Simple Two-Stage Damping Modulation Procedure in Linear System Feedback Designs

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Abstract— **Control design by state feedback in linear time-invariant (LTI) systems is approached by a new perspective, enabling to meet some basic design demands of many physical systems. The proposed approach is based on the separation of state feedback design in two sequential stages. As proven in the paper, in each of the two stages a reduced-order state feedback control problem has to be solved. Based on the results of both stages, a direct formula of constructing the state feedback closed-loop gain-matrix for the original system is established where the closed-loop eigenvalues are exactly the whole set of the particular eigenvalues as determined in each stage. While the assignment of each closed-loop eigenvalue in accurate position on the left-half plane is not always meaningful, especially for large-scale systems, a novel, Lyapunov-based method is adopted that decisively simplifies the procedure by assigning in each stage the corresponding eigenvalues as a set with common real parts. Except for its simplicity, the deployed two-stages procedure provides the basis for assigning part of the eigenvalues at dominant and others at non-dominant places, a significant fact since closed-loop slow and fast modes coincide with the nature of many real-world systems. Furthermore, in specific cases, even more simplified solutions can be obtained that modify state feedback into output feedback loops. To demonstrate the proposed scheme and validate its effectiveness, the interesting, standard flight cruise control problem is examined.**

I. Introduction

LINEAR time invariant (LTI) systems are the cornerstone in Systems Theory and Automatic Control designs [1-3]. For multivariable LTI systems with multi-inputs and multi-outputs (MIMO) the state-space representation is rather used, leading in state- or output-feedback methods of design. One of the most widely adopted methods that is certainly used to provide the suitable feedback gain-matrix, is referred to the well-known eigenvalue assignment technique [3-5]. Early works referred to the necessary and sufficient conditions and to methodologies of assigning all the closed-loop eigenvalues on the left half-plane [6,7]. Also, often, the study of nonlinear systems results in handling their linearized models to benefit from the strong theoretical background of LTI system feedback designs.

In more advanced designs, eigenstructure (eigenvalue-eigenvector) assignment methods have been proposed [8-10] that aim to extend system performance specifications beyond stability [11] under the cost of complex and cumbersome calculations. Other popular and easily implemented methods are based on linear quadratic regulator (LQR) designs [12]. LQR clearly provides a simpler approach for state feedback designs since it results from the solution of a Ricatti equation. However, it suffers from the fact that the damping characteristics cannot be predefined. For this reason, some attempts use advanced combined methods to focus on the minimization of control effort in the eigenvalue assignment process by proposing specific algorithms as in [13-15]. Other interesting studies mainly aim to achieve arbitrary pole-placement in preselected regions in the left-half complex plane through cumbersome procedures [16], or in combination with linear quadratic regulator (LQR) methods [17-21]. Therefore, as methods based on LQR concept can efficiently guarantee system stability with low calculation cost, when the latter methods are combined with eigenstructure assignment demands, the task becomes very difficult [17,22]. As a result, the development of more efficient and simplified feedback control design methodologies still remains an open problem.

While the principles and fundamentals for feedback designs based on eigenvalue assignment have been fully established, resulting however in quite complex procedures with high calculation effort, other characteristics coming from real-world applications are not taken into account. For example, for MIMO systems it is not clear in what positions one must decide to exactly assign all the closed-loop eigenvalues. Therefore, it is expended much more effort in using a demanded and cumbersome tool for a detailed design without any real reason. Nevertheless, for many real-world large or very large systems, such as power systems, other electromechanical systems, flight and aircraft models, biological systems, etc., there is not any need for exact pole-placement. On the contrary, precise eigenvalue assignment may lead to ineffective designs and to a confused handling far from the actual system specifications. Typically, the response of most large systems, [21-25], is simply characterized by slow and fast modes as a whole; specific units or system parts are categorized by their nature to follow one of these modes and it is unrealistic to push them beyond their capabilities. Therefore, it is of reasonable importance to exploit this natural property to simplify and to propose more beneficial and satisfactory control solutions.

In this paper, a different approach is proposed that permits the state feedback design for LTI systems in two sequential stages. The aim is to separately assign in each stage the

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corresponding set of the desired closed-loop eigenvalues. As proven in the paper, the first stage is fully independent from the second and establishes a reduced-order state feedback control problem. Based on the results of the first stage, in the second stage, a similar problem is solved; combining the two stage results the state feedback closed-loop gain-matrix of the original system is directly constructed. Hence the partitioning of the initial assignment problem into sequential two stages provide the basis for the separate assignment of the eigenvalues into slow and fast ones. The proposed deployment enables further simplification that is to effectively assign in each stage the eigenvalues as a set with common real parts. This task is implemented by a novel method capable of ensuring the sufficiency of a Lyapunov based design that satisfies, in each stage, the stable real part pole-placement at a desired common position. Also, certain other system features can be easily exploited to provide dominant eigenvalue designs or/and other interesting solutions, such as modifying the state feedback to output feedback design. The proposed scheme is particularly useful in most power systems [23,24] or standard flight cruise control problems [21,22]. Hence, the present work constitutes a decisive evolution of previous works of the authors [10,15,17] and especially [25], providing substantial novelties in pole assignment methods for LTI systems.

The paper is organized as follows. In Section II, some preliminaries for LTI systems are given. In III, the proposed design procedure is deployed in detail. In Sections IV and V, a flight control example and some conclusions are presented.

II. Preliminaries For LTI Systems And Problem Formulation

Linear time invariant (LTI) systems are described in the time domain by the following general state-space equation

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ represent the state and input vectors of the system with time invariant, plant matrix $A \in \mathbb{R}^{n\times n}$ and input matrix $B \in \mathbb{R}^{n\times m}$, respectively.

For the LTI system of (1), the following is considered.

Assumption 1. The pair (*A, B*) is a completely controllable pair.

It is also assumed that in (1), the following state feedback control law is applied

$$u = Kx \qquad (2)$$

where $K \in \mathbb{R}^{m\times n}$ is defined as the state feedback control gain matrix.

By incorporating the above controller into the original system, the closed-loop system is easily obtained as

$$\dot{x}(t) = (A + BK)x(t) \qquad (3)$$

with $A + BK$ representing the closed-loop system matrix.

In the next section, some novel theoretical results are established, and their characteristic properties are analyzed with aim to provide an efficient and computationally easy way for feedback designs of LTI systems.

III. State-Feedback Control Design Of LTI Systems By Establishing A Simplified Two Stage Procedure

A. Novel Sequential Approach for State-Feedback Control Design of LTI Systems

In this section, a fundamental Theorem is proven that enables us to establish a sequential approach for obtaining the closed-loop state feedback gain-matrix K by arbitrarily assigning the closed-loop eigenvalues. Furthermore, from the deployment of the proof of Theorem 1, it becomes clear how the particular procedure can be implemented into two sequential stages.

Theorem 1. For the LTI system of the form of (1), there exists a two-stage eigenvalue assignment approach, realized through the design of a state feedback control, as follows: In the first stage, the arbitrary partial assignment of the (*n-m*) closed-loop eigenvalues is performed. In the second stage, without affecting the eigenvalue assignment of the first stage, the remaining *m* eigenvalues are arbitrarily assigned as a sequel of the first stage procedure, and a unique state feedback gain-matrix K of the form of (2), is directly provided from the combined two stage procedure with assigned closed-loop eigenvalues of (3) the whole set.

Proof: Consider $\tilde{B}$ an $n\times(n-m)$ arbitrary constant matrix such that $[B \;\; \tilde{B}]$ is invertible. Let

$$Y_{sq} \triangleq \begin{bmatrix} Y \\ \tilde{Y} \end{bmatrix} = [B \;\; \tilde{B}]^{-1} \qquad (4)$$

where Y and $\tilde{Y}$ are of appropriate dimensions, $m\times n$ and $(n-m)\times n$, respectively.

We consider a similarity transformation $T^{-1}A_{cl}T$ of the closed-loop matrix $A_{cl} = A + BK$ by constructing T as follows

$$T = [B \;\; \tilde{B}]\begin{bmatrix} I_m & F \\ 0 & I_{n-m} \end{bmatrix} = \begin{bmatrix} B & \tilde{B} + BF \end{bmatrix} \qquad (5)$$

with T^{-1} calculating as

$$T^{-1} = \begin{bmatrix} Y - F\tilde{Y} \\ \tilde{Y} \end{bmatrix} \qquad (6)$$

where I_m and I_{n-m} are the *m* and (*n-m*) identity matrices, respectively, and F a constant matrix that will be determined later.

Applying the proposed transformation, it is obtained

$$T^{-1}A_{cl}T = \begin{bmatrix} [Y - F\tilde{Y}]A_{cl}B & [Y - F\tilde{Y}]A_{cl}[\tilde{B} + BF] \\ \tilde{Y}A_{cl}B & \tilde{Y}A_{cl}[\tilde{B} + BF] \end{bmatrix} \qquad (7)$$

and after algebraic manipulations wherein, according with the definition it holds true: $YB = I_m$, $\tilde{Y}B = 0$, it is implied

$$T^{-1}A_{cl}T = \begin{bmatrix} A_{11} + KB & A_{12} + A_{11}F - FA_{22} + KBF + K\tilde{B} \\ A_{21} & A_{22} + A_{21}F \end{bmatrix} \qquad (8)$$

where $A_{11} = YAB - F\tilde{Y}AB$, $A_{12} = YA\tilde{B}$, $A_{21} = \tilde{Y}AB$ and $A_{22} = \tilde{Y}A\tilde{B}$.

Now, defining the mx(n-m) $K\tilde{B}$ matrix by $K_2 \triangleq K\tilde{B}$, and in a similar manner matrix KB which is an mxm square matrix, by $K_1 \triangleq KB$, and constraining K_2 to satisfy the following relation

$$K_2 = -A_{12} - A_{11}F + FA_{22} - K_1 F \tag{9}$$

the last expression (8) of the closed-loop similarity transformation $T^{-1}A_{cl}T$ becomes

$$T^{-1}A_{cl}T = \begin{bmatrix} A_{11} + K_1 & 0 \\ A_{21} & A_{22} + A_{21}F \end{bmatrix} \tag{10}$$

Expression (10) clearly reveals that the (n-m) closed-loop eigenvalues of $T^{-1}A_{cl}T$ and obviously of A_{cl}, can be arbitrarily assigned at preselected positions, as the eigenvalues of

$$A_{22} + A_{21}F \tag{11}$$

by suitably determining the auxiliary gain-matrix F, while the remaining m eigenvalues of A_{cl} can be arbitrarily assigned at preselected positions, as the eigenvalues of

$$A_{11} + K_1 \tag{12}$$

by suitably determining the auxiliary gain-matrix K_1.

Therefore, it is evident that the eigenvalue assignment procedure is partitioned into two independent stages:

In the first stage, we determine F by solving a (n-m) eigenvalue assignment problem by state feedback for the reduced-order system (A_{22}, A_{21}). To that end, it is necessary for the pair (A_{22}, A_{21}) to be completely controllable. Nevertheless, this condition holds true due to Assumption 1, since it can be trivially proven from the structure of (A_{22}, A_{21}) that this pair is completely controllable, if and only if, the pair (A, B) is completely controllable [3].

In a second stage, we have all prerequisites to determine K_1 by solving an m eigenvalue assignment problem by state feedback for the reduced-order system (A_{11}, I_m) where matrix A_{11}, depending on F, is known at the present stage since F has been calculated in the first stage, and I_m is the mxm identity matrix. To that end, it is necessary for the pair (A_{11}, I_m) to be completely controllable, a fact that obviously holds.

Now, after the calculation of F and K_1, restriction (9) can be satisfied, enabling us to analytically calculate K_2. This is a crucial point since the whole two stage procedure is valid under the strict condition of (9) that relates F and matrices K_1 and K_2, as these are previously defined by

$$\begin{bmatrix} K_1 & K_2 \end{bmatrix} = K\begin{bmatrix} B & \tilde{B} \end{bmatrix} \tag{13}$$

and explicitly provide the closed-loop system gain-matrix as

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}\begin{bmatrix} Y \\ \tilde{Y} \end{bmatrix} = K_1 Y + K_2 \tilde{Y} \tag{14}$$

Indeed, according to the previous analysis, K preserves the positions of the n closed-loop system eigenvalues exactly at the positions where the (n-m) and m eigenvalues of (11) and (12), respectively, have been sequentially assigned. ■

With reference to Theorem 1 and particularly to the two-stages eigenvalue assignment procedure, we make the following Remarks.

Remark 1. In the first stage, the arbitrary partial assignment of the (n-m) closed-loop eigenvalues as achieved through (11), is fully independent for the next stage. However, from the structure of (10), it is apparent that it provides the capability to inspect, through matrix A_{11}, in what positions the remaining m eigenvalues are posed before continuing with the second stage. This is a new result with respect to the conventional partial eigenvalue assignment methods that cannot separate the partial eigenvalue assignment from the remaining eigenvalues.

Remark 2. With respect to the computational effort of assigning the whole set of the n closed-loop eigenvalues, if one adopts conventional pole-placement techniques, in both stages, there is not any effort reduction. Therefore, the proposed separation in two stages alone does not in general provide significant computational and handling improvements if it is not followed by other more advanced and easily applied methods.

Since the design complexity problem still exists, in the next subsection, some recent novel techniques are incorporated into the proposed design to substantially improve the effectiveness and the computational effort.

B. Simplifying the Two Stages Approach of State-Feedback Control Design

Our aim is to combine the two-stages procedure presented in the previous subsection with computationally and practically efficient methods that can assign independently the (n-m) and m closed-loop eigenvalues on the basis of the desired performance and the capabilities of the closed-loop system. For example, there are systems with their linearized model having in open-loop, some dominant eigenvalues with low real part near to the imaginary axis and the others more to the left. Such systems are many power systems where the mechanical response is much slower than the one of the electromagnetic part, or other electromechanical systems, or flight and aircraft control cases, etc. with similar characteristics. In most of these cases, it is needed to maintain the slow and fast characteristics in closed-loop, and, if possible, to improve the assignment of the dominant eigenvalues in slightly more stable positions, according to the actuators' capabilities, without taking special care for the fast dynamics. To this end, a feedback design based on two sequential stages, as in the proposed procedure, is especially useful and becomes as effective as it is implemented easier.

To satisfy the previously mentioned criteria, some recent design novelties proposed by the authors, are considered in the form stating by Theorem 2, recalled from [25].

Theorem 2. Consider the LTI system of (1) and the corresponding feedback control law (2) and let matrix A have distinct eigenvalues. Then, the controller gain-matrix,

$$K = -B^T P, \quad P = X^{-1} \tag{15}$$

guarantees global exponential stability (GES), by assigning all the eigenvalues $\lambda\left(A_{cl}\right)$ of the closed-loop matrix $A_{cl} = A + BK$ on the left half-plane with all real parts, having a prescribed damping rate ρ: $\operatorname{Re}\lambda\left(A_{cl}\right) = -\rho$, where $X = X^T > 0$, always exists as the positive definite solution of the Lyapunov equation

$$[-A - \rho I]X + X[-A - \rho I]^T = -2BB^T \tag{16}$$

under the obvious condition

$$\rho > \left|\operatorname{Re}\lambda(A)\right|_{\max}. \tag{17}$$

As Theorem 2 provides a direct Lyapunov-based batch and therefore efficient method for assigning the real part of a set of eigenvalues, our prime intension is to use it at each of the two stages proposed. Particularly, in the first stage to assign the real parts of the *n-m* closed-loop eigenvalues and in the sequel, second stage, to assign the remaining *m* eigenvalues with real parts to another position.

C. The Simplified Design Procedure

According to the analysis presented in subsections III-A and III-B, the following design procedure is proposed:

<u>Step 1</u>: Select suitably matrix $\tilde{B}$ and conduct the transformations indicated in the proof of Theorem 1.

<u>Step 2</u>: Apply the procedure indicated in Theorem 2 by corresponding the pair $(A_{22},\ A_{21})$ to (*A, B*); select the desired real part of all the (*n-m*) eigenvalues at $-\rho$, under restriction (17) and calculate F from (15) by solving (16).

<u>Step 3</u>: With F coming from Step 2, calculate A_{11}. Apply the procedure of Theorem 2 by corresponding the pair $(A_{11},\ I_m)$ to (*A, B*); select the desired real part of all the *m* eigenvalues at a new $-\rho$, under the restriction of (17) and calculate, in a similar way, gain K_1.

<u>Step 4</u>: Given F and K_1, calculate K_2 from eq. (9).

<u>Step 5</u>: For the calculated K_1 and K_2, the closed-loop gain matrix K is obtained through (14).

The proposed design procedure, in Step 3, may be substituted by another even easier pole assignment method due to the square form of the gain-matrix K_1. Thus, continuing the comment of Remark 1, it becomes clear that in the second stage (Step 3) one has the possibility either to move the remaining *m* eigenvalues in new desired positions without affecting the eigenvalue assignment of the first stage, or to leave them $(K_1 = 0)$, if the eigenvalues of A_{11} are in satisfactory stable positions.

Therefore, taking into account the previous comments and the pole-assignment partitioning methodology shown in Theorem 1, we propose the following Corollary.

Corollary 1. For the LTI system of the form of (1), a one-stage assignment of the (*n-m*) closed-loop eigenvalues can be applied leading to output feedback control designs, provided that the remaining *m* eigenvalues preserve stability.

According to Corollary 1, if the *m* eigenvalues of A_{11} are in satisfactory stable positions, then selection of $K_1 = 0$ in (12), does not move the *m* eigenvalues of A_{11} and does not affect the assignment of the (*n-m*) closed-loop eigenvalues while it provides from (14) the output feedback control law,

$$u = K_2 C x \tag{18}$$

with output matrix $C = \tilde{Y}$.

Now, in cases where matrix A_{22} can have eigenvalues with their real parts near the imaginary axis, which allows restriction (17) to be satisfied for small enough ρ, then it is apparent that, in Step 2, the assignment of the dominant closed-loop eigenvalues becomes easy. In the case of large-scale systems with fast and slow modes, it is significant to exploit this sequential and separate assignment procedure with strict priority to assign the dominant eigenvalues at desired positions. In practice, under normal conditions, if in the first stage the whole set of dominant eigenvalues (slow modes) is assigned in stable enough places but near the original slow modes, far from the remaining *m* fast modes, then in most of the cases the fast eigenvalues are slightly moved often near the open-loop stable positions.

In the next Section, the proposed procedure is validated on a classic example of aircraft cruise control application.

IV. Implementation Of The Proposed Approach To Aircraft Cruise Control

The proposed procedure is applied to enhance the dynamic response of an L-1011 aircraft at cruise conditions.

A. Aircraft Model at Cruise Conditions

The standard linearized model of the aircraft in state space, has been widely used as basic cruise model [21,22] for feedback control designs; it is given in the form of (1), by a 6th order system with two inputs.

Plant matrix A is given as

$$A = \begin{bmatrix} -20 & 0 & 0 & 0 & 0 & 0 \\ 0 & -25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -0.744 & -0.032 & 0 & -0.154 & -0.0042 & 1.54 \\ 0.337 & -1.12 & 0 & 0.249 & -1 & -5.2 \\ 0.02 & 0 & 0.0386 & -0.996 & -0.0003 & -0.117 \end{bmatrix}$$

whereas the input matrix B is

$$B = \begin{bmatrix} 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

The 6th-order state vector is as follows

$$x = [\delta_r \quad \delta_a \quad \phi \quad r \quad p \quad \beta]^T$$

where states δ_r, δ_a, ϕ, r, p, β denote the rudder deflection, aileron deflection, bank angle, yaw rate, roll rate and sideslip angle, respectively. The system input is $u = [u_r \quad u_a]^T$ with u_r and u_a the rudder command input and the aileron command input, respectively.

The open-loop eigenvalues of state matrix A are given in Table I. One can immediately see that the first four open-loop eigenvalues have small or very small real part values while the last two are relatively far to the left on the real axis. It is noted that the system states' response of ϕ, r, p, β which are affected by the slow modes, are of prime importance. Therefore, the aim is to move the first four eigenvalues in more stable positions to the left but not far to the left since the input effort must be reasonably implemented by the actuators.

B. Applying the Proposed Design Procedure

Following the Steps of the design procedure presented in Subsection III-C, we first select

$$\tilde{B} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T .$$

We calculate the pair (A_{22}, A_{21}). The eigenvalues of A_{22} are exactly the first four open-loop eigenvalues of A. We select the real part of the first four closed-loop eigenvalues at the value: -1.5 which means $\rho = 1.5$.

We calculate gain F:

$$F = \begin{bmatrix} -0.0044 & 0.1841 & 0.0064 & -0.2852 \\ 0.0952 & 0.0483 & 0.0671 & -0.1460 \end{bmatrix}$$

Then, we calculate A_{11}. The eigenvalues of A_{11} are: {-17.1708, -23.1002}, which are far to the left and near enough to the non-dominant open-loop eigenvalues of A.

Now, to further proceed, the following two different cases are considered.

Case A: The methodology of Theorem 2 is followed for designing $A_{11} + K_1$. To calculate K_1, we select the real part of the two last closed-loop eigenvalues at the negative value: -25, which means $\rho = 25$. Then

$$K_1 = \begin{bmatrix} -7.8113 & -0.3254 \\ -0.3254 & -1.9177 \end{bmatrix}$$

and

$$K_2 = \begin{bmatrix} -0.1202 & 4.8610 & 0.1476 & -6.8472 \\ 2.3753 & 1.3632 & 1.7057 & -3.9063 \end{bmatrix}$$

Finally, the closed-loop gain matrix: $K=$

$$\begin{bmatrix} -0.3906 & -0.0130 & -0.1202 & 4.8610 & 0.1476 & -6.8472 \\ -0.0163 & -0.0767 & 2.3753 & 1.3632 & 1.7057 & -3.9063 \end{bmatrix}.$$

Case B: Matrix K_1 is selected to be: $K_1 = 0$.

This means that the last two closed-loop eigenvalues remain at the positions of the A_{11} eigenvalues with K_2 calculated:

$$K_2 = \begin{bmatrix} -0.1171 & 3.4068 & 0.0761 & -4.5716 \\ 2.1941 & 1.2106 & 1.5749 & -3.5336 \end{bmatrix}$$

which, according to Corollary 1, constitutes the output feedback gain-matrix with output matrix C, from (18):

$$C \triangleq \tilde{Y} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} .$$

Then, the closed-loop system matrix is $A_{cl} = A + BK_2C$.

In Table I, the open-loop and closed-loop eigenvalues are given for the two cases examined. As expected, the first four closed-loop eigenvalues are the same in both cases with real parts as desired at -1.5 to remain again the dominant ones.

TABLE I
SYSTEM EIGENVALUES

Examined case	Open- and Closed-loop system eigenvalues
Open-loop	$\{-0.0092 \quad -0.0882 \pm j1.2695 \quad -1.0855 \quad -20 \quad -25\}$
Case A	$\{-1.5 \pm j0.7380, \quad -1.5 \pm j1.9424, \quad -25, \quad -25\}$
Case B	$\{-1.5 \pm j0.7380, \ -1.5 \pm j1.9424, \ -17.1708, \ -23.1002\}$

C. Simulation Results

The aircraft model at cruise conditions is simulated and the responses of the system states are addressed in open-loop and closed-loop operation for the two cases examined.

The initial state values are defined as:

$$x(0) = \begin{bmatrix} 2.5 & 1.3 & 0.5^o & 0 & 0 & -0.5^o \end{bmatrix}^T .$$

In Figure 1, the open-loop response of the system is presented. It is easily seen that due to the weak damping caused by the slow eigenvalues, a slow swing convergence of the state is observed. As shown in Figure 2, which is inserted for comparison reasons to provide in detail the open

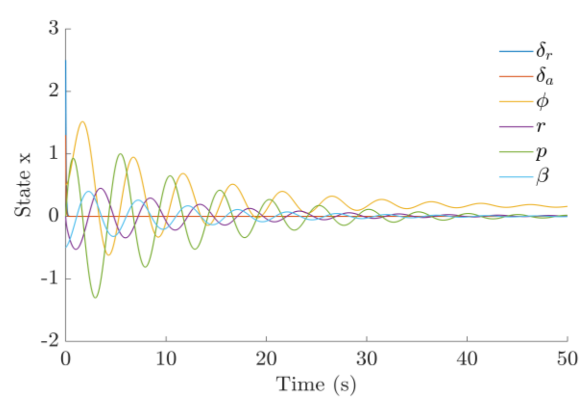

Fig. 1. Open-loop state response.

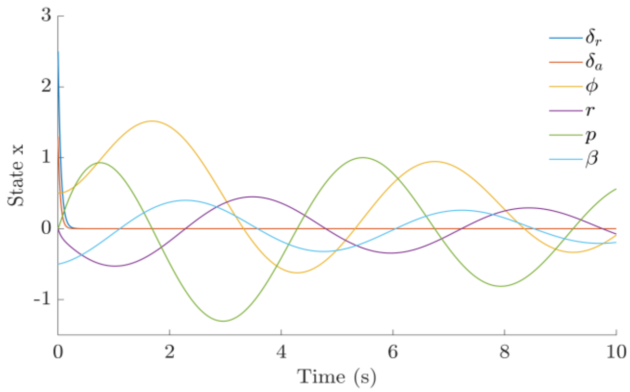

Fig. 2. Open-loop state response (detail: from t=0 to t=10s).

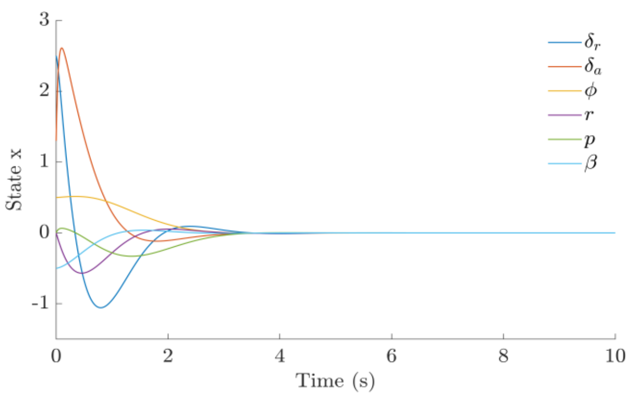

Fig. 3. Closed-loop state response.

loop state response during the first 10 seconds, unacceptable swings for the aircraft performance are observed in all the critical states: ϕ, r, p, β while the other two states converge very fast to zero.

Figure 3 is taken from simulating the closed-loop system performance for Case B that provides the assignment of only the dominant eigenvalues through output feedback control. Case A is not exhibited because the state responses do not involve any observable difference with respect to that of Figure 3. This is an excepted result caused by the assignment of the four dominant poles in both the examined cases at the same positions while the other two poles are placed in almost same positions far to the left. The system responses of all the critical states: ϕ, r, p, β are clearly enhanced with smooth and fast enough convergence due to the common assignment of the dominant eigenvalues' damping. The cost seems to be now the slower convergence of the other two states, which however is of low importance.

V. Conclusions

In the frame of the eigenvalue assignment techniques, a novel and efficient two-stages state-feedback design method is analytically developed. The proposed sequential design approach is extended to incorporate simple Lyapunov-based techniques that assign the eigenvalues as a set in each stage, with common real parts. The method is easily implemented with main contribution to allow under conditions, the separate assignment of the slow and fast eigenvalues. The procedure is demonstrated on a standard aircraft flight cruise model, which fully verifies its capabilities to provide certain design and performance enhancements.

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