficiency of OPF algorithms in adverse system conditions. In a power network, stress usually appears in one or more the magnitudes of the bus voltage are close to their lower limits and generators are forced to their maximum limits of active or reactive output. Together they form a problem space in which it is a real challenge for any optimization algorithm to remain feasible.

Stress in a power network typically manifests in one or more ways. Bus voltage magnitudes drift toward their permissible lower bounds and generators are pushed toward their maximum active or reactive output limits. Together, they create a scenario where maintaining feasibility becomes a genuine challenge for any optimization algorithm. It is precisely under such conditions that the real-world value of a relaxation technique like SOCP-III must be tested not just under comfortable base-case loading, but under conditions that truly stress the underlying mathematical formulation. The method is test on IEEE bus systems under increasing load conditions (100%–125%). Generation cost increases with load demand, Larger systems like the IEEE 118-bus handle stress better.The method becomes infeasible at very high stress loading condition in smaller systems.

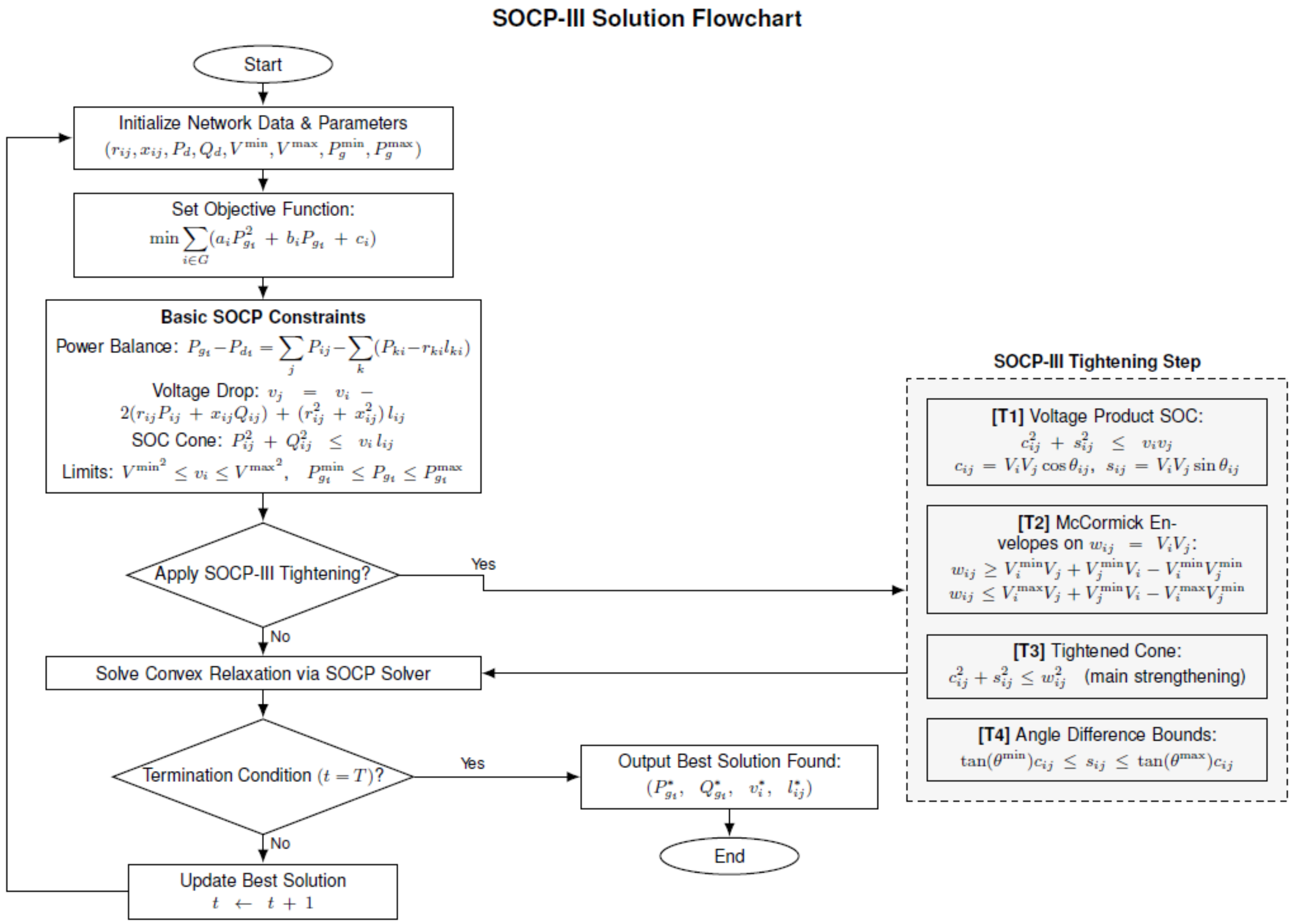

Fig. 1. SOCP-III Formulation Flowchart

TABLE II
SOCP-III AC-OPF RESULTS FOR IEEE 30-BUS SYSTEM

δ	Load (%)	Gen Cost ($/hr)	CPU (s)	Solver Status
0.000	100.0	573.58	5.7607	Solved
0.020	102.0	588.33	5.9461	Solved
0.050	105.0	610.70	5.9709	Solved
0.100	110.0	648.92	5.5979	Solved
0.150	115.0	688.92	5.5720	Inaccurate/Solved
0.200	120.0	Infeasible	5.7706	Infeasible
0.250	125.0	Infeasible	5.4061	Infeasible

TABLE III
SOCP-III AC-OPF RESULTS FOR IEEE 57-BUS SYSTEM

δ	Load (%)	Gen Cost ($/hr)	CPU (s)	Solver Status
0.000	100.0	41711.01	16.2398	Solved
0.020	102.0	42785.48	5.8155	Solved
0.050	105.0	44409.37	5.5121	Solved
0.100	110.0	47171.11	5.4178	Solved
0.150	115.0	50049.11	5.3840	Solved
0.200	120.0	Infeasible	5.7398	Failed
0.250	125.0	Infeasible	5.5908	Infeasible

IV. RESULTS & DISCUSSION

A. Impact of Load Variation on Generation Cost

Load variation occurs when the electricity demand in the power system changes over time consumption patterns, seasonal effects, or operational conditions. Load variation affects generation cost because generators must produce more power when system demand increases.
Table I, II and III shows how the generation cost changes when the system load is slightly increased.

The first column represents δ (load increase factor). This value indicates how much the system demand is increased

TABLE IV
SOCP-III AC-OPF RESULTS FOR IEEE 118-BUS SYSTEM

δ	Load (%)	Gen Cost ($/hr)	CPU (s)	Solver Status
0.000	100.0	129339.54	9.1111	Solved
0.020	102.0	132705.18	6.7365	Solved
0.050	105.0	137770.29	6.3815	Solved
0.100	110.0	146253.15	6.6210	Solved
0.150	115.0	154786.43	6.6126	Solved
0.200	120.0	163368.50	6.3503	Inaccurate/Solved
0.250	125.0	171992.55	6.1187	Inaccurate/Solved

compared to the base load. When $\delta = 0$, the system operates at 100% of the base load. As δ increases, the load in the network increases proportionally.

The second column shows the load level in percentage. For example, when $\delta = 0.000$, the system load becomes 100% of the base load, and when $\delta = 0.020$, the load reaches 102% of the base load.

The third column represents the total generation cost in dollars per hour ($\$/hr$) obtained from the optimal power flow solution. CPU (s) measures the total computation time in seconds taken by the solver to solve the SOCP optimization problem for each load level. Solver status reports the outcome returned by CVX after solving, with four possible values: Solved (optimal solution found), Inaccurate/Solved (solution found but with reduced numerical accuracy), Infeasible (no feasible solution exists at that load level), or Unbounded (rarely occurs in OPF).The results show the generation cost gradually increases as load increases. This happens because generators must produce more power to satisfy the higher demand, which leads to higher operating and fuel costs.

All three plots show the same fundamental as the load level increases (from 100% up to the maximum feasible percentage), the optimal generation cost ($/hr) rises consistently in a smooth, upward curve for the IEEE bus test systems solved using the SOCP-III method. The x-axis represents the scaled demand level as a percentage of the base case load, and the y-axis represents the minimum achievable generation cost found by the SOCP solver meaning each point is the cheapest possible way to meet that load while satisfying all power flow, voltage, and generator constraints. The trend in all three plots is almost linear but slightly steepens. This implies that as load increases, generators are gradually being pushed towards their capacity limit and the system is being forced to dispatch more expensive units which results in the cost increasing at an increasing rate. The 118-bus system can accommodate all 7 load levels (100%–125%) before becoming infeasible, whereas the 30-bus and the 57-bus system become infeasible at lower load levels (around 115%–120%), indicating that the larger 118-bus network has more generation capacity and flexibility to cope with heavier loading conditions.

B. Variation of Generator Constraint Violations

Variation in generator constraint violations is defined as the difference in the level of generator output violations against their maximum/minimum limits for different system conditions. In the framework of Optimal Power Flow (OPF) each generator is required to operate within its given active power limits and these can be written as

$$P_g^{\min} \leq P_g \leq P_g^{\max} \tag{11}$$

A constraint violation is said to occur when the computed generation value P_g lies outside its permissible operating range. As operating conditions change, such as variations in the loading factor δ, total system demand, or uncertainty in the network, the magnitude of these violations may either increase or decrease. The variation of generator constraint violations represents the change in the magnitude of limit violations under different operating scenarios.

Analysis of this variation is useful for understanding the impact of system stress on generator feasibility and overall operational security.

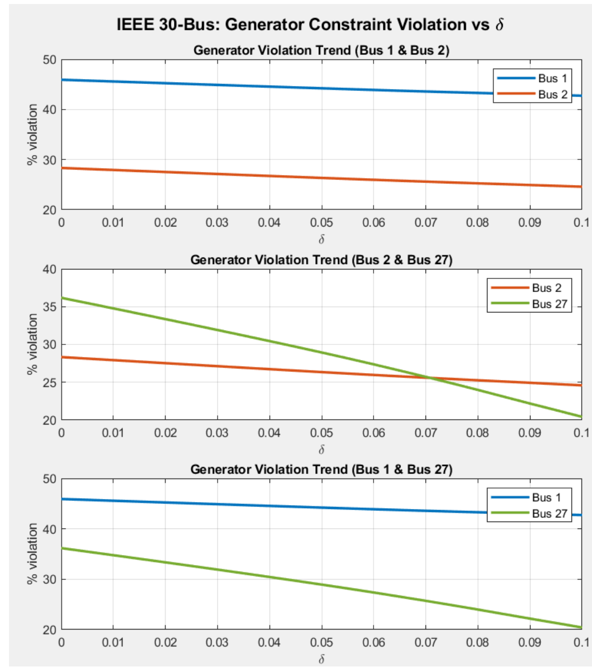

Fig. 2. Generator constraint violation observed in IEEE 30-bus system

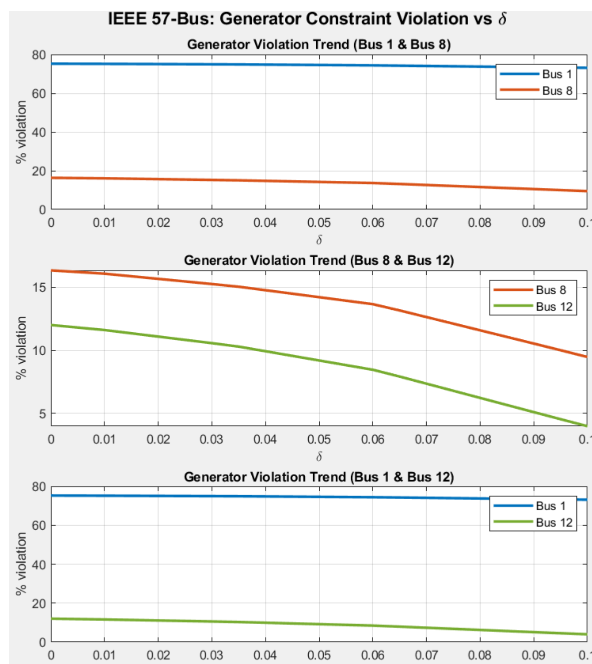

Fig. 3. Generator constraint violation observed in IEEE 30-bus system

The plots show the variation of generation constraint violations with parameter δ. As the value of increases, the violation

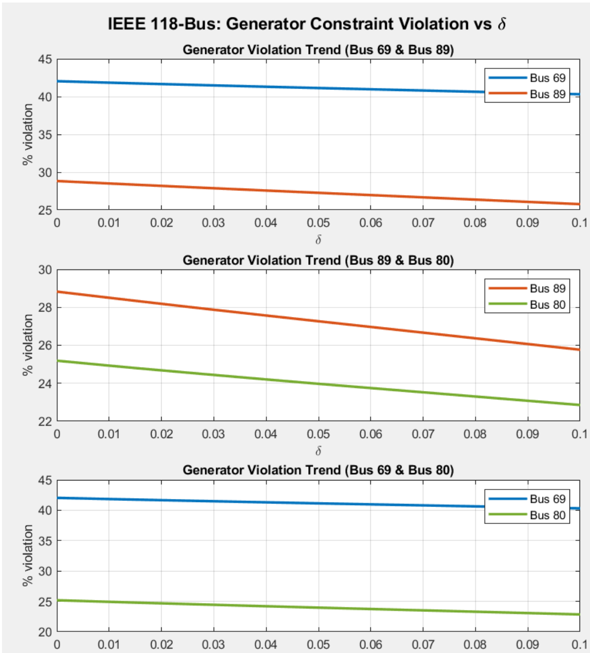

Fig. 4. Generator constraint violation observed in IEEE 30-bus system

decreases for all generators, implying increased feasibility. The IEEE bus test systems and the generator constraint violations show a consistent decrease as the perturbation parameter is increased from 0 to 0.1, although the magnitude and rate of decrease varies significantly with the bus and the system size. In the 30-Bus system, Bus 1 has the highest violation (45%), which drops only modestly, while Bus 27 has the steepest drop from 36% to 20% suggesting that it is the most sensitive to . Bus 1 has a consistently high violation (75%) that is almost flat in , while the violation of Bus 12 drops sharply from 12% to near 0%, indicating great responsiveness. The violation of Bus 8 is in between with a slow drop from 16% to 10%. In the 118-Bus system, a more compressed and uniform behavior is observed, as all three buses (69, 89, 80) show relatively moderate violations in the 22-42% range and slowly decreasing, suggesting that larger and more interconnected networks distribute the effect more evenly. The overall results show that increasing is an effective way to reduce the violations of the generator constraints. However, the impact of is focused on some specific buses disproportionately, especially those with steeper curves, while the high-violation buses such as Bus 1 (30-Bus) and Bus 1 (57-Bus) are still not much improved, regardless of the system.

V. Conclusion

This paper presented a performance evaluation of the SOCP-III relaxation technique to solve the AC Optimal Power Flow (AC-OPF) problem under stressed operating conditions.

The IEEE bus systems are respectively used to investigate the behavior of the proposed approach under increased loading levels. Various performance indices were considered such as impact of load variation on generation cost, generator constraint violations.

Simulation results show that the SOCP-III relaxation can produce solutions that are very close to the AC-OPF solutions with a competitive computational performance. The observed variation of generation cost is consistent with the expected trend with increased system demand, implying that the method effectively captures the economic behavior of the power network. In addition, the analysis of generator operation range shows that the optimization maintains the reasonable generator dispatch with small deviation.

The overall results demonstrate that the SOCP-III formulation offers an effective balance between computational speed and solution precision. The proposed method is capable of producing stable and reliable OPF solutions while successfully meeting the essential operational constraints required for secure and efficient power system operation. Therefore, SOCP-III can be considered as a useful tool for the study of power system operation cases in which fast and reliable optimization methods are required.

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