An Experimental Methodology for LuGre Friction Identification and Validation in Precision Motion Systems

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Abstract— **This paper evaluates the systematic nature and experimental applicability of a recently proposed LuGre friction model identification methodology for MKS Instruments sub-micron precision motion systems with diverse mechanical and guidance characteristics. The methodology combines experiment-based parameter initialization, derived from a classical two-step strategy, with gradient-based nonlinear optimization under a single-objective criterion. A dedicated friction-focused experimental protocol is implemented to ensure data consistency and identification reliability. The accuracy of the resulting LuGre model is demonstrated through the evaluation of the normalized root-mean-square error (NRMSE) between estimated and measured friction forces. Additionally, the model is applied in a feedforward friction compensation scheme on the industrial positioning system "XML-350 Ultra-Precision Linear Motor Stage." Experimental results illustrate its beneficial impact on trajectory tracking performance in terms of positioning accuracy.**

I. INTRODUCTION

Friction-induced phenomena degrade tracking performance on high-precision positioning systems driven by linear motors and supported by rolling bearings. Achieving submicrometer positioning accuracy requires accurate friction estimation and compensation, for which model-based approaches are considered the state of the art [1].

Among dynamic friction formulations, the LuGre model [2] reproduces essential asperity-scale phenomena [3] while preserving a reasonable trade-off between modeling fidelity and parameter parsimony [1]. It has therefore become a common choice for friction estimation and compensation in precision positioning applications [3]-[4]. However, effective use of this model for compensation relies on accurate parameter identification, which remains a non-trivial task due to its nonlinear dynamics, parameter interdependence, and the presence of an unmeasurable internal state.

A widely adopted LuGre model identification approach is the two-step strategy introduced in [4] and extended in [5]. This method separates friction regimes according to the level of applied force, allowing subsets of parameters to be identified within a sequential optimization procedure. Although a wide variety of excitation signals has been reported in the literature [4]-[8], no standardized experimental protocol has emerged. In parallel, numerous optimization strategies have been explored for parameter estimation, including heuristic and metaheuristic methods such as genetic algorithms, differential evolution, and SINDy-based approaches [9]–[13]. Despite these developments, two practical issues remain central within the nonlinear optimization framework: the selection of informative excitation signals and the initialization of the LuGre parameters.

The present work investigates LuGre parameter identification for characterizing friction in a high-precision roller-bearing linear stage. The identification methodology employed in this study was originally introduced in [14], where it was validated on a ball-bearing-guided experimental stage. The contribution of this paper is to experimentally validate the applicability of this identification methodology to a motion system with different mechanical and guiding characteristics.

The methodology is applied here to an XML350 linear stage from MKS Instruments [15]. In addition, the identified LuGre model is implemented in a feedforward friction-compensation scheme. This provides a control-oriented validation of the identification strategy by demonstrating that the estimated LuGre parameters can be directly used within a feedforward friction-compensation scheme, which in turn leads to improved positioning performance of a high-precision motion system.

The paper is structured as follows. Section II introduces the LuGre model and the parameter decomposition underlying the proposed methodology. Section III recalls the identification procedure and the experimental protocol. Section IV presents the experimental setup, while Section V describes the implementation of the proposed methodology for this experimental setup. Finally, Section VI reports the LuGre identification and LuGre-based feedforward compensation results.

II. FRICTION MODELING AND PROPERTIES

A. Positioning Stage with Rolling-Element Bearing

In a one-dimensional (1-D) positioning stage driven by an ironless permanent magnet linear synchronous motors (PMLSMs). The permanent magnet assembly forms the moving element traveling along a stationary coil track and is guided by lubricated rolling-element bearings. Within this configuration, the primary contribution to friction comes from the rolling contact between the bearings and the guide surface. The motion along a single axis is modeled as a lumped inertia M actuated by the control effort f_e, where the friction force f_r represents the dominant disturbance to

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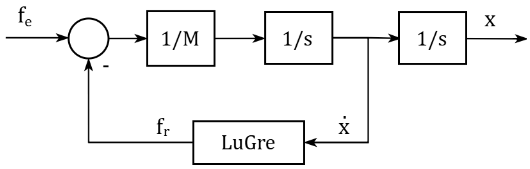


Fig. 1: 1-D positioning stage with rolling-element bearing

be accurately modeled. The dynamics are given by:

$$M\ddot{x} = f_e - f_r, \tag{1}$$

where x denotes the slider position. A schematic representation is provided in Fig. 1. The friction force f_r is described using the LuGre model in a state–force formulation, with the velocity $\dot{x}$ as the input and the friction force f_r as the output. This model is detailed in the following section, and its parameter decomposition forms the basis of the identification methodology applied in this paper. The resulting LuGre model is subsequently used for feedforward compensation.

B. LuGre Model

The LuGre model, as formulated by Canudas de Wit et al. (1995) [2], describes friction at the asperity level by representing the contact between surfaces as a distribution of elastic bristles.

When the external force f_e remains below the static friction threshold F_s, the bristles deform elasto-plastically, entering the pre-sliding regime, where friction exhibits position-dependent behavior. When f_e reaches and exceeds F_s, the bristles break away, and sustained motion occurs, corresponding to the gross sliding regime. The bristle dynamics are then given by:

$$\dot{z} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} z, \tag{2}$$

where the internal state $z[m]$ denotes the average bristle deflection and $\dot{x}$ denotes the sliding velocity.

The function $g(\dot{x})$ $[N]$ denotes the Stribeck curve describing the steady-state friction–velocity relation, which may exhibit directional asymmetry, as reported in [3]. In this study, the Lorentzian formulation is adopted for $\sigma_0 g(\dot{x})$:

$$\sigma_0 g(\dot{x}) = F_c + (F_s - F_c)\frac{1}{1+\left(\frac{|\dot{x}|}{v_s}\right)^2}, \tag{3}$$

where F_c is the kinetic friction threshold and v_s denotes the Stribeck velocity. The resulting LuGre friction force is given by:

$$f_r = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x} + b, \tag{4}$$

where σ_0 [N/m] denotes the elastic stiffness, σ_1 [Ns/m] the micro-damping coefficient associated with energy dissipation during pre-sliding, and σ_2 [Ns/m] the viscous friction coefficient. The additional term b [N] represents a constant offset introduced to capture the directional asymmetry of the friction force observed experimentally.

For constant sliding velocities, the friction operates in steady state ($\dot{z} = 0$) and is governed by the Stribeck curve:

$$z_{ss} = g(\dot{x})\,\mathrm{sgn}(\dot{x}), \text{ where } \mathrm{sgn}(\dot{x}) = \begin{cases} 1 & \text{if } \dot{x} > 0, \\ -1 & \text{if } \dot{x} < 0. \end{cases} \tag{5}$$

C. Parameter Space Decomposition

The LuGre model identification methodology relies on parameter-space decomposition derived from the model equations (2)-(5).

1) Force-based Regime Decomposition: The magnitude of the applied force $|f_e|$ relative to the static friction threshold F_s allows to distinguish the pre-sliding and steady-state sliding regimes, both governed by different parameter sets.

In the steady-state sliding regime, where $|f_e| > F_s$ under constant velocity conditions, friction follows the Stribeck curve parameterized by $\theta_{stat} = [F_c, F_s, v_s, \sigma_2, b]$:

$$f_{r_{ss}} = g(\dot{x}, [F_c, F_s, v_s])\, sgn(\dot{x}) + \sigma_2 \dot{x} + b. \tag{6}$$

When $|f_e| < F_s$, the system operates in the pre-sliding regime where $f_r = f_e$ and $\dot{x} \approx \dot{z}$; in this scenario, the friction behavior is determined by the dynamic parameters $\theta_{dyn} = [\sigma_0, \sigma_1]$. The viscous term σ_2 and the friction offset b are negligible in this regime:

$$f_{r_{dyn}} = \sigma_0 x + \sigma_1 \dot{x}. \tag{7}$$

2) Linear–Nonlinear Decomposition: The LuGre model can be reformulated as a combination of a linear-in-parameters output equation and a nonlinear state evolution. It is expressed as

$$\mathrm{LuGre}(\dot{x}, \theta_{\mathrm{L}}, \theta_{\mathrm{NL}}) : \begin{cases} f_r = \theta_{\mathrm{L}}^T \cdot [z, \dot{z}, \dot{x}, 1]^T, \\ \dot{z} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x}, \theta_{\mathrm{NL}})} z. \end{cases} \tag{8}$$

The parameter vectors are defined as

$$\theta_{\mathrm{L}} \triangleq [\sigma_0, \sigma_1, \sigma_2, b]^T, \quad \theta_{\mathrm{NL}} \triangleq [a_1, a_2, v_s]^T,$$

where $a_1 = \frac{F_c}{\sigma_0}$ and $a_2 = \frac{F_s - F_c}{\sigma_0}$. In this formulation, θ_{L} groups the parameters entering linearly in the friction force expression, while θ_{NL} governs the nonlinear bristle-state dynamics through the function $g(\dot{x})$.

III. Identification Methodology

The comprehensive LuGre friction identification methodology introduced in our previous work [14], combines an experiment-based initialization, inspired by the approach in [4], with a gradient-based nonlinear optimization using the cost function introduced in [11]. The method is recalled in this section for completeness.

A. The Comprehensive LuGre Friction Identification Methodology

The methodology relies on a grey-box formulation of the LuGre model (8), in which the unknown parameter vector is $\theta = [\theta_L, \theta_{NL}]$. It comprises the strictly positive parameters $[\sigma_0, \sigma_1, \sigma_2, a_1, a_2, v_s]^T$, and the bias term $b \in \mathbb{R}$.

1) Alternating Optimization Scheme: The identification of the model parameters is expressed as a nonlinear least-squares problem aimed at minimizing the squared deviation between the measured friction force $f_r(t)$ and the LuGre-estimated friction $\hat{f}_r(t,\theta)$ over N recorded samples:

$$\mathscr{J} = \sum_{j=1}^{N} (f_r(t_j) - \hat{f}_r(t_j,\theta))^2. \tag{9}$$

Although a single cost function is employed, the refinement phase uses an alternating optimization scheme in which nonlinear and linear parameters are updated sequentially. The total number of refinement iterations, denoted by k, is selected empirically and acts as the stopping criterion of the optimization procedure. At iteration i, the two successive optimization problems are carried out as:

$$\begin{aligned} \hat{\theta}_{\mathrm{NL}}^{(i)} &= \arg\min_{\theta_{\mathrm{NL}}} \mathscr{J}\left(\theta_{NL} \,/\, \theta_{\mathrm{L}}^{(i-1)}\right), \\ \hat{\theta}_{\mathrm{L}}^{(i)} &= \arg\min_{\theta_{\mathrm{L}}} \mathscr{J}\left(\theta_{L} \,/\, \theta_{\mathrm{NL}}^{(i)}\right). \end{aligned} \tag{10}$$

This iteration yields the updated grey-box model $\mathrm{LuGre}\big(\hat{\theta}^{(i)}\big)$, which is successively refined over the k predefined iterations. The resulting cost function in (9) is generally non-convex. As a consequence, gradient-based optimization may converge to local minima, depending on the initialization and the sequence of parameter updates.

The final estimate $\hat{\theta}^{(k)}$ therefore corresponds to a local minimum of the cost function associated with the best model fit achieved within the considered refinement process.

Owing to the nonlinear and non-convex nature of the least-squares cost function, suitable initial parameter estimates are necessary to avoid convergence to non-physical local minima when using local optimization methods. The refinement phase is therefore initialized as

$$\hat{\theta}^{(0)} = \big(\hat{\theta}_{\mathrm{L}}^{(0)}, \hat{\theta}_{NL}^{(0)}\big).$$

The parameter space is defined as $\Theta(\delta)$, where $\delta \in [0,1]$ is a tolerance factor controlling the extent of variation around the initial estimates. The Stribeck velocity v_s is not scaled by δ and is instead assigned a fixed physically motivated range to ensure sufficient exploration of the transition between static and kinetic friction. The resulting parameter space is defined as follows:

$$\Theta(\delta) \triangleq \begin{cases} \sigma_k \in \left[(1-\delta)\hat{\sigma}_k^{(0)}; (1+\delta)\hat{\sigma}_k^{(0)}\right], & k \in \{0,1,2\}, \\ a_k \in \left[(1-\delta)\hat{a}_k^{(0)}; (1+\delta)\hat{a}_k^{(0)}\right], & k \in \{1,2\}, \\ b \in \left[(1-\delta)\hat{F}_s^{(0)}; (1+\delta)\hat{F}_s^{(0)}\right], & \\ v_s \in \left[1\times 10^{-5}; 1\times 10^{-2}\right]. & \end{cases}$$

The initialization strategy detailed in the following section yields a suitable initial LuGre model.

2) Parameter Initialization: The initial parameter values $\hat{\theta}^{(0)}$ and the parameter space $\Theta(\delta)$ are obtained using the two-step identification method proposed in [4], which leverages the friction regime decomposition introduced in Section II-C.1.

a) Static Parameter Initialization: At steady-state velocity $\dot{x}_{ss}$, the friction force is approximated by $f_{r_{ss}} = f_e - M\ddot{x}$, where the inertial term compensates for residual acceleration effects. The static parameters are identified by fitting the Stribeck relation in (6) to the experimental data $(\dot{x}_{ss}, f_{r_{ss}})$. This yields the steady-state friction representation:

$$\hat{f}_{r_{ss}}(\dot{x}_{ss}) = g\left(\dot{x}_{ss}, [\hat{F}_c, \hat{F}_s, \hat{v}_s]\right)\,\mathrm{sgn}(\dot{x}_{ss}) + \hat{\sigma}_2 \dot{x}_{ss}. \tag{11}$$

b) Dynamic Parameter Initialization: Dynamic parameters $\hat{\theta}_{dyn}$ are obtained through frequency-domain identification using the linearized LuGre formulation of [5]. In the pre-sliding regime, i.e., under the constraint $|f_e| < F_s$, the system dynamics reduce to a second-order linear model:

$$f_e = M\ddot{x} + \sigma_1 \dot{x} + \sigma_0 x. \tag{12}$$

This leads to the pre-sliding transfer function:

$$\frac{X(s)}{F_e(s)} = \frac{1}{Ms^2 + \sigma_1 s + \sigma_0}. \tag{13}$$

The identification is performed in terms of the equivalent second-order parameters, the damping ratio $\hat{\zeta}$ and the natural frequency $\hat{\omega}$, from which the LuGre coefficients are reconstructed as: $\hat{\sigma}_0 = M\hat{\omega}_n^2, \hat{\sigma}_1 = 2M\hat{\zeta}\hat{\omega}_n$. The estimation is carried out in the frequency domain, which offers two main advantages: reduced sensitivity to measurement noise and no prior assumption on the damping ratio of the linearized system, an aspect that can lead to inaccurate identification in time-domain approaches if such assumptions are poorly defined.

B. Friction-oriented Experimental Protocol

The identification methodology relies on a friction-oriented experimental protocol composed of two successive steps. Each step is associated with a specific excitation strategy and requires a dedicated parameter estimation procedure. The computational and optimization tools employed for parameter estimation are detailed in our previous work [14].

1) Initialization Step: This step aims at independently exciting the static and dynamic friction phenomena, as detailed in Section II-C.1.

a) Static parameters: The experimental Stribeck curve (11) is obtained under closed-loop control. A low-frequency trapezoidal velocity reference $\dot{x}_d$, including constant-velocity plateaus within the range $\dot{x}_{ss} \in [\dot{x}_{\min}, \dot{x}_{\max}]$, is applied. The corresponding steady-state velocity and friction samples $(\dot{x}_{ss}, f_{r_{ss}})$ are used to reconstruct the experimental Stribeck curve.

The static friction parameters, $\hat{\theta}_{stat}$, are then identified by nonlinear least-squares fitting of the steady-state friction data to the Stribeck model.

b) Dynamic parameters: An acceleration input in the form of a pseudo-random binary sequence (PRBS) is applied to the open-loop system, while ensuring that the pre-sliding condition $f_e < F_s$ is satisfied. The resulting time-series data (f_e, x) are used to estimate the experimental frequency response function $G(j\omega)$.

This enables the identification of the pre-sliding transfer function, hereafter denoted $\hat{G}(j\omega)$, from which the dynamic friction parameters are estimated using a second-order process model with complex-conjugate poles, as described in (13).

2) Refinement Step: In this step, the excitation signal is designed such that the measured friction reflects the intrinsic frictional behavior of the system, without interference from structural dynamics. To this end, the velocity reference $\dot{x}_d$ is generated as low-frequency colored noise obtained by low-pass filtering a zero-mean, unit-variance Gaussian white noise signal.

The grey-box identification problem described in (10) is implemented in MATLAB using the `idnlgrey` constructor and the `nlgreyest` parameter estimation function. At iteration i, the nonlinear parameters $\hat{\theta}_{\mathrm{NL}}$ are estimated using the Levenberg–Marquardt algorithm, while the linear parameters $\hat{\theta}_{\mathrm{L}}$ are identified through constrained least-squares regression.

IV. Experimental Setup

A. XML-350 precision stage

The experimental setup consists of an XML-350 linear motor stage [15] from MKS Instruments (see Fig. 2), designed for high-precision positioning. An ironless PMLSM actuates the linear stage with a force constant of $27.5\,\mathrm{N/A}_{RMS}$, driven by an XPS-D motion controller with the driver card XPS-DRV11. Motion is guided by crossed roller bearings, introducing rolling friction. The stage offers a 350 mm travel range with a 11.8 kg moving mass, enabling velocities up to 300 mm/s. A Heidenhain linear encoder provides position feedback with a minimum incremental motion of 1 nm. The motor driver operates in closed-loop current control with a bandwidth of 400 Hz. Given the electrical dynamics of the motor, the control effort is proportional to the motor current, allowing the analysis to focus solely on the position control system.

B. Position Control System

1) Controller Architecture: The control law implemented in the manufacturer's controller is presented below for completeness. Fig. 3 illustrates this law without the feedforward friction compensation, together with the motion profile $(x_d, \dot{x}_d, \ddot{x}_d)$ used to excite the system.

$$u_{SD}(t) \triangleq k_i(x)\int_0^t (x_d - x)\,d\tau + k_p(x)(x_d - x) + k_d(x)(\dot{x}_d - \dot{x}) + \ddot{x}_d. \tag{14}$$

The baseline acceleration control implements a linear PID, defined by the gain triplet (K_p, K_i, K_d) with an acceleration feedforward term. The controller structure can be extended

Fig. 2: MKS Instruments - XML-350 Ultra-Precision Linear Motor stage used in the experimental setup [15].

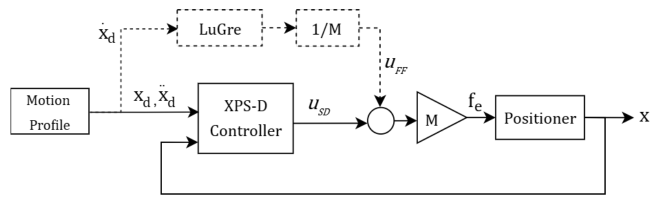


Fig. 3: Block diagram for the XPS-D controller with the add-on LuGre-based feedforward compensator.

to a state-dependent mode developed by the manufacturer to enhance control performance near the target position x_f [16]. This configuration yields the state-dependent PID (SD-PID), in which each gain (k_p, k_i, k_d) is modulated by an error-dependent factor as

$$k_j(x) \triangleq \left(1 + \alpha_j \frac{\beta}{|x_f - x| + \beta}\right) K_j \quad \text{with} \quad j \in \{p, i, d\}, \tag{15}$$

where $\alpha_j \in \mathbb{R}$ and $\beta \in \mathbb{R}^+$. In the vicinity of the target position ($|x_f - x| \to 0$), the state-dependent gain converge to $(1+\alpha_j)K_j$. Setting $\alpha_j = 0$ yields the linear PID.

2) Offline Friction Estimation and Model-Based Compensation: For this work, the linear PID gains are set to the XPS-D nominal values $K_p = 3 \times 10^6$, $K_i = 10 \times 10^6$, and $K_d = 8 \times 10^2$, while the SD-PID parameters are determined empirically as $\beta = 4 \times 10^{-4}$, $\alpha_p = -0.66$, $\alpha_i = 9$, and $\alpha_d = 0$.

For each experimental task, namely, identification and compensation, a dedicated controller configuration is employed, with motion trajectories generated independently to meet each task's specific objective.

a) Identification task: The linear PID with acceleration feedforward is used to ensure stable excitation and reproducible data for LuGre friction identification.

b) Compensation task: The SD-PID controller is employed to reduce the remaining control error caused by unmodeled dynamics. The identified LuGre model is then evaluated through a feedforward compensation scheme implemented as an add-on to the existing controller.

A key advantage of this strategy is its ease of deployment in the industrial motion controller, as the compensation term is added in feedforward. Moreover, the nominal feedback structure and its stability properties remain unchanged. However, the approach is limited by the use of the velocity reference. In contrast, feedback-based friction compensation strategies require an internal state observer [4], [8], which in turn requires access to lower-level control layers of the industrial controller. This perspective is left for future work.

The LuGre-based feedforward term estimates the friction force as $\hat{f}_r \triangleq LuGre(\dot{x}_d)$, computed from the desired velocity. The overall control law is therefore

$$u = u_{SD} + u_{FF} \quad \text{with} \quad u_{FF} = \frac{1}{M} \cdot \hat{f}_r, \tag{16}$$

as illustrated in Fig. 3. The LuGre-based feedforward compensator is evaluated on a point-to-point trajectory defined by an S-curve motion profile [16], implemented in the XPS-D controller, with a velocity of 250 mm/s and acceleration of 4000 mm/s^2.

V. Experimental Application of the Identification Methodology

In this section, the LuGre identification methodology previously introduced is applied to the high-precision XML-350 stage. The experimental protocol is adapted to account for the mechanical constraints and operating limits of this motion system. The data acquisition and processing procedures are then detailed.

A. Configuration of the Friction-oriented Experimental Protocol

1) Initialization Step: The static parameters set $\hat{\theta}_{stat}$ is identified using a trapezoidal velocity profile $\dot{x}_d$ that includes steady-state segments within the range $\dot{x}_{ss} \in [-30;30]$ mm/s. Subsequently, the dynamic parameters $\hat{\theta}_{dyn}$ are estimated using a PRBS excitation with a bandwidth up to 500 Hz and an RMS level of 5 mm/s^2.

2) Refinement Step: The velocity reference $\dot{x}_d$ is generated using low-frequency colored noise obtained through an 8th-order Chebyshev I low-pass filter with a cutoff frequency of 10 Hz. Two separate experiments, each generated with a distinct noise realization, are performed to build the identification and validation datasets $(\dot{x}, f_r)$. The resulting datasets consist of $N = 14000$ and $N = 10000$ samples, respectively. The single-objective optimization is executed for $k = 5$ iterations within the parameter domain $\Theta(\delta = 0.5)$.

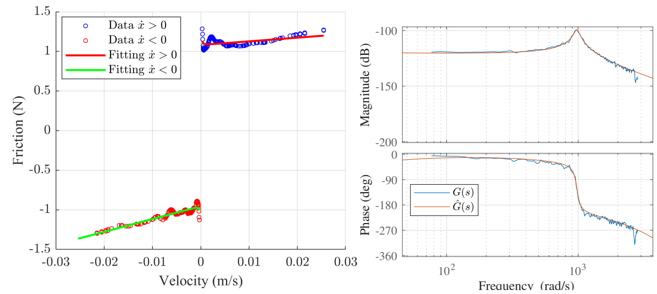
B. Data Gathering and Processing

Data are acquired at 10 kHz and, after filtering, decimated to 1 kHz. The measured signals are the controller output converted into the applied force and the slider position, yielding the pair (f_e, x). A zero-phase, second-order Butterworth low-pass filter with a cutoff frequency of 100 Hz is applied to reduce measurement noise. Velocity $\dot{x}$ and acceleration $\ddot{x}$ are computed from the filtered position signal using a second-order finite-difference scheme. The experimental friction force f_r is then obtained using the inertial model in (1). The processed dataset $(\dot{x}, f_r)$ forms the input to the identification procedure.

VI. Experimental Results

A. LuGre Friction Identification Results

The initialization step provides a reliable starting point for the LuGre parameter refinement. The initial parameter set $\hat{\theta}^{(0)}$ is derived from the estimated Stribeck curve shown in Fig. 4(a), combined with the identification of the linearized LuGre transfer function $\hat{G}(s)$ depicted in Fig. 4(b). The parameters identified at each stage, initialization and refinement, are reported in Table I. The final LuGre model accurately estimates the friction force, achieving 82.8% accuracy on the identification dataset and 74.4% on the validation dataset, as measured by the normalized root mean square error (NRMSE) between estimated and experimental friction forces. These results demonstrate that the identification procedure can be successfully applied to a motion system with different mechanical characteristics.



(a) (b)

Fig. 4: Initialization step of the LuGre identification procedure for the experimental linear stage: (a) Fitted Stribeck curve to the experimental data used for the static parameters estimation. (b) Experimental frequency response function $G(j\omega)$ and the linearized LuGre transfer function $\hat{G}(j\omega)$, used for dynamic parameters estimation.

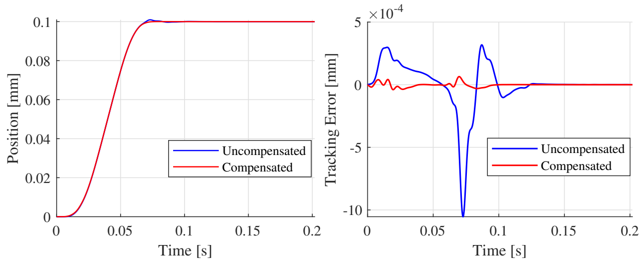
TABLE I: Identified LuGre Parameters for the XML-350 Stage

Parameter	Initialization Step			Refinement Step
	$\dot{x} < 0$	$\dot{x} > 0$	Initial	Final
F_s [N]	1.21	1.3	1.3	1.1
F_c [N]	0.94	1.08	1.02	0.95
σ_2 [kg/s]	17.03	4.66	10.41	15.62
v_s [m/s]	1.15×10^{-4}	1.53×10^{-4}	1.34×10^{-4}	1×10^{-5}
σ_0 [N/m]	-	-	11.28×10^6	11.02×10^6
σ_1 [kg/s]	-	-	9.76×10^2	1.46×10^3
b [N]	-	-	0	1×10^{-2}

B. LuGre-Based Compensation Friction Results

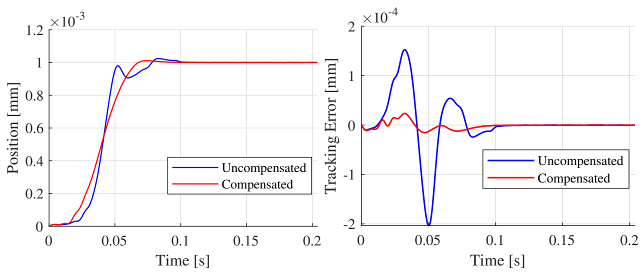
The identified LuGre model, integrated into the feedforward compensation scheme with the existing controller SD-PID (Fig. 3), is evaluated through point-to-point motions of the linear stage at 100 μm and 1 μm scales. The objective is to demonstrate the effectiveness of LuGre-based friction compensation in a high-precision motion system. These trajectories provide a natural test bench, encompassing kinetic friction during velocity transitions and pre-sliding stiction effects during settling.

The resulting trajectory tracking performance for each point-to-point motion is compared in Figs. 5 and 6. The tracking performance is significantly improved for both motion scales in terms of RMS and peak error reductions, as presented in Table II. This improvement serves as a practical means of validating the identification accuracy, since effective compensation reflects an accurate LuGre model.



(a) (b)

Fig. 5: Friction compensation results for a 100 μm point-to-point motion: (a) Measured positions with/without feedforward friction compensation. (b) Tracking error of the friction-uncompensated (blue) and feedforward compensated (red) systems.



(a) (b)

Fig. 6: Friction compensation results for a 1 μm point-to-point motion: (a) Measured positions with/without feedforward friction compensation. (b) Tracking error of the friction-uncompensated (blue) and feedforward compensated (red) systems.

VII. CONCLUSIONS

This paper extends the authors' LuGre friction model identification methodology to high-precision motion platforms with significantly distinct technological characteristics. The approach integrates a friction-focused experimental protocol with gradient-based nonlinear optimization, incorporating physics-based parameter initialization and refined calibration under colored-noise excitation.

Unlike existing LuGre identification methods—typically reliant on ad hoc excitation signals and conventional nonlinear optimization—this methodology introduces a structured experimental protocol combining complementary excitation signals. The refinement step, based on alternating estimation of linear and nonlinear parameter subsets, enables systematic parameter adjustment within the optimization framework.

Experimental validation on a high-precision MKS linear stage demonstrated the method's effectiveness, the LuGre model's robustness across diverse operating conditions and guidance types, and its practical utility in friction compensation strategies, as applied to the XML-350 Ultra-Precision Linear Motor Stage. Integrated into a feedforward compensation scheme, the identified model significantly enhances trajectory tracking performance and positioning accuracy.

The results complement prior work on a distinct technological platform, confirming the methodology's generality for linear positioning systems with varied mechanical and guidance characteristics. This approach provides a practical, reproducible, and transferable framework for LuGre model identification and industrial integration.

TABLE II: Effect of LuGre-Based Friction Compensation in Tracking Error Reduction Compared to the Uncompensated Case

Motion	RMS error reduction (%)	Peak error reduction (%)
100 μm	92.3	93.9
1 μm	87.3	88.4

VIII. ACKNOWLEDGMENT

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