A Comparative Numerical Evaluation of PID-Based Controllers on a Twin Rotor MIMO System*

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Abstract— **This paper presents a comparative numerical evaluation study of PID-based control strategies on a twin rotor multiple-input multiple-output system (TRMS). The main objective is to evaluate the tracking performance and control effort of different PID structures under a common numerical testing setting. In particular, a proposed self-tuned PID-based controller is compared with representative nonlinear/adaptive PID approaches from the literature and also with a classical fixed-gain PID controller. In the numerical evaluations reported in this work, single-channel structure is applied separately to the horizontal and vertical axes of the TRMS while the full coupled multiple-input multiple-output system (MIMO) plant dynamics are preserved. All controllers are tested on the same TRMS model, reference trajectory, noise conditions, and performance criteria. The comparative assessment is carried out using root mean square error, integral of absolute error, integral of time-weighted absolute error, and integral of absolute control input. Numerical results show that the proposed method yields the best overall tracking performance, while the fixed-gain PID controller provides lower control effort in some cases. These results demonstrate the practical potential of self-tuned PID structures for nonlinear MIMO systems such as the TRMS.**

I. INTRODUCTION

The three-term control method known as PID (proportional-integral-derivative) was introduced about a century ago by Minorsky [1], [2] and has since remained one of the most widely used feedback control tools in industrial practice. The main reasons for its popularity are its simple structure, ease of implementation, and satisfactory performance for a broad range of applications. Despite these advantages, selecting appropriate PID gains is not straightforward, especially for nonlinear systems, systems operating under varying conditions, and systems subject to disturbances [3]–[5].

To address this difficulty, a large body of the relevant literature has focused on tuning and self-tuning strategies for PID-type controllers. Numerical approaches such as neural networks, fuzzy systems, evolutionary algorithms, and particle swarm optimization are commonly used to improve the performance of PID controller structures [6]–[18]. These approaches are often effective in practice, yet they are generally assessed through numerical evidence rather than through a unified comparative study on a challenging nonlinear plant. Analytical and nonlinear PID-type approaches have also been proposed in the literature [19]–[36]. These methods differ in structure, adaptation mechanism, and complexity, and therefore their performance may vary considerably depending on the considered system.

The twin rotor multiple-input multiple-output system (TRMS) is a well-known nonlinear test platform with coupled dynamics and nontrivial control characteristics [37], [38]. Because of these properties, it provides a meaningful setting for evaluating the practical behavior of PID-based controllers under a common framework. In this paper, the TRMS is considered specifically as a common MIMO plant for comparing PID-based control methodologies. The main purpose of this study is to investigate the closed-loop performance of a proposed self-tuned PID-based method against representative PID-type controllers from the literature and against a classical fixed-gain PID controller under the same conditions. Accordingly, the main contribution of this paper is the comparative study itself and the resulting performance insights on the TRMS.

The main highlights of this study are summarized as follows:

- A common framework is established for evaluating multiple PID-based controllers on the TRMS under identical operating conditions.
- A proposed self-tuned PID-based controller is compared with representative adaptive/nonlinear PID methods from the literature and a classical fixed-gain PID controller.
- The compared methods are quantitatively evaluated in terms of tracking performance and control effort by using RMSE (root mean square error), IAE (integral of absolute error), ITAE (integral of time-weighted absolute error) and integral absolute control input criteria.
- The study highlights the performance–effort trade-off of PID-type controllers on a nonlinear MIMO system with coupled dynamics.

The remainder of the paper is organized as follows. The TRMS model is presented in Section II. The controller structures included in the comparative studies are summarized in Section III. The numerical evaluation setup and performance criteria are described in Section IV. Comparative results are presented in Section V. Finally, concluding remarks are given in Section VI.

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II. Mathematical Model

In this paper, the comparison plant is selected as the TRMS, which is a nonlinear MIMO system. The compared controllers are applied separately to the horizontal and vertical channels while the full coupled MIMO system dynamics are preserved in the simulation model. The mathematical model of the TRMS is given by

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) = \tau \tag{1}$$

where $\theta(t) = [\theta_h\ \theta_v]^T \in \mathbb{R}^2$ is the angular position vector, $\dot{\theta}(t)$, $\ddot{\theta}(t) \in \mathbb{R}^2$ are the corresponding velocity and acceleration vectors, and $\tau(t) = [\tau_h\ \tau_v]^T \in \mathbb{R}^2$ is the control input vector. In (1), $M(\theta) \in \mathbb{R}^{2\times 2}$ represents the inertia matrix and $N(\theta, \dot{\theta}) \in \mathbb{R}^2$ denotes the remaining nonlinear dynamics such as Coriolis, centrifugal, and gravitational effects. These terms are structured as

$$M \triangleq \begin{bmatrix} f_1 & f_2 \\ f_2 & J \end{bmatrix}, \qquad N \triangleq \begin{bmatrix} f_3 + f_4 \\ f_5 + f_6 \end{bmatrix} \tag{2}$$

where the nonlinear functions $f_1(\theta_v)$, $f_2(\theta_v)$, $f_3(\theta_v, \dot{\theta}_v)$, $f_4(\theta_v, \dot{\theta}_v, \dot{\theta}_h)$, $f_5(\theta_v, \dot{\theta}_h)$, and $f_6(\theta_v) \in \mathbb{R}$ are defined as

$$f_1 = \phi_1 \cos^2(\theta_v) + \phi_2 \sin^2(\theta_v) + \phi_3 \tag{3}$$
$$f_2 = \phi_4 \sin(\theta_v) - \phi_5 \cos(\theta_v) \tag{4}$$
$$f_3 = [\phi_4 \cos(\theta_v) + \phi_5 \cos(\theta_v)]\, \dot{\theta}_v^2 \tag{5}$$
$$f_4 = 2\phi_6 \sin(\theta_v)\cos(\theta_v)\dot{\theta}_v\dot{\theta}_h \tag{6}$$
$$f_5 = -\phi_6 \sin(\theta_v)\cos(\theta_v)\dot{\theta}_h^2 \tag{7}$$
$$f_6 = \phi_7 \cos(\theta_v) + \phi_8 \sin(\theta_v). \tag{8}$$

The constant model parameters are selected as $\phi_1 = 0.0591$ kgm^2, $\phi_2 = 0.0058$ kgm^2, $\phi_3 = 0.0132$ kgm^2, $\phi_4 = 0.0019$ kgm^2, $\phi_5 = 0.022$ kgm^2, $\phi_6 = -0.0533$ kgm^2, $\phi_7 = 0.1535$ kgm^{2}rad^2/s^2, $\phi_8 = 0.022$ kgm^{2}rad^2/s^2, and $J = 0.0649$ kgm^2.

III. Controllers Under Comparison

This section briefly summarizes the controller structures included in the comparative study. Since the main purpose of the paper is the simulation-based comparison, only the implementation-level forms necessary for reproducibility are presented. Due to page limitations, for detailed derivations of the proposed controller and complete theoretical developments of the compared methods, the readers are referred to the references.

A. Proposed self-tuned PID-based controller

Let $x(t) \in \mathbb{R}$ denote the individual entries of the angular position vector, *i.e.*, $x = \theta_h$ or $x = \theta_v$. The tracking error is defined as

$$e \triangleq x_d - x \tag{9}$$

where $x_d(t) \in \mathbb{R}$ denotes the desired trajectory. The proposed PID-based controller is implemented as [39]

$$\begin{aligned} u(t) = {} & k(t)\big(\dot{e}(t) + \beta e(t)\big) - k(0)\big(\dot{e}(0) + \beta e(0)\big) \\ & + \alpha \int_0^t k(\sigma)\big(\dot{e}(\sigma) + \beta e(\sigma)\big)\, d\sigma \end{aligned} \tag{10}$$

where α, $\beta \in \mathbb{R}$ are positive gains, and $k(t) \in \mathbb{R}$ is a time-varying gain updated according to

$$k(t) = k_c + \frac{1}{2}\gamma(\dot{e} + \beta e)^2 + \alpha\gamma \int_0^t (\dot{e}(\sigma) + \beta e(\sigma))^2\, d\sigma \tag{11}$$

with $\gamma \in \mathbb{R}$ denoting the adaptation gain and $k_c \in \mathbb{R}$ being the constant offset.

Although the controller in (10)–(11) is written for a single channel, in the simulations, it is applied separately to the horizontal and vertical axes of the TRMS. Accordingly, the control input vector is written as

$$\tau(t) = \begin{bmatrix} u_h(t) \\ u_v(t) \end{bmatrix} \tag{12}$$

where $u_h(t)$ and $u_v(t)$ are obtained from (10) by using the axis-wise errors

$$e_h \triangleq \theta_{d,h} - \theta_h \tag{13}$$
$$e_v \triangleq \theta_{d,v} - \theta_v. \tag{14}$$

Separate adaptive gains $k_h(t)$ and $k_v(t)$ are used for the two channels. In this manner, a SISO control law is employed on each channel while the closed-loop simulations are carried out on the full, coupled MIMO model of the twin rotor system.

B. Other compared controllers

The proposed method is compared with three additional PID-based controllers, namely, the adaptive PID-type controller in [7], the nonlinear PID controller in [20], and a classical fixed-gain PID controller tuned according to the Ziegler–Nichols approach [40]. For completeness, the main control laws of these benchmark methods are briefly summarized here.

The adaptive PID-type controller in [7] is formulated as

$$u(t) = u_{\mathrm{PID}}(t) + u_s(t), \tag{15}$$

where

$$u_{\mathrm{PID}}(t) = \frac{1}{b}\left[K_P(t)e(t) + K_I(t)\int_0^t e(\tau)\, d\tau + K_D(t)\dot{e}(t)\right], \tag{16}$$

and the switching term $u_s(t)$ is introduced to improve robustness against uncertainty and disturbances. In this method, the PID gains are updated online through following adaptive laws

$$\dot{K}_P(t) = -\,\eta_1\sigma(t)e(t), \tag{17}$$
$$\dot{K}_I(t) = -\,\eta_2\sigma(t)\int_0^t e(\tau)\, d\tau, \tag{18}$$
$$\dot{K}_D(t) = -\,\eta_3\sigma(t)\dot{e}(t), \tag{19}$$

with $\sigma(t)$ denoting the sliding variable. In (16), b is input gain, and in (17)–(19), η_1, η_2, η_3 are defined as learning rate.

The nonlinear PID controller in [20] employs a conventional PID structure driven by a scaled error signal, and can be written as

$$u(t) = k_p f(e(t)) + k_i \int_0^t f(e(\tau))\, d\tau + k_d \frac{d}{dt} f(e(t)), \tag{20}$$

where the nonlinear mapping is defined as $f(e) = k(e)e$, with $k(e)$ being a sector-bounded nonlinear gain used to increase control action for large tracking errors while reducing it near the equilibrium.

Finally, the classical benchmark controller is taken as the standard fixed-gain PID law,

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e}(t), \qquad (21)$$

where K_p, K_i, and K_d are constant parameters selected according to the Ziegler–Nichols tuning rule [40]. In the TRMS simulations, all controllers are used within the same channel-wise comparison framework so that the comparison remains consistent across methods.

IV. Numerical Evaluation Setup and Performance Metrics

All controllers are evaluated on the same TRMS model under identical conditions to ensure a fair comparison. The desired trajectory is selected as

$$\theta_d(t) = [0.2 \sin(0.2\pi t) \;\; 0.6 \sin(0.2\pi t)]^T \text{ rad}. \qquad (22)$$

For the proposed self-tuned PID-based controller, the gains are selected as $\beta = 5$, $\alpha = 5$, $\gamma = 100$, and $k_c = 0$. The same initial conditions, reference trajectory, simulation horizon, and noise profile are used for all compared controllers. For the adaptively tuned controllers, the total simulation interval is divided into two parts: an initial adaptation/training phase and a subsequent test phase. In the first 60 seconds of the simulations, additive white Gaussian noise with a signal-to-noise ratio of 10 dB is injected into the measured velocity signals. After this phase, the noise level is reduced to 20 dB, and the remaining interval is used for performance evaluation.

The compared controllers are assessed by using the following performance measures: “RMSE”, “IAE”, “ITAE”, “integral of the absolute control input”. These criteria are used to jointly evaluate tracking accuracy and control effort. The first three criteria quantify the quality of trajectory tracking, while the last one reflects the amount of control action required by each controller.

V. Comparative Simulation Results

This section presents the comparative simulation results obtained for the PID-based controllers included in the study. The objective is to evaluate the closed-loop tracking behavior and control effort of each method under the same scenario. The performance of the proposed self-tuned PID-based controller is first illustrated, and then quantitative comparisons with the other methods are provided.

The obtained results for the proposed controller are presented in Figures 1-4. The state variables and their corresponding reference trajectories are presented in Figure 1. The tracking error obtained by the proposed method is shown in Figure 2. The time evolution of the adaptive gains is given in Figure 3, and the control inputs are shown in Figure 4. From Figures 1 and 2, it can be observed that the proposed method achieves satisfactory tracking performance on both channels of the TRMS. The adaptive gains increase rapidly during the initial training phase and then tend to settle during the test phase, which indicates that the controller gradually reaches a near-stationary operating regime.

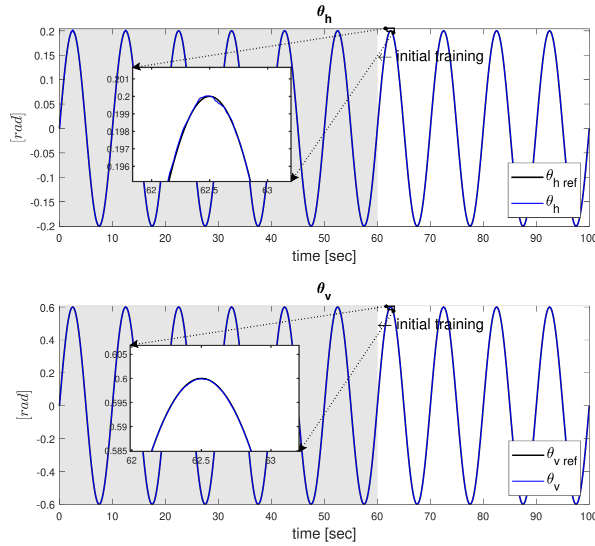

Fig. 1. State variables and reference trajectories for the proposed self-tuned PID-based controller with h and v denoting horizontal and vertical motions.

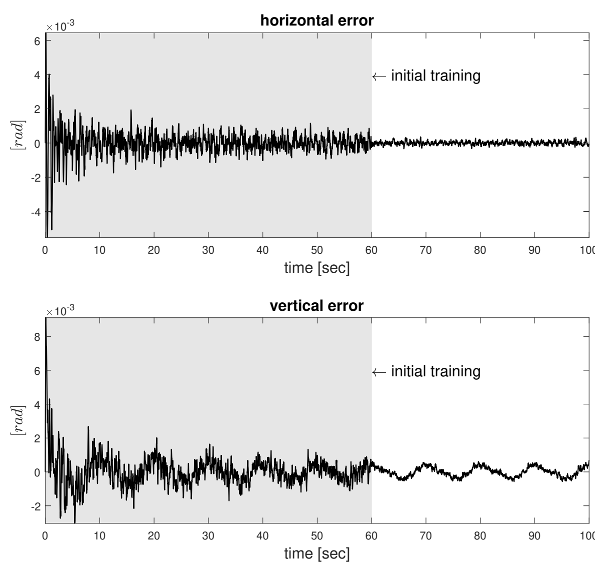

Fig. 2. Tracking error obtained by the proposed self-tuned PID-based controller.

The comparative tracking performances of all methods are shown in Figure 5, while the corresponding control efforts are given in Figure 6. The quantitative comparison results are delineated in Table I. As seen from the results, the proposed method provides the best overall tracking performance among the compared controllers. In particular, it yields the lowest error values according to RMSE, IAE, and ITAE for both channels. The adaptive and nonlinear PID methods respectively from [7] and [20] exhibit considerably larger tracking errors under the same simulation conditions.

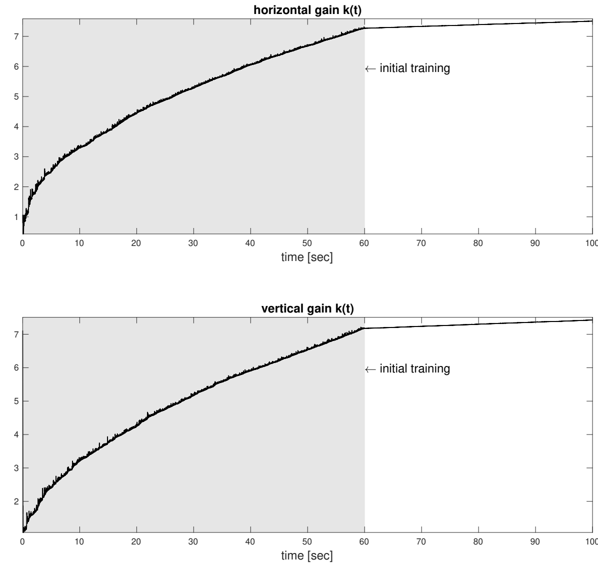

Fig. 3. Time evolution of the adaptive gains for the proposed self-tuned PID-based controller.

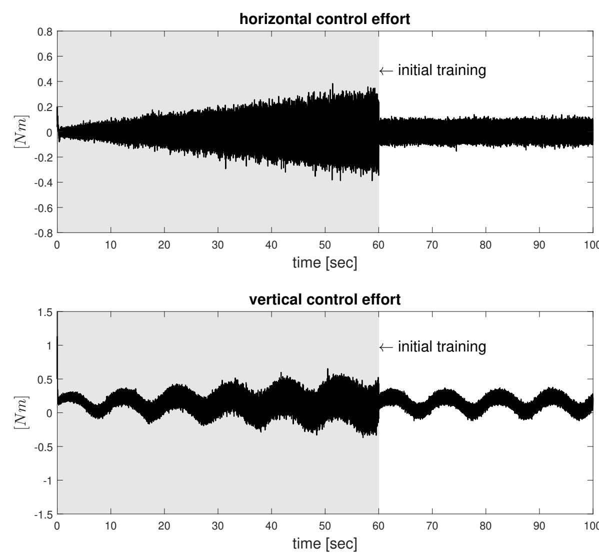

Fig. 4. Control inputs of the proposed self-tuned PID-based controller.

The classical fixed-gain PID controller performs better than these two methods in some cases, yet it remains significantly inferior to the proposed method in terms of tracking accuracy. In terms of control effort, the fixed-gain PID controller exhibits a lower input magnitude in one channel and a comparable effort in the other. However, this reduction in control action is accompanied by a substantial loss in tracking performance. Therefore, the simulation results suggest that the proposed self-tuned PID-based controller offers a more favorable balance between tracking quality and control effort for the considered TRMS scenario.

TABLE I

COMPARISON OF PERFORMANCE MEASURES

		Proposed	[7]	[20]	PID
RMSE	e_1	1.2785e-4	6.7894e-1	2.4644e-1	1.9971e-2
	e_2	1.6191e-3	2.6155e-1	6.2553e-1	3.4217e-1
IAE	e_1	6.5303e-4	4.2939	1.2641	1.0300e-1
	e_2	8.8111e-3	1.6541	3.4972	1.8879
ITAE	e_1	5.1905e-2	343.54	103.73	8.1207
	e_2	7.0638e-1	132.37	285.53	150.71
$\int \lvert\tau_1\rvert \, dt$		0.9894	11.64	7.586	0.1346
$\int \lvert\tau_2\rvert \, dt$		5.668	14.26	17.87	5.659

VI. CONCLUSIONS

This paper presented a comparative simulation study of PID-based controllers on the TRMS. The paper aimed to compare the proposed self-tuned PID-based method with representative PID-type controllers from the literature and also with a classical fixed-gain PID controller under a common simulation framework. The controller design itself was introduced in a SISO, and its applicability to the TRMS was demonstrated by applying the channel-wise structure to the horizontal and vertical dynamics while preserving the full, coupled MIMO plant model.

The simulation results showed that the proposed method achieved the best overall tracking performance for both channels of the TRMS according to RMSE, IAE, and ITAE measures. Although the classical fixed-gain PID controller required lower control effort in some cases, its tracking accuracy was significantly worse than that of the proposed method. These findings indicate that the proposed self-tuned PID-based structure can provide an effective compromise between tracking quality and control effort for nonlinear MIMO systems with coupled dynamics.

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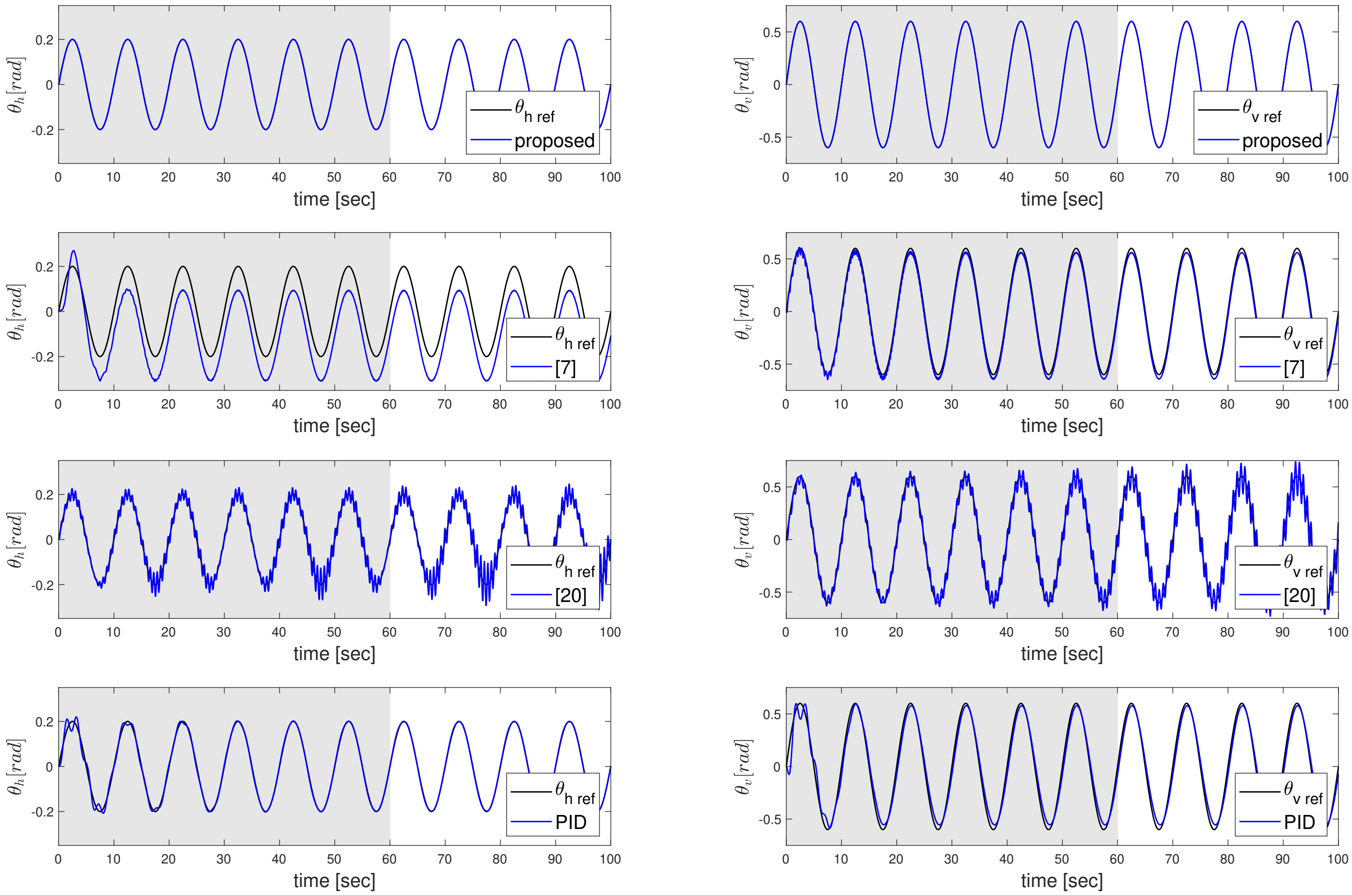

Fig. 5. Comparative tracking performances of the compared controllers with h and v denoting horizontal and vertical motions.

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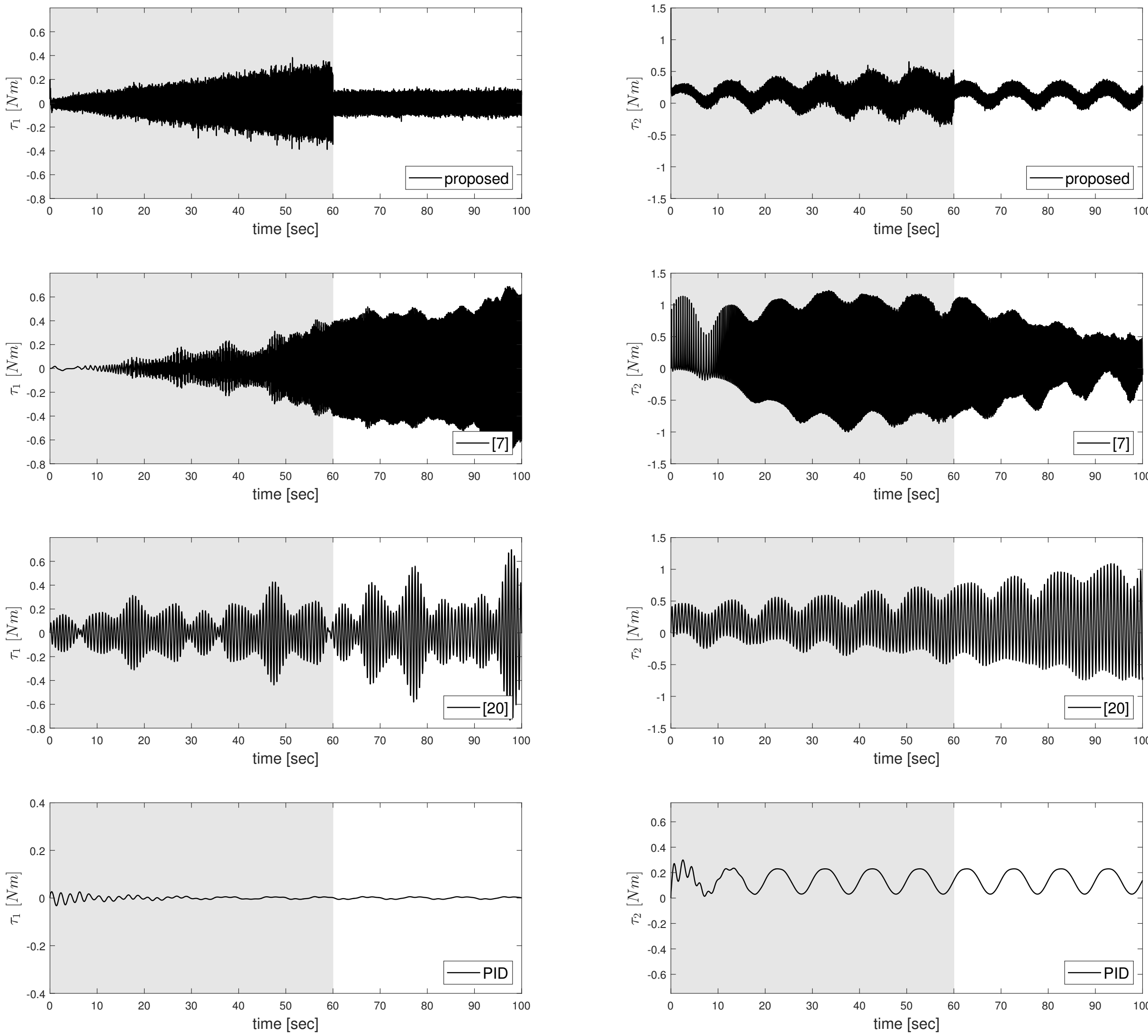

Fig. 6. Comparative control efforts of the compared controllers.

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