A Search and Rescue Problem with Path-Planning under Uncertainty*

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***Abstract*— The Search and Rescue Problem (SARP) can be formulated in an environment characterized by subjective uncertainty, i.e. a lack of knowledge about the state of the world. In this paper, we present an interval arithmetic interpretation of the uncertainty related to parameters based on the calculation of shortest paths, computed through uncertainty propagation over terrain parameters. A crisis scenario is modeled as an assignment and optimization problem with interval-valued parameters and constraints. A branch-and-bound algorithm is used to explore the solution space. Interval arithmetic is employed to compute bounds and obtain feasible assignments. We show here how path planning can, for a certain map, serve as input to an uncertain assignment problem, and at the same time, for another map according to a safety criterion, aid in the execution of the plan.**

I. Introduction

In crisis-response and humanitarian assistance scenarios, the efficient allocation of limited medical resources is critical. Coordinating caregivers with diverse skills to meet heterogeneous needs under urgent conditions requires structured decision-making models.

In this paper, we consider the **Search and Rescue Problem** (SARP), which includes several topics of study such as information perception and fusion, path planning, and, more particularly, what interests us here, the assignment of caregivers to wounded individuals.

The problem of **assigning resources to tasks** has been widely studied in the literature and concerns many domains where resources must be optimized, such as industry, logistics, computing, military domain, etc. In all these domains, resources are assigned to tasks under various **constraints** (e.g., a resource cannot be assigned to two tasks simultaneously). Under these constraints, it is necessary to compute, ideally, the optimal assignments according to several parameters.

However, in emergency situations, the environment is subject to both objective uncertainty (randomness inherent to nature) and subjective uncertainty (lack of knowledge about the state of the world). Regarding the latter, some parameters are not precisely known and are therefore uncertain. To model this uncertainty, we propose an interval-based approach [5], rather than a probabilistic one. The goal is then to solve this problem of assigning caregivers to wounded individuals using a constraint optimization algorithm with interval parameters.

For the assignment of caregivers to wounded individuals, two main classes of uncertain parameters affect the likelihood that a caregiver will treat a wounded person. The first class, called **utility**, corresponds to the necessity of assigning a caregiver to a wounded individual according to several other parameters such as the wounded person's injury level or the team's proximity to the wounded person. The second class of parameters is the **adequacy** between a caregiver's skills/equipment and a wounded person's injury type. Furthermore, the effects of assigning several caregivers to wounded individuals must be considered cumulative.

In this work, we therefore model the crisis scenario as a well-known problem in operations research and defense systems, namely the classical Weapon Target Assignment (WTA) problem [1]. It originally aims to assign the best weapons to the most relevant targets, but it has many dual cases. Thus, according to its canonical formulation, a real-valued parameter, as a particular case of the utility class, denoted by v_j, represents the (intrinsic) **initial injury level** of wounded individual j. This initial injury level depends only on the wounded person and is fixed a priori. On the other hand, the real value p_{ij}, belonging to the adequacy class, represents the probability that caregiver i successfully treats wounded individual j. In the canonical formulation, a treatment of caregiver i reduces the injury level of wounded individual j by a factor $(1 - p_{ij})$. When several caregivers are assigned to the same wounded person, their effects are assumed to be independent. As a consequence, each additional caregiver i' further reduces the residual injury of wounded individual j by a factor $(1 - p_{i'j})$. This multiplicative structure is a key feature of the classical WTA formulation.

Let m be the number of caregivers and n the number of wounded individuals. In the WTA problem, the decision variables are defined as: $x_{ij} = 1$ if caregiver i is assigned to wounded individual j, $x_{ij} = 0$ otherwise. The objective of the problem is to minimize the **total residual injury** after all assignments:

$$\min \sum_{j=1}^{n} v_j \prod_{i=1}^{m} (1 - p_{ij})^{x_{ij}}.$$

The main constraint of this problem is that each caregiver is assigned to at most one wounded individual. It is well known for its combinatorial nature and its computational difficulty: the WTA problem is NP-hard [10]. To solve it,

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many exact and approximate methods have been proposed in the literature [13]. But the computation of v_j can be enriched beyond the simple quantification of the initial injury level.

Thus, in the weapon-target analogy, and according to a more general paradigm, the parameter v_j can be viewed as a threat decomposed into **Capability-Opportunity-Intent** (COI) [9]. This principle is applied in a more detailed model for the computation of v_j as a combination of several factors, such as the capability of target j (i.e., lethality), its intention (i.e., behavior), and the opportunity to act (i.e., proximity) [3], where Bayesian networks are employed to manage probabilistic uncertainties. These parameters are further extended in [4], where weighted combinations are used, and introducing a formation parameter that reflects how targets are grouped.

In our SARP, relying on the same COI analogy, the parameter related to the necessity of assignment v_j could be assimilated to the only opportunity (O) parameter as the proximity of the wounded person [2]. In this case, v_j is defined as $1/d_j$, a function of the inverse of the distance between the caregivers (assumed to be at the same position) and wounded individual j. The distance is computed using an A* algorithm that selects nodes in a graph based on the sum of the known path cost between the starting node and the current node and a heuristic function to guide the search towards the destination node [11]. The WTA-like problem is then solved using an approximate method (a genetic algorithm). The proximity v_j still depends only on the wounded index j.

However, in practice, caregivers are **not necessarily located at the same position**. Moreover, in real-time situations, subjective uncertainty means that the parameters v_j and p_{ij} are often **imperfectly known**. When the parameters of an assignment problem are represented by intervals, they are considered indirectly because the problem is reduced to real numbers [12]. In addition, to support decision-making, operators may require that the best solution be guaranteed with an **exact resolution algorithm**. In this context, we begin by presenting how the utility defined by [2], as $v_j = 1/d_j$, can be seen as a special case of a generic utility function that depends on multiple variables. We then extend this approach by making several contributions.

First, for injury-level computation, we assume that the m caregivers are not all located at the same base, but are distributed among k **teams**. This leads to a distance matrix indexed by teams and wounded individuals, and to a new expression of the objective function compared to the canonical formulation. Second, and unlike existing approaches, the parameters v_j and p_{ij} are uncertain and represented as **intervals**, as described in [5]. Using interval arithmetic, the output values are guaranteed with respect to the input uncertainties. Third, we solve this problem with uncertainties using an exact **Branch and Bound** algorithm by directly computing interval propagation instead of indirect methods.

II. Interval Assignment Problem Formulation

This section specifies the problem of assigning caregivers, organized into teams, to wounded individuals with uncertain interval parameters. First, it shows how the problem involves minimizing the residual utility of treatment, i.e., ensuring that after the assignments, the necessity for **further treatment is minimal**. Second, it describes the constraints of the model. Third, it explains what it means to compare intervals in this context.

A. Minimizing the total residual utility of treatment

We consider m **caregivers** indexed by $i \in \{1, \dots, m\}$, to treat n **wounded individuals** indexed by $j \in \{1, \dots, n\}$. The caregivers are grouped into k **teams** indexed by e, stored in the vector K, where $K[i] = e$ means that caregiver i belongs to team e. An example is given in Figure 1. According to the

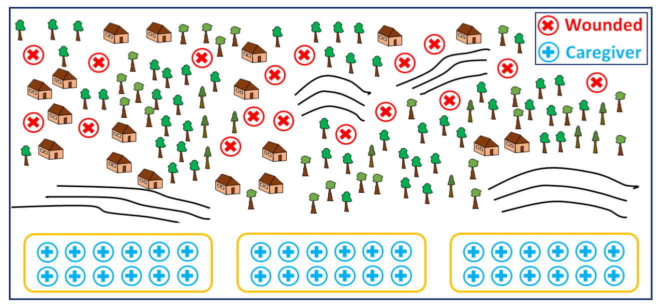

Fig. 1. A crisis scenario.

COI paradigm, the initial utility function $u(\cdot)$ takes as input the matrix of uncertain distances $[d_{ej}]$ between a team e and a wounded j, the uncertain injury level $[v_j]$ of wounded j, and his uncertain behaviour $[q_j]$. The return values of $u(\cdot)$ are stored in the interval matrix $[U]$, indexed by e (and not by i), since caregivers belonging to **the same team are assumed to remain together**.

The objective function also depends on the interval matrix of **reduction ratios** $[P]$, whose elements are the intervals $[p_{ij}]$. We note that $[P]$ is indexed by i and not by e, since caregivers are **heterogeneous** within teams.

By extension with respect to the WTAP problem, we aim to determine the best assignment, i.e. we seek an assignment matrix $X = (x_{ij})_{1 \le i \le m,\ 1 \le j \le n}$ that minimizes the total residual utility of treatment $F(X)$ defined by:

$$F(X) = \sum_{e=1}^{k} \sum_{j=1}^{n} u([v_j], [d_{ej}], [q_j]) \prod_{i=1}^{m} \big(1 - [p_{ij}]^{x_{ij}}\big). \quad (1)$$

However, in the following, to focus on calculation, influence and interpretation of parameters based on the shortest paths, we consider that $u(\cdot)$ depends only on $[d_{ej}]$ and that the elements $[u_{ej}]$ take values $[u_{ej}] = 1/[d_{ej}]$ ($[v_j]$ and $[q_j]$ are assumed to be constants).

Therefore, under the constraints, the determination of the best assignment depends solely on the **best global compromise** between all the shortest distances $[d_{ej}]$ and the strongest caregiver-wounded pairs $[p_{ij}]$.

B. Under Constraints

To complete our minimization problem, we need to consider five constraints.

On the one hand, we consider the two canonical constraints introduced earlier. First, the **binary constraint** (C.1), which defines the domain of the decision variables. Second, the **at-most-one-wounded-per-caregiver constraint** (C.2), which follows from the definition of the assignment problem (this constraint is enforced by design in the algorithm).

On the other hand, we introduce problem-specific constraints. Third, the **minimum treatment constraint** (C.3): if caregiver i is assigned to wounded j, then the treatment value $[u_{ej}]\cdot[p_{ij}]$ must be greater than or equal to a minimum threshold $w_j^{\min}$ (collected in the vector $W^{\min}$). This is an active filtering constraint used to avoid negligible treatments. Fourth, the **team coherence constraint** (C.4): caregivers belonging to the same team (confer vector K) cannot be assigned to different wounded individuals (however, some caregivers in a team may remain unassigned while others are assigned to a wounded). Fifth, the **maximum residual utility constraint** (C.5): the total residual utility for a wounded must be less than or equal to a maximum threshold $w_j^{\max}$ (collected in the vector $W^{\max}$).

$$\forall (i,j) \in \{1,\dots,m\} \times \{1,\dots,n\}, \quad x_{ij} \in \{0,1\} \tag{C.1}$$

$$\forall i \in \{1,\dots,m\}, \quad \sum_{j=1}^{n} x_{ij} \leq 1 \tag{C.2}$$

$$\forall (i,e,j), \quad x_{ij} = 1 \Rightarrow [u_{ej}]\cdot[p_{ij}] \subset [w_j^{\min}, +\infty) \tag{C.3}$$

$$\forall (i,i'),\ (K[i] = K[i']) \ \Rightarrow\ \forall j \neq j', \quad x_{ij}\, x_{i'j'} = 0 \tag{C.4}$$

$$\forall (e,j), \quad [u_{ej}] \prod_{i=1}^{m} (1-[p_{ij}])^{x_{ij}} \subset [0, w_j^{\max}] \tag{C.5}$$

C. Hurwicz optimism–pessimism criterion

Since, as described above, the objective of this minimization problem is interval-valued, we choose to follow the Hurwicz optimism–pessimism criterion [7] to scalarize it using the upper and lower bounds of each interval, denoted respectively by $\mathrm{ub}(\cdot)$ and $\mathrm{lb}(\cdot)$, as follows. The function $F_\alpha(X)$ is defined by:

$$F_\alpha(X) = \alpha \cdot \mathrm{ub}(F(X)) \ + \ (1-\alpha)\cdot \mathrm{lb}(F(X)) \tag{2}$$

where the parameter $\alpha \in [0,1]$ expresses the relative weight given to the optimistic (lower bound) versus pessimistic (upper bound) evaluation of the interval objective of the minimization problem. When $\alpha = 0$, it is fully optimistic because it assumes that the best cases will occur regarding the uncertain parameters. When $\alpha = 1$, the decision is fully robust because it considers that the worst cases will occur. Intermediate values yield a compromise between these two extremes. In this work, we adopt the **robust case** $\alpha = 1$ (pessimistic approach), so that $F_\alpha(X) = \mathrm{ub}(F(X))$.

III. A Branch and Bound Algorithm with Path-Planning under Uncertainty

In our model, the terrain is represented by a grid in which uncertain data, as perceived and understood, are stored and updated in real-time. This data concerns both the caregivers and the wounded. Furthermore, for the terrain, the uncertain navigation costs and, due to various risks, the uncertain safety costs are also stored. First, we detail the calculation of these uncertain terrain costs, as well as the degrees of freedom in the grid. Then, we show how interval arithmetic allows us to compute the guaranteed minimum-cost path according to all possible values of these uncertain terrain costs. Finally, we explain how our Branch and Bound algorithm allows us to obtain all optimal assignments based on all uncertain inputs.

A. Computation of the uncertain terrain parameters

The grid consists of cells with coordinates noted as (x,y) and associated with an uncertain terrain cost

$$[c_{xy}] = \beta[c_{xy}^{\mathrm{nav}}] + \gamma[c_{xy}^{\mathrm{saf}}], \tag{3}$$

where $[c_{xy}^{\mathrm{nav}}]$ denotes the uncertain **navigability** cost of the cell (i.e. the terrain roughness and the obstacles captured by the sensors from the real terrain), and $[c_{xy}^{\mathrm{saf}}]$ denotes the uncertain **safety** cost of the cell (according to various risks such as rockfalls). These uncertain terrain costs are of relative importance, weighted respectively by the navigability coefficient β and the safety coefficient γ, depending on the type of mission and the subjectivity of the caregivers.

B. Movement in the Grid and weighted distance

In the grid, each admissible path from a team e to a wounded j is denoted by $\pi_{ej} = \{(x_t,y_t)\}_{t=1}^{L}$, where t is the index of each step and L is the total number of steps (i.e. the path length). The path is restricted to Manhattan moves i.e. for all $t \in \{1,\dots,L-1\}$:

$$(x_{t+1},y_{t+1}) - (x_t,y_t) \in \{(1,0),(-1,0),(0,1),(0,-1)\}. \tag{4}$$

Based on the uncertain cost of each cell stored in the interval matrix $[C]$, a weighted distance of an admissible path π_{ej} from the position of e to the position of j is

$$[d_{\mathrm{w}}(\pi_{ej})] = \sum_{(x,y)\in\pi_{ej}} [c_{xy}]. \tag{5}$$

Then the goal is to compute all elements of the matrix $[D]$, whose entries $[d_{ej}]$ are the minimum values of $[d_{\mathrm{w}}(\pi_{ej})]$, i.e. the distances of the best paths π_{ej}^*, gathered in Π_{ej}^*, over all admissible paths π_{ej} connecting team e to wounded j.

C. Pathfinding Algorithm

To achieve this, the pathfinding algorithm takes as input the path-planning domain $\Sigma_{\mathrm{path}} = (O^e, O^w, [C])$, containing the teams' position matrix O^e, the wounded' position matrix O^w, and an interval-valued cost matrix $[C]$. The search is performed using the A* algorithm as described in [8], which minimizes $[cost(\pi)] + h(x_n,y_n)$, where $[cost(\pi)]$ denotes the uncertain accumulated cost of the path from (x_e,y_e)

to (x_n, y_n), and $h(x_n, y_n)$ is a heuristic estimate of the remaining cost to reach the wounded. Each move adds the global cost associated with entering the destination cell. In a pessimistic approach (as for $F(\cdot)$ as discussed in Section II-C), the algorithm considers the upper bounds of the interval costs in order to minimize the worst-case traversal cost.

As a heuristic, we use the Manhattan distance. This heuristic computes the minimum number of horizontal and vertical moves required to reach the wounded cell, ignoring obstacles, terrain costs, and uncertainty. It is admissible (according to [8]) because it never overestimates the true shortest path cost on a 4-connected grid. As a result, A* is guaranteed to return a robust and optimal path.

This function, named `SHORTEST_PATHS`, is used to compute shortest paths in Algorithm 1 and is called by the main `SOLVE` function in Algorithm 2.

Algorithm 1 A* search as forward search for a team–wounded pair

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Input: team index e, wounded index j, path-planning domain $\Sigma_{\text{path}} = (O^e, O^w, [C])$
Output: interval shortest-path cost $[d_{ej}]$ from team e to wounded j, optimal path π^*_{ej}
<u>**Function**</u> SHORTEST_PATHS$(e, j, \Sigma_{\text{path}})$

Let $(x_e, y_e) \leftarrow$ grid cell in O^e corresponding to the initial position of team e
Let $(x_j, y_j) \leftarrow$ grid cell in O^w corresponding to the position of wounded j
Initialize $[cost(x, y)] \leftarrow [+\infty]$ for all cells (x, y), $[cost(x_e, y_e)] \leftarrow 0$
Initialize $parent(x, y) \leftarrow \perp$ for all cells (x, y)
Frontier $\leftarrow \{(\langle\rangle, (x_e, y_e))\}$
Expanded $\leftarrow \emptyset$
while Frontier $\neq \emptyset$ **do**
  select node $(\pi, (x_n, y_n)) \in$ Frontier with minimal $[cost(\pi)] + h(x_n, y_n)$
  remove $(\pi, (x_n, y_n))$ from Frontier
  if $(\pi, (x_n, y_n)) \in$ Expanded **then**
    continue
  end if
  add $(\pi, (x_n, y_n))$ to Expanded
  if $(x_n, y_n) = (x_j, y_j)$ **then**
    $\pi^*_{ej} \leftarrow \pi$
    $[d_{ej}] \leftarrow [cost(\pi)]$
    return $\big([d_{ej}], \pi^*_{ej}\big)$
  end if
  Children $\leftarrow \emptyset$
  for each 4-neighbor (x_v, y_v) of (x_n, y_n) inside the grid **do**
    if $[c_{x_v, y_v}] < 0$ **then**
      continue {i.e. impassable cell}
    end if
    $[tentative] \leftarrow [cost(\pi)] + [c_{x_v, y_v}]$
    if $[tentative] < \min\{[cost(\pi')], \pi'$ to $(x_v, y_v)\}$ **then**
      $\pi' \leftarrow \pi \cdot (x_v, y_v)$
      $[cost(\pi')] \leftarrow [tentative]$
      $parent(\pi', (x_v, y_v)) \leftarrow (\pi, (x_n, y_n))$
      add node $(\pi', (x_v, y_v))$ to Children
    end if
  end for
  prune dominated or inconsistent nodes
  Frontier $\leftarrow$ Frontier $\cup$ Children
end while
return $\big([+\infty, +\infty], \emptyset\big)$

<u>**Heuristic**</u> $h(x, y)$
return $|x - x_j| + |y - y_j|$

D. Branch and Bound algorithm calling Pathfinding

The main `SOLVE` function takes as input the parameters of our problem: the number of caregivers m and wounded n, the number of teams k, the interval matrix of initial utility of treating $[U]$, the interval matrix of **reduction ratios** $[P]$, the parameters $(W^{\min}, K, W^{\max})$ associated with the constraints (C.3)–(C.5), the x- and y-coordinates of the teams gathered in the vector O^e, the x- and y-coordinates of the wounded gathered in the vector O^w, the uncertain cost of each cell gathered in the interval matrix $[C]$, a greedy heuristic first solution X_{gr} (here, pairs (i, j) that maximize $\text{ub}([u_{ej}]\,[p_{ij}])$) and its interval objective $F(X_{\text{gr}})$ denoted $[F_{\text{gr}}]$. The $[u_{ej}]$ values are updated according to `SHORTEST_PATHS` (Algorithm 1), as well as the minimal treatment thresholds associated with the constraint (C.3).

Based on these inputs, the branch-and-bound algorithm is executed. The branches are evaluated with the lowest possible residual utilities for each remaining caregiver to be assigned, and pruned if the upper bound of the best solution already found is smaller than the lower bound achievable.

Then `SOLVE` function returns the best assignment X^* of caregivers to wounded individuals, the best interval objective value $[F^*]$, and the paths Π followed by each team to reach its assigned wounded.

IV. RESULTS

In this section, these algorithms are applied to a schematic crisis scenario.

A. Use case with uncertain parameters

For the use case, we consider $m = 18$ caregivers distributed among $k = 6$ teams who need to minimize the residual utility of treating $n = 6$ wounded individuals. The interval matrix of **reduction ratios** $[P]$ is given in Table I.

As explained in Section III-A, navigability and safety are weighted and combined according to Equation (3). We consider $\beta = 0.8$ for navigability and $\gamma = 0.2$ for safety. The resulting **interval costs** are shown on the **global map** in Figure 2.

Algorithm 2 An interval B&B algorithm for IPWTAP.

Input: $\Sigma_{\text{wta}} = (m, n, k, [U], [P], W^{\min}, K, W^{\max})$, $\Sigma_{\text{path}} = (O^e, O^w, [C])$, X_{gr}, $[F_{\text{gr}}]$
Output: best assignment X^*, best objective value $[F^*]$, best paths of assigned pairs Π (not necessary Π^*)

Function SOLVE($\Sigma_{\text{wta}}, \Sigma_{\text{path}}, X_{\text{gr}}, [F_{\text{gr}}]$)

for $e = 1$ **to** k **do**
  for $j = 1$ **to** n **do**
    $([d_{ej}], \pi^*_{ej}) \leftarrow$ SHORTEST_PATHS($e, j, \Sigma_{\text{path}}$)
  end for
end for
update $[U]$ and $W^{\min}$ with $[D]$
if $X_{\text{gr}} \neq \perp$ **then**
  return OPT($\Sigma_{\text{wta}}, [-1, \ldots], X_{\text{gr}}, [F_{\text{gr}}], \Pi$)
else
  return OPT($\Sigma_{\text{wta}}, [-1, \ldots], [-1, \ldots], [+\infty, +\infty], \Pi$)
end if

Function OPT($\Sigma, X, X^*, [F^*], \Pi$)

Perform an interval branch-and-bound search that recursively assigns caregivers (not only teams) to wounded under constraints, updates $(X^*, [F^*])$, and records in Π^* the paths of all final team–wounded assignments.
return $(X^*, [F^*], \Pi^*)$

Our problem was implemented in `C++` with the `Ibex` library to handle interval arithmetic [6], compute rigorous bounds on the objective function, and return the solutions.

	$W01$	$W02$	$W03$	$W04$	$W05$	$W06$
$C01$	$[0.10, 0.12]$	$[0.80, 0.82]$	$[0.50, 0.52]$	$[0.90, 0.92]$	$[0.20, 0.22]$	$[0.70, 0.72]$
$C02$	$[0.90, 0.92]$	$[0.50, 0.52]$	$[0.20, 0.22]$	$[0.70, 0.72]$	$[0.10, 0.12]$	$[0.30, 0.32]$
$C03$	$[0.60, 0.62]$	$[0.10, 0.12]$	$[0.50, 0.52]$	$[0.30, 0.32]$	$[0.70, 0.72]$	$[0.20, 0.22]$
$C04$	$[0.90, 0.92]$	$[0.10, 0.12]$	$[0.40, 0.42]$	$[0.60, 0.62]$	$[0.70, 0.72]$	$[0.80, 0.82]$
$C05$	$[0.40, 0.42]$	$[0.10, 0.12]$	$[0.50, 0.52]$	$[0.30, 0.32]$	$[0.90, 0.92]$	$[0.60, 0.62]$
$C06$	$[0.20, 0.22]$	$[0.90, 0.92]$	$[0.10, 0.12]$	$[0.60, 0.62]$	$[0.50, 0.52]$	$[0.30, 0.32]$
$C07$	$[0.50, 0.52]$	$[0.60, 0.62]$	$[0.70, 0.72]$	$[0.30, 0.32]$	$[0.40, 0.42]$	$[0.80, 0.82]$
$C08$	$[0.10, 0.12]$	$[0.50, 0.52]$	$[0.40, 0.42]$	$[0.80, 0.82]$	$[0.70, 0.72]$	$[0.20, 0.22]$
$C09$	$[0.60, 0.62]$	$[0.90, 0.92]$	$[0.30, 0.32]$	$[0.80, 0.82]$	$[0.10, 0.12]$	$[0.40, 0.42]$
$C10$	$[0.80, 0.82]$	$[0.50, 0.52]$	$[0.20, 0.22]$	$[0.90, 0.92]$	$[0.70, 0.72]$	$[0.30, 0.32]$
$C11$	$[0.20, 0.22]$	$[0.30, 0.32]$	$[0.60, 0.62]$	$[0.90, 0.92]$	$[0.50, 0.52]$	$[0.80, 0.82]$
$C12$	$[0.70, 0.72]$	$[0.10, 0.12]$	$[0.80, 0.82]$	$[0.90, 0.92]$	$[0.50, 0.52]$	$[0.60, 0.62]$
$C13$	$[0.90, 0.92]$	$[0.40, 0.42]$	$[0.20, 0.22]$	$[0.60, 0.62]$	$[0.10, 0.12]$	$[0.50, 0.52]$
$C14$	$[0.90, 0.92]$	$[0.10, 0.12]$	$[0.80, 0.82]$	$[0.60, 0.62]$	$[0.20, 0.22]$	$[0.40, 0.42]$
$C15$	$[0.10, 0.12]$	$[0.60, 0.62]$	$[0.80, 0.82]$	$[0.90, 0.92]$	$[0.70, 0.72]$	$[0.40, 0.42]$
$C16$	$[0.20, 0.22]$	$[0.50, 0.52]$	$[0.10, 0.12]$	$[0.40, 0.42]$	$[0.70, 0.72]$	$[0.30, 0.32]$
$C17$	$[0.50, 0.52]$	$[0.80, 0.82]$	$[0.90, 0.92]$	$[0.60, 0.62]$	$[0.10, 0.12]$	$[0.70, 0.72]$
$C18$	$[0.70, 0.72]$	$[0.20, 0.22]$	$[0.30, 0.32]$	$[0.90, 0.92]$	$[0.60, 0.62]$	$[0.40, 0.42]$

TABLE I

INTERVAL REDUCTION RATIOS p_{ij} (CAREGIVER C, WOUNDED W).

B. Output Representations

Based on these inputs, Algorithm 2 allows for the calculation of optimal assignments. These are provided in Table II.

For these globally optimal assignments, the **best paths** of the solution pairs (e, j) are displayed in Figure 3 on the global costs map. It is clear in this figure that the computed paths clearly avoid the roughest areas with the highest costs (in descending order of cost: violet, pink, beige, green).

It can be seen (in magenta) that caregivers from team 4 are assigned to wounded individual 3 along path π_{43}, whereas the sole criterion of the best path π^*_{45} would lead them to wounded individual 5. Indeed, the optimal assignment also depends on all combinations of the best paths π^*_{ej} and all the ratios $[p_{ij}]$. Thus, the paths displayed correspond to those induced by the optimal solution to the problem of globally assigning caregivers to the wounded where it is most useful, and at the same time where the caregiver-wounded adequacy is the strongest.

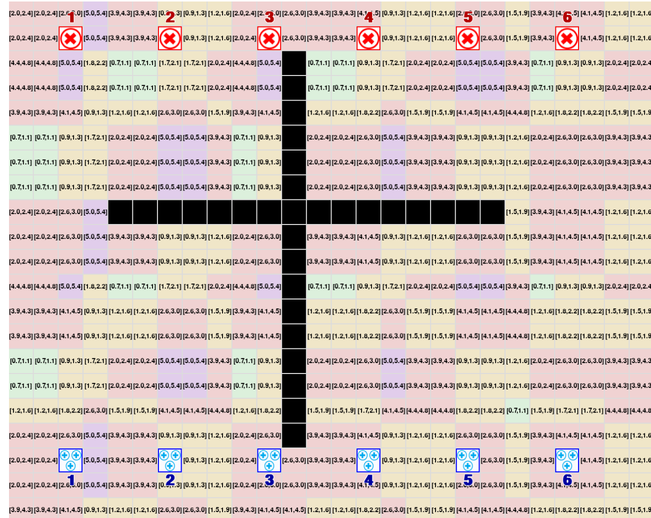

Fig. 2. Global map of combined costs with $\beta = 0.8$ and $\gamma = 0.2$ (green if $[c_{xy}] \subset [0.7, 1.1]$), beige if $[c_{xy}] \subset [0.9, 2.2]$, pink if $[c_{xy}] \subset [2.0, 4.8]$, violet if $[c_{xy}] \subset [5.0, 5.4]$).

Furthermore, it is observed that caregivers always move with their teams. However, according to the problem constraints (such as the minimal treatment constraint (C.3)), some caregivers may not treat a wounded for this iteration, thus conserving their equipment for the next iteration.

Careg.	Wound.
01	∅
02	01
03	01

Team $E01$.

Careg.	Wound.
04	05
05	05
06	05

Team $E02$.

Careg.	Wound.
07	02
08	02
09	02

Team $E03$.

Careg.	Wound.
10	∅
11	03
12	03

Team $E04$.

Careg.	Wound.
13	04
14	04
15	04

Team $E05$.

Careg.	Wound.
16	06
17	06
18	06

Team $E06$.

TABLE II

BEST ASSIGNMENT BY TEAM.

C. Information based on a particular criterion or bound

Within this global framework, each caregiver now knows which wounded individual is assigned to them for the global minimization of residual utility of treatment. These assignments depend on the navigability coefficient β and the safety coefficient γ, which were introduced in the Equation (3). However, independently of this weighting for the assignments, the caregivers may have more freedom of choice at their decision level and, in parallel, be informed based on one of these criteria if they want to adjust their path

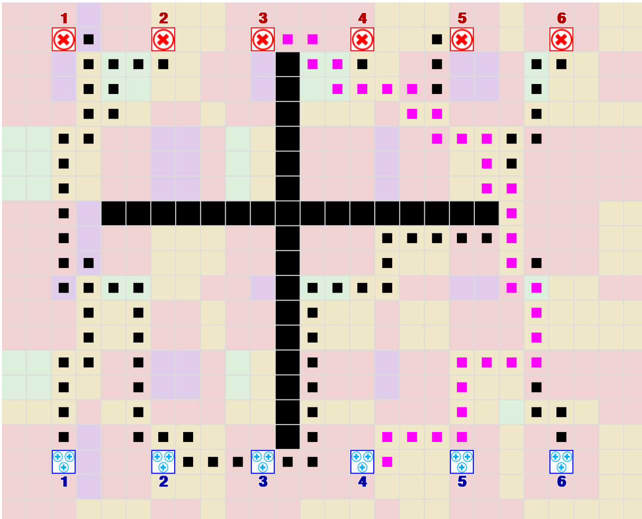

Fig. 3. Optimal paths on global map with $\beta = 0.8$ and $\gamma = 0.2$ (green if $[c_{xy}] \subset [0.7, 1.1]$), beige if $[c_{xy}] \subset [0.9, 2.2]$, pink if $[c_{xy}] \subset [2.0, 4.8]$, violet if $[c_{xy}] \subset [5.0, 5.4]$).

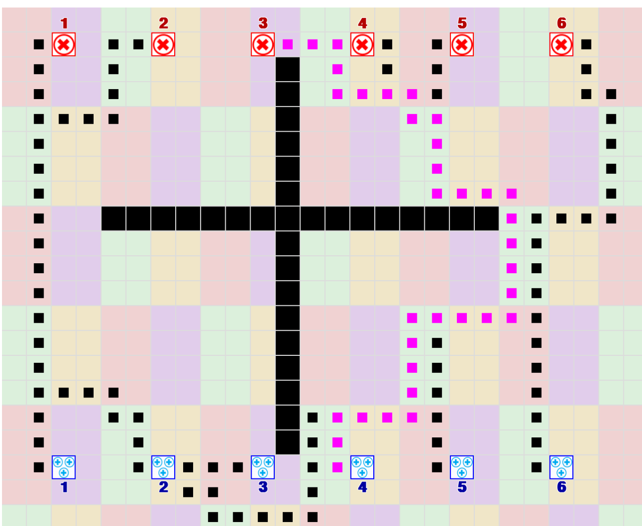

Fig. 4. Optimal paths on safety map with $\beta = 0.0$ and $\gamma = 1.0$ (green if $[c_{xy}] = [0.9, 1.1]$), beige if $[c_{xy}] = [1.9, 2.1]$, pink if $[c_{xy}] = [2.9, 3.1]$, violet if $[c_{xy}] = [4.9, 5.1]$).

towards the assigned wounded. On the one hand, the team can decide to adjust the path according to the subjective and contingent importance they assign between navigability and safety. Figure 4 (on the **safety map**, which is on a different frame from the global map) thus shows that, by prioritizing safety (with $\beta = 0.0$ and $\gamma = 1.0$), the team will avoid the cells with the highest safety costs (in particular the violet ones). On the other hand, let us recall that the approach is initially pessimistic: the worst-case scenarios of the terrain costs (upper bounds of the intervals) are considered. The team can then be in an optimistic approach that would consider the lower bounds of the terrain costs.

To go further, a sensitivity analysis would show how the paths and assignments are influenced by the trade-offs between navigability/safety and optimism/pessimism.

V. Conclusion and Future Works

In this paper, we address a crisis scenario in which caregivers are assigned to wounded individuals. Our optimization problem is characterized by two classes of parameters: those giving the utility of treating the wounded, and those giving the adequacy of the caregivers' skills according to the needs of the wounded. As in the WTA problem model (initially for military applications), the treatments are cumulative.

However, unlike WTA formulations, the caregivers are organized into teams. The teams' abilities to move are an important parameter of the global optimization. These parameters are not known exactly and are represented as intervals, therefore standard real-valued optimization approaches are unsuitable. Thus, we proposed a branch-and-bound algorithm that, with the guarantee of interval arithmetic, finds the best solution to the problem of achieving a global compromise between shortest distances and adequacies. The distances are obtained using an A* path planning algorithm operating on uncertain cost maps. The input uncertainties are rigorously propagated via interval arithmetic.

In the application study, we also analyzed how the subjectivity of the caregiver can be parameterized. The set of caregivers thus chooses the balance between a pessimistic and an optimistic approach. A team of caregivers can also choose between navigating quickly and safely.

As future work, we aim to investigate the interdependence between planning and assignment. The assignment process depends on an initial state, which itself results from a planning phase, while planning is in turn influenced by assignment outcomes. This interdependence becomes more complex with uncertainty, especially in a dynamic environment. It will be explored in the context of integrating Hierarchical Task Network planning (confer [8]), assignment, and uncertainty into a coherent decision-making framework.

References

[1] A. S. Manne, A target-assignment problem, Operations research, 6(3), 346-351, 1958.

[2] M. Chabanenko, *et al*, Combining pathfinding and Weapon-Target Assignment for air defence, In 7th IEEE APUAVD 2026, 2024.

[3] F. Johansson, & G. Falkman, A Bayesian network approach to threat evaluation with application to an air defense scenario, In 11th International conference on information fusion, 2008.

[4] A. Naseem, S. A. Khan, & A. W. Malik, A real-time man-in-loop threat evaluation and resource assignment in defense, Journal of the Operational Research Society, 68(6), 725-738, 2017.

[5] R. E. Moore, R. B. Kearfott, & M. J. Cloud, Introduction to interval analysis, Society for Industrial and Applied Mathematics, 2009.

[6] https ://ibex-team.github.io/ibex-lib/

[7] V. Kreinovich, Decision making under interval uncertainty, In Human-centric decision-making models for social sciences, 2013.

[8] M. Ghallab, *et al*, Acting, planning, and learning, 2025.

[9] E. Little, *et al*, An ontological analysis of threat and vulnerability, In 2006 9th International Conference on Information Fusion, 2006.

[10] S. P. Lloyd, & H. S. Witsenhausen, Weapons allocation is NP-complete, In 1986 summer computer simulation conference, 1986.

[11] P. E. Hart, N. J. Nilsson, & B. Raphael, A formal basis for the heuristic determination of minimum cost paths, IEEE transactions on Systems Science and Cybernetics, 4(2), 100-107, 1968.

[12] S. Majumdar, Interval linear assignment problems, Universal Journal of Applied Mathematics, 1(1), 14-16, 2013.

[13] A. Kline, D. Ahner, & R. Hill, The weapon-target assignment problem, Computers & Operations Research, 105, 226-236, 2019.