String Stabilization of Predecessor-Following Vehicle Platoons Using Linear Quadratic Integral Control

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***Abstract*— This paper studies homogeneous vehicle platoons modeled as continuous-time LTI systems under a predecessor-following topology and a constant time-headway spacing policy. The local controller is designed through a Linear Quadratic Integral framework augmented with a double integrator in order to achieve zero steady-state error for ramp-type references. For this setup, we show that internal stability of the local closed loop does not in general imply string stability. We then characterize string stability through the $\mathcal{H}_\infty$ norm of the transfer function between consecutive spacing errors and provide a state-space bounded-real Riccati condition based on a reduced-order closed-loop realization. Numerical examples illustrate both string-stable and string-unstable platoons under different choices of time headway and weighting matrices.**

***Index Terms*— String stability, platooning system, LQI, double integrator.**

I. Introduction

The growing interest in vehicle platoon systems has driven the development of advanced control techniques to ensure safety, efficiency, and robustness in cooperative driving [1]. In particular, vehicle platooning (the coordinated motion of vehicle formations) has emerged as a promising strategy to improve traffic flow, reduce energy consumption, and enhance road safety [2], [3].

Vehicle platoons operate in a coordinated manner by maintaining precise inter-vehicular distances. However, achieving stable and scalable platoon control presents significant challenges, primarily due to the need to guarantee string stability. This critical property prevents disturbances from amplifying along the vehicle string; without it, even minor perturbations in the leader's motion could propagate and escalate downstream, potentially causing dangerous oscillations or collisions [4]–[6].

To address these challenges, researchers have explored various control architectures and communication topologies. Common approaches include, among others, Predecessor-Follower (PF), Leader-Predecessor-Follower (LPF), Bidirectional (BD), and Bidirectional-leader (BDL) schemes, each offering distinct trade-offs regarding communication complexity and performance [4], [7]. This work focuses specifically on the PF topology, where maintaining string stability requires careful selection of the Constant Time Headway (CTH), denoted by h, to ensure adequate inter-vehicle spacing [8]. This parameter adjusts the distance between consecutive vehicles based on the vehicle's instantaneous velocity, such that the steady-state following distance is affine to h. It is well established that achieving string stability is impossible if h is set to 0 [9].

In this topology, a double-integrator structure is required to track ramp-type position references, which naturally arise when the platoon moves at a common constant velocity. In the homogeneous case, a classical condition for string stability is that the complementary sensitivity function of each local loop satisfies $|T_i(j\omega)| \leq 1$ for all $\omega \geq 0$, or equivalently $\|T_i\|_\infty \leq 1$ [10].

Although control schemes based on double integrators are common, optimal control approaches remain underexplored in this context. A classic optimal controller is the well-studied LQR controller [11], [12]. An extension of this controller is the LQI controller, which is an LQR controller with integral action enabling effective constant reference tracking. In continuous time, it uses an augmented state based on the original system, including the integral of the error as an additional state. The LQR controller is then applied to this augmented system [13], [14]. This controller has been widely used in different works (see e.g., [15]–[19]).

To track ramp references, it is necessary to include a double integrator in the LQR controller and define a new augmented system, applying the LQR controller to this system. In particular, several works have used this controller in platooning systems. For example, [20] proposes a Linear Quadratic Integral control with a double integrator (hereafter referred to as LQI for simplicity) for vehicles with Adaptive Cruise Control (ACC). This work achieves tracking and string stability via the CTH obtained through Nyquist/pole-zero analysis of the spacing error transfer functions using fixed weighting matrices Q and R. Similarly, [21] develops an LQI for homogeneous ACC platoons using Nyquist diagrams of spacing error transfer functions. Likewise, [22] proposes an LQI controller for ACC systems with stability analysis in the frequency domain under variable CTH values (with fixed controller gain), yet without examining how the selection of the weighting matrices Q and R affects stability when the CTH is constant.

These works [20]–[22] first derive an LQI controller for a prescribed choice of Q, R, and h, and then assess string sta-

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bility by varying only the time-headway parameter. However, since the feedback gain depends on the joint selection of Q, R, and h, this approach does not characterize their combined influence on string stability. Moreover, in those studies, the control-loop reference is the inter-vehicle distance measured by onboard sensors, which is consistent with ACC architectures without vehicle-to-vehicle communication.

A preliminary numerical study of this joint dependence was presented in [23], where the effect of the LQI weighting matrices and the time-headway constant on string stability was explored through simulations. The present work extends that study by providing a theoretical characterization of ramp tracking and string stability for continuous-time homogeneous platoons under a constant time-headway spacing policy, a predecessor-following topology, and Cooperative Adaptive Cruise Control technology.

The main contributions are twofold. First, we show that a standard LQI structure with single integral action is insufficient to achieve zero steady-state error for ramp-type references, even when the plant already contains integral action. This motivates the use of a double-integrator augmentation. Second, we show that internal stability of the resulting LQI closed loop does not, in general, imply string stability. To address this issue, we derive a tractable string-stability condition based on the bounded real lemma and a reduced-order closed-loop realization. Numerical simulations illustrate both string-stable and string-unstable behaviors under different choices of time headway and weighting matrices.

The remainder of this paper is organized as follows. Section II defines the platoon system configuration and the LQI controller with a double integrator. Section III presents the analysis of the control loop sensitivity functions and establishes a condition for the LQI controller that guarantees string stability. Section IV provides numerical examples illustrating our results. Finally, Section V presents the concluding remarks.

II. Problem formulation

The system under study is a collection of N homogeneous vehicles, where each vehicle is modeled by a continuous-time LTI plant $G(s)$, described in state space by the matrices (A, B, C, D), together with its local controller. These vehicles operate on a one-dimensional track and employ a predecessor-following (PF) topology, that is, where each vehicle communicates only with its immediate predecessor [24], [25]. We assume that the i-th agent, $1 \leq i \leq N$, has access to its own state $x_i(t)$, and that each vehicle transmits its absolute position $y_i(t)$ (ramp signal) to the immediate follower. The control objective is to maintain the inter-vehicle distances $\ell_i(t)$ equal to a desired reference $r_i(t)$ whenever possible. This implies that the agents move together at a constant speed or remain stationary in a desired formation.

To reduce collision probability, we assume a spacing policy such that as the speed of the i-th vehicle increases, so does the desired spacing $r_i(t)$. Specifically, we consider

$$r_i(t) = \varepsilon_i + hv_i(t),$$

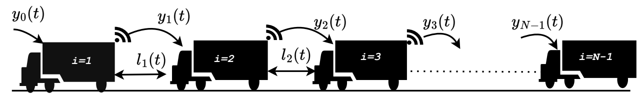

Fig. 1. One-dimensional platooning configuration.

where $\varepsilon_i > 0$ is a constant representing the minimum desired inter-vehicle spacing, and $h \geq 0$ is a constant weighting the importance of vehicle speed. The constant h is called the constant time headway. For $h = 0$, the system implements a constant spacing policy, aiming to maintain a fixed distance ε_i between agents. If $h > 0$, the control maintains distances that increase proportionally with agent speed [8].

For simplicity, we assume that each vehicle length is zero, $\varepsilon_i = 0$, and that h and the initial conditions $y_i(0)$ and $v_i(0)$ for $i = 1, \dots, N$ are compatible, i.e., chosen such that $e_i(0) = 0$ for all i. In this paper we focus on studying the effect of the controller structure LQI (Linear Quadratic integrator with Double Integrator) on the spacing error $e_i(t)$. Since $\ell_i(t) = y_{i-1}(t) - y_i(t)$, we have

$$e_i(t) = y_{i-1}(t) - y_i(t) - r_i(t), \tag{1}$$

and thus, the resulting local control loop is modeled as shown in Figure 2, where each loop is implemented as a state-feedback scheme with double integral action on the output error. The system considers the plant $\tilde{G}$, where the gain k weights the states $x_i(t)$, and the gains k_z and k_g weight the augmented state from the single and double integral of the tracking error e_i, respectively. In this framework, let $x_i(t)$ represent the state vector and $u_i(t)$ denote the control input of the i-th vehicle. The position output is given by $y_i(t) = Cx_i(t)$. Additionally, we define an auxiliary output of interest as $w_i(t) = hv_i(t) + y_i(t)$. It is explicitly assumed that the state vector $x_i(t)$ is structured such that this auxiliary output can be obtained via a linear transformation defined by a matrix C_w, yielding $w_i(t) = C_w x_i(t)$. The vehicle's overall dynamics are represented by the state-space model $G : (A, B, C, D)$, while the notation $G_x : (A, B, I_n, D)$ refers to the realization where the full state vector is considered the output.

The main goal of this paper is to establish conditions for the LQI controller that guarantee string stability of the platoon. It is not difficult to verify that using a single-integrator LQI controller leads to a steady-state tracking error because the reference signal is a ramp signal. Hence, in this work we consider an LQI controller with a double integrator, enabling ramp reference tracking and thus achieving zero steady-state tracking error.

A. LQI with double integrator

The LQI controller structure emerges from solving an optimal control problem that simultaneously minimizes both the energy of system states and control signals [11], [26]. Through appropriate design of the LQI cost functional, this approach enables precise state regulation to desired operating points while explicitly controlling their transient trajectories and convergence rates. The double integrator version of

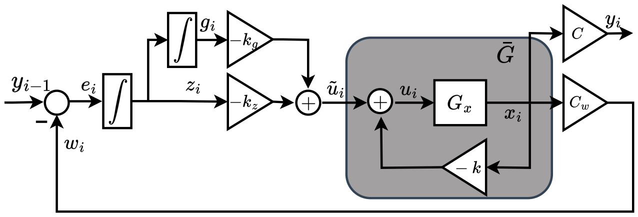

Fig. 2. LQI with double integrator control loop.

the LQI controller inherently incorporates both single and double integral actions of the tracking error, guaranteeing zero steady-state error for ramp references and effective rejection of low-frequency disturbances.

To properly account for these integral actions in the LQI optimization framework, we define an augmented system $G_a : (A_a, B_a, C_a, D_a)$ for vehicle i, which includes both the first and second integral of the tracking error as additional states of the system, and then apply standard LQR techniques to solve the underlying minimization problem. This optimization problem is formulated through the following cost function:

$$\mathcal{J} = \int_0^\infty \left(\mathcal{X}_i(t)^T Q \mathcal{X}_i(t) + u_i(t)^T R u_i(t)\right) dt, \qquad (2)$$

where $Q \geq 0$ and $R > 0$ represent the state and control weighting matrices, respectively, and $\mathcal{X}_i(t)$ denotes the state vector of the augmented system. For our specific platooning application, we define the state vector as

$$\mathcal{X}_i(t)^T = [x_i(t) \quad z_i(t) \quad g_i(t)]^T$$

where x_i denotes the states of the system G, $z_i(t) = \int e_i(t)dt$ and $g_i(t) = \iint e_i(t)dtdt$.

The optimal feedback gain matrix $\mathcal{K}$ is obtained through:

$$\mathcal{K} = R^{-1} B_a^T P \qquad (3)$$

where P represents the solution to the continuous-time algebraic Riccati equation:

$$A_a^T P + P A_a - P B_a R^{-1} B_a^T P + Q = 0. \qquad (4)$$

This yields the optimal control input $u_i^{opt}(t) = -\mathcal{K}\mathcal{X}_i(t)$, with $\mathcal{K} = [k \;\; k_z \;\; k_g]$.

Figure 2 illustrates the resulting control architecture. The state-feedback gain $-k$ closes the loop around the plant state dynamics, yielding the internally stabilized subsystem $\bar{G}$. This composite subsystem takes $\tilde{u}_i(t)$ as input and produces $w_i(t) = C_w x_i(t) = y_i(t) + h v_i(t)$ as output, thereby defining the complete LQI control loop.

The controller design process begins with the state-space representation of the augmented system:

$$\underbrace{\begin{bmatrix}\dot{x}_i(t)\\ \dot{z}_i(t)\\ \dot{g}_i(t)\end{bmatrix}}_{\dot{\mathcal{X}}_i(t)} = \underbrace{\begin{bmatrix}A & 0 & 0\\ -C_w & 0 & 0\\ 0 & 1 & 0\end{bmatrix}}_{A_a}\begin{bmatrix}x_i(t)\\ z_i(t)\\ g_i(t)\end{bmatrix} + \underbrace{\begin{bmatrix}B\\ 0\\ 0\end{bmatrix}}_{B_a} u_i(t) + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix} y_{i-1}(t)$$

$$w_i(t) = \underbrace{\begin{bmatrix}C_w & 0 & 0\end{bmatrix}}_{C_a}\begin{bmatrix}x_i(t)\\ z_i(t)\\ g_i(t)\end{bmatrix}$$

III. Sensitivity Functions and String Stability Condition

This section addresses two fundamental requirements for vehicle platooning: zero steady-state error and string stability. First, we analyze the steady-state tracking performance for ramp references, comparing the standard LQI controller against the double-integrator version in order to justify the need for double integral action. Subsequently, we derive a string-stability condition for the LQI controller with double integrator, based on the $\mathcal{H}_\infty$ norm of the transfer function between consecutive spacing errors and expressed through a bounded-real Riccati inequality.

A. Sensitivity function analysis

The objective of the control system is to achieve zero steady-state error for the reference signal of interest. In the present platooning setup, the predecessor position acts as a ramp-type reference. Therefore, it is natural to analyze the transfer function from $Y_{i-1}(s)$ to the internal spacing error $E_i(s)$. This transfer function is given by

$$E_i(s) = \frac{1}{1 + C_w \bar{G}(s)\left(\frac{-k_g - k_z s}{s^2}\right)} Y_{i-1}(s). \qquad (5)$$

The following proposition formalizes the role of the double-integrator term in achieving exact tracking of ramp references.

Proposition 1: Assume that the local closed-loop system is internally stable and that the corresponding limits below exist. Then, for a ramp reference $Y_{i-1}(s) = 1/s^2$:

- if $k_g = 0$ (standard LQI structure with single integral action), the steady-state tracking error is in general nonzero;
- if $k_g \neq 0$ (double-integrator LQI structure), the steady-state tracking error is zero.

Proof: Given that the local closed-loop system is internally stable by design of k, the final value theorem can be used to characterize the steady-state tracking error as $\lim_{t\to\infty} e_i(t) = \lim_{s\to 0} sE_i(s)$.

For the standard LQI controller, the double-integrator gain is absent, i.e., $k_g = 0$. Then, for a ramp input

$$\lim_{s\to 0} sE_i(s) = -\frac{1}{C_w \bar{G}(0) k_z}.$$

Hence, the steady-state error is, in general, a nonzero constant. In contrast, for the LQI controller with double integrator ($k_g \neq 0$), we have

$$\lim_{s\to 0} sE_i(s) = \lim_{s\to 0} \frac{s}{s^2 + C_w\bar{G}(s)\left(-k_g - k_z s\right)} = 0,$$

provided that $C_w\bar{G}(0)k_g \neq 0$. Therefore, the double-integrator augmentation is the appropriate LQI structure for exact ramp tracking in the present platooning problem. ■

Proposition 1 shows that double integral action is not merely a controller-design choice, but a structural requirement for achieving zero steady-state tracking error under ramp-type predecessor signals.

B. String stability condition

String stability requires that the transfer function between consecutive vehicle errors satisfy the following condition:

$$\left\| \frac{E_{i+1}(s)}{E_i(s)} \right\|_\infty \leqslant 1 \tag{6}$$

where $\|\cdot\|_\infty$ denotes the infinity norm and $\frac{E_{i+1}(s)}{E_i(s)}$ represents the error propagation dynamics.

From the local closed-loop relation derived in the previous subsection, the spacing error satisfies

$$E_i(s) = \frac{1}{1 + C_w\bar{G}(s)\left(\dfrac{-k_g - k_z s}{s^2}\right)} Y_{i-1}(s), \tag{7}$$

whereas the vehicle position output is given by

$$Y_i(s) = C\,\bar{G}(s)\left(\frac{-k_g - k_z s}{s^2}\right) E_i(s). \tag{8}$$

By homogeneity, the same local relation holds for the next vehicle. Therefore, substituting (8) into the corresponding expression for $E_{i+1}(s)$ yields the transfer function between consecutive spacing errors:

$$\frac{E_{i+1}(s)}{E_i(s)} = \frac{C\,\bar{G}(s)\left(\dfrac{-k_g - k_z s}{s^2}\right)}{1 + C_w\bar{G}(s)\left(\dfrac{-k_g - k_z s}{s^2}\right)}. \tag{9}$$

An equivalent reduced-order state-space realization of (9) is obtained by considering the augmented closed-loop dynamics of a single vehicle

$$\dot{\mathcal{X}}_i(t) = \underbrace{\begin{bmatrix} A - Bk & -Bk_z & -Bk_g \\ -C_w & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{A_c} \mathcal{X}_i(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{B_c} y_{i-1}(t), \tag{10}$$

$$y_i(t) = \underbrace{\begin{bmatrix} C & 0 & 0 \end{bmatrix}}_{C_y} \mathcal{X}_i(t). \tag{11}$$

Furthermore, substituting (7) into (8) yields the exact expression derived in (9), proving both transfer functions are identical. Then the transfer function between consecutive spacing errors admits the reduced realization

$$\frac{E_{i+1}(s)}{E_i(s)} = C_y(sI - A_c)^{-1}B_c. \tag{12}$$

Therefore, instead of using a state-space model involving two consecutive vehicles, string stability can be characterized directly through the reduced-order closed-loop realization (A_c, B_c, C_y).

The following theorem provides a condition for the closed-loop system in (10)–(11) that guarantees string stability.

Theorem 1: Consider the homogeneous predecessor-following platoon controlled by the LQI law with double integrator. Let $h > 0$ and let $Q \geq 0$, $R > 0$ be such that the associated Riccati equation

$$A_a^T P + PA_a - PB_aR^{-1}B_a^T P + Q = 0 \tag{13}$$

admits a solution P yielding the feedback gain

$$\mathcal{K} = [\,k \;\; k_z \;\; k_g\,] = R^{-1}B_a^T P. \tag{14}$$

Assume furthermore that the reduced closed-loop transfer function in (12) is well posed and strictly proper. Then the platooning system is string stable if the bounded-real Riccati inequality

$$A_c^T X + XA_c + XB_cB_c^T X + C_y^T C_y \leq 0 \tag{15}$$

admits a solution $X = X^T \geq 0$.

Proof: Hence, the string stability condition (6) is equivalent to

$$\left\|C_y(sI - A_c)^{-1}B_c\right\|_\infty \leq 1.$$

Since this transfer function is strictly proper, its direct term is $D = 0$. Therefore, by the continuous-time bounded real lemma with $\gamma = 1$, the previous condition holds if there exists $X = X^\top \geq 0$ satisfying (15). This proves the result. ■

Theorem 1 shows that the proposed analysis involves two coupled Riccati conditions. The first one determines the LQI feedback gain from the weighting matrices Q, R, and the time-headway parameter h, while the second one characterizes string stability by applying the bounded-real lemma to the reduced-order error-propagation realization. Under the assumptions of internal stability, well-posedness, and strict properness, the Riccati inequality in (15) provides a state-space test for the classical frequency-domain string-stability condition.

In the following section, this criterion will be used to design string-stable platoons and simulate the platoon behavior.

IV. Numerical Example

This section illustrates the theoretical framework through numerical analysis of a vehicle platoon. We examine how string stability depends on the LQI design parameters and the constant time headway policy. We consider a homogeneous platoon where each vehicle follows the second-order dynamics (see for instance [27]):

$$\underbrace{\begin{bmatrix} \dot{x}_{p_i}(t) \\ \dot{x}_{v_i}(t) \end{bmatrix}}_{\dot{x}_i(t)} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -7.1873 \end{bmatrix}}_{A} \begin{bmatrix} x_{p_i}(t) \\ x_{v_i}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u_i(t)$$

$$y_i(t) = \underbrace{\begin{bmatrix} 0.5235 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_{p_i}(t) \\ x_{v_i}(t) \end{bmatrix}$$

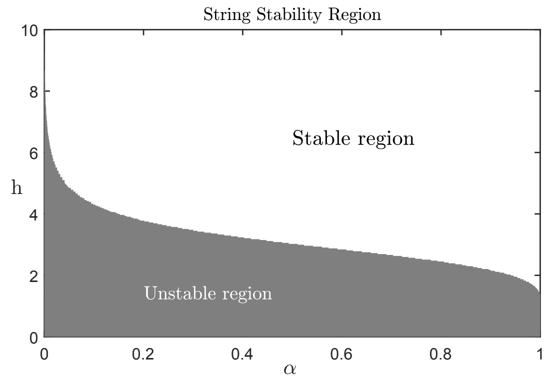

Fig. 3. String stability region as a function of spacing policy h and controller design parameter α.

Here, $x_{p_i}(t)$ corresponds to scaled position and $x_{v_i}(t)$ to scaled velocity. For this numerical example, given the state structure $x_i = [x_{p_i}, x_{v_i}]^T$, the matrix C_w is defined as $C_w = [C_{11} \quad C_{11}h]$ to ensure that the output matches the definition $w_i(t) = y_i(t) + hv_i(t)$, where C_{11} is the first element of the output matrix C.

To explicitly characterize the relationship between the weighting matrices, time headway, and string stability, we employ a diagonal parameterization:

$$Q = \alpha I_4, \quad R = 1 - \alpha, \quad \alpha \in [0, 1)$$

This specific parameterization serves two important purposes: it reduces the multidimensional design space to a single scalar parameter α, and it establishes a direct trade-off between state regulation ($\alpha \to 1$) and control effort ($\alpha \to 0$). This approach enables clear visualization of how the balance between state weighting and control penalty interacts with the time headway h to determine string stability.

We evaluate Theorem 1 across the parameter space $\alpha \in [0, 1)$ and $h \in [0, 10]$. Figure 3 shows the string-stability region obtained for the specific parametrization $Q = \alpha I_4$, $R = 1 - \alpha$. For intermediate values $\alpha \in [0.2, 0.8]$, string stability shows a nearly linear dependence on h, requiring headways between 2 and 4 seconds. As α decreases, the stability region shrinks significantly within the explored range of headway values, indicating that small values of α require substantially larger time headways to satisfy the string-stability condition. Conversely, higher α values expand the stability region, permitting smaller headways through increased controller gains.

Furthermore, to extend the numerical study, we validate these findings through time-domain simulations of a 20-vehicle platoon with a fixed time headway of $h = 3$ s under four distinct design scenarios. The first two cases employ controllers derived from the previously proposed parameterization, where the states are equally weighted using a diagonal matrix Q. Case 1 utilizes $\alpha = 0.9$, while Case 2 uses $\alpha = 0.1$, resulting in string-stable and string-unstable platoons, respectively (consistent with Figure 3). The last two cases are included as illustrative examples for non-diagonal choices of Q and R, rather than as the outcome of a systematic tuning procedure. The third and fourth cases utilize arbitrary weighting matrices $Q \geq 0$ and $R > 0$, which yield string-unstable and string-stable behaviors, respectively:

Case 3 (string-unstable): Using a full weighting matrix to test an "arbitrary" regulatory scenario with a high control penalty.

$$Q = \begin{bmatrix} 10 & 1 & 4 & -2 \\ 1 & 5 & 3 & -1 \\ 4 & 3 & 20 & 4 \\ -2 & -1 & 4 & 15 \end{bmatrix}, \quad R = 100$$

Case 4 (string-stable): The structured choice prioritizing velocity and integral states to ensure error attenuation.

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 10 \end{bmatrix}, \quad R = 1$$

Figure 4 illustrates the vehicle position signals and their corresponding tracking errors for the four aforementioned cases. Due to the LQI controller with double integration, all vehicles achieve zero steady-state tracking error. The fundamental difference lies in their transient behavior: string-stable configurations attenuate the spacing-error magnitudes along the string, whereas unstable cases amplify the perturbations from predecessors.

The numerical analysis illustrates how Theorem 1 can be used to assess string stability for different controller parameters. In particular, the parameterization $Q = \alpha I_4$, $R = 1 - \alpha$ highlights the relationship between controller tuning and string stability, while the time-domain simulations are consistent with the theoretical predictions for the cases considered.

V. Conclusion

This paper studied homogeneous vehicle platoons under a predecessor-following topology and a constant time-headway spacing policy, controlled through an LQI law with double integral action. First, it was shown that the standard single-integrator LQI structure is not sufficient to achieve zero steady-state tracking error for ramp-type references, which motivates the double-integrator augmentation. Second, string stability was characterized through the transfer function between consecutive spacing errors and expressed via a bounded-real Riccati inequality associated with a reduced-order closed-loop realization. This framework highlights a fundamental coupling between a Riccati equation for internal stabilization and a Riccati inequality for string stability.

The numerical examples illustrated how the time headway and the weighting matrices affect the string-stability property. In particular, the results show that internal stability of the local closed loop does not automatically imply attenuation of spacing errors along the platoon.

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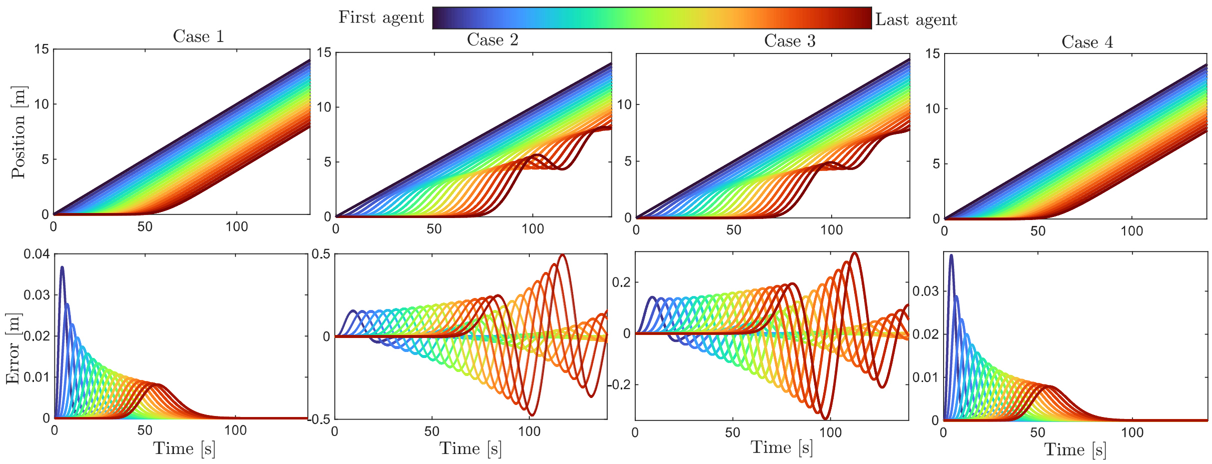

Fig. 4. Simulation results for a 20-vehicle platoon under different control parameters. The top row displays the vehicle positions, while the bottom row shows the tracking errors

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