Lyapunov-Based Adaptive Control for Constrained Kinematic Tracking of Robot Manipulators under Parameter Uncertainty

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***Abstract*— This study addresses the trajectory tracking control of non-redundant robot manipulators under kinematic parametric uncertainties. The primary objective is to ensure high-precision tracking while strictly maintaining the transient and steady-state performance of the task-space error. To achieve this, an Adaptive Kinematic Control strategy is developed by integrating the Prescribed Performance Control (PPC) framework. The PPC mechanism, formulated via a Barrier Lyapunov Function (BLF), constrains the tracking error within a predefined time-varying envelope, thereby guaranteeing specific overshoot limits and convergence rates. An adaptive update law is utilized to identify uncertain kinematic parameters, while a smooth joint velocity control input is implemented via the estimated Jacobian matrix. Using Lyapunov stability theory, the closed-loop system is proved to be Uniformly Bounded with the Cartesian tracking error achieving Asymptotic Convergence to zero. Numerical results validate the effectiveness of the proposed strategy in satisfying prescribed performance boundaries despite parametric uncertainties.**

I. Introduction

Precise trajectory tracking of robot manipulators remains a fundamental challenge in robotics, particularly in the presence of kinematic parametric uncertainties such as unknown or inaccurately modeled link lengths. Although many classical control approaches rely on exact kinematic information, practical robotic systems are often subject to structural variations, manufacturing tolerances, and environmental influences that hinder accurate model identification. To address these challenges, numerous adaptive control strategies have been proposed in the literature. However, a significant portion of existing methods, including those reported in [1], [2], [3], either assume perfectly known kinematic parameters or focus primarily on parameter estimation. Such approaches commonly impose stringent persistence of excitation (PE) conditions and provide limited guarantees on the transient behavior of the tracking error, which is crucial for ensuring reliable and safe motion during the initial response phase.

In recent years, the Prescribed Performance Control (PPC) framework has gained considerable attention as an effective means of explicitly shaping the transient and steady-state performance of tracking errors. The study in [4] proposes a kinematic controller that employs the prescribed performance framework to handle joint positioning constraints. By transforming constrained joint variables into an unconstrained space. Specifically, the work in [5] utilizes a preset trajectory approach combined with quadratic programming to guarantee time-constrained tracking for aerial manipulators while simultaneously handling physical joint and base limitations. However, while effective in handling coordinate constraints, this class of methods does not explicitly consider the combined effects of kinematic parametric uncertainties on task-space tracking performance. In contrast to conventional control schemes, prescribed performance control (PPC) guarantees that the tracking error evolves strictly within a predefined, time-varying performance envelope, thereby ensuring bounded overshoot during the transient response as well as prescribed steady-state convergence. Within this framework, several studies have explored the application of PPC to robotic systems. In [6], a position control strategy with prescribed performance was developed for robot manipulators with partially known dynamics, while a task-space regulation approach ensuring prescribed performance was proposed in [7]. In parallel, barrier Lyapunov function (BLF)–based techniques have been widely employed to address system constraints under various structural forms, including Brunovsky canonical form [8], strict-feedback systems [9], strict-feedback systems with time-varying output constraints [10], and pure-feedback configurations [11]. Moreover, a BLF-based approach was utilized in [12] to handle joint limit avoidance as a secondary task in the control of redundant robot manipulators. In a related study, a desired model–based joint position–constrained controller was proposed in [13], guaranteeing that the joint tracking error remains within a predefined region and converges asymptotically to zero, even in the presence of a partially known dynamic model.

This paper proposes an integrated Adaptive Kinematic Control (AKC) strategy for non-redundant robot manipulators. The core contribution of this work is the development of a robust adaptive law that compensates for kinematic uncertainties without the explicit necessity of identifying the actual values of the unknown parameters. By incorporating the Prescribed Performance Control (PPC) framework, **implemented through a Barrier Lyapunov Function (BLF)**, the proposed controller maintains the task-space tracking error within prescribed boundaries regardless of model mismatches

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for a two-degree-of-freedom, non-redundant robotic system composed of two revolute joints. Through a rigorous Lyapunov-based stability analysis, it is demonstrated that the proposed scheme ensures the Uniform Boundedness of all closed-loop signals and achieves Asymptotic Convergence of the Cartesian space tracking error. The effectiveness of the method is further validated through numerical simulations, highlighting its superiority in maintaining tracking quality under parametric uncertainties.

II. Kinematic Model and System Assumptions

The forward kinematics of an n degree-of-freedom robot manipulator operating in an n-dimensional workspace is given by the nonlinear transformation:

$$x \triangleq f(q), \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the end-effector position vector in Cartesian space, $q \in \mathbb{R}^n$ represents the joint position vector in joint space, and $f : \mathbb{R}^n \to \mathbb{R}^n$ specifies the forward kinematic model. The velocity kinematics is obtained as:

$$\dot{x} = J(q)\,\dot{q}, \tag{2}$$

In this formulation, $\dot{x} \in \mathbb{R}^n$ represents the end-effector velocity vector, $\dot{q} \in \mathbb{R}^n$ is the joint velocity vector, and $J(q) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the Jacobian matrix, which relates the joint and Cartesian velocities and is defined as $J(q) \triangleq \frac{\partial f(q)}{\partial q}$.

Property 1 (Linear Parameterization of the Jacobian): The Jacobian matrix permits a linear parameterization form when multiplied by an arbitrary velocity vector $\zeta \in \mathbb{R}^n$ [14]:

$$J(q)\,\zeta = W(q,\zeta)\,\phi, \tag{3}$$

where $W \in \mathbb{R}^{n\times p}$ is the regressor matrix composed of known/measurable quantities, and $\phi \in \mathbb{R}^p$ is the vector containing constant kinematic parameters (e.g., link lengths, joint offsets, and twist angles) [15].

Assumption 1: It is assumed that all kinematic singularities are asumed to be avoided throughout the entire operation. Hence, the inverse of the Jacobian matrix, J^{-1}, is guaranteed to be available for all relevant joint positions q [16].

Assumption 2: The forward kinematics function f and the Jacobian matrix J are assumed to be bounded, provided that the joint position vector q remains bounded within its limits [17].

III. Control Development

The primary control objective is to design a kinematic controller that ensures the end-effector position vector x accurately tracks a given desired trajectory $x_d : [0,\infty) \to \mathbb{R}^n$. This must be accomplished despite the presence of uncertainty in the kinematic model parameter vector ϕ as introduced in (3). Specifically, the joint velocity vector is chosen as the control input, $u(t) \triangleq \dot{q} \in \mathbb{R}^n$, and an adaptive control approach is pursued to simultaneously achieve accurate trajectory tracking and enforce prescribed performance constraints on the task-space tracking error through the PPC framework. The subsequent control design hinges on the availability of joint position q and end-effector position x. We note that x is assumed to be obtained via alternative sensing techniques, as the forward kinematics formulation in (1) cannot be utilized to compute x due to the parametric uncertainties. The desired task-space trajectory x_d is assumed to be sufficiently smooth, meaning x_d and $\dot{x}_d$ are bounded functions of time.

To quantify the control objective, the task-space tracking error signal $e(t) \in \mathbb{R}^n$ is defined as:

$$e \triangleq x_d - x. \tag{4}$$

By substituting the control input $u = \dot{q}$ into the velocity kinematics equation (2), the time derivative of the tracking error in (4) is derived as:

$$\dot{e} = \dot{x}_d - Ju. \tag{5}$$

The control law is designed based on the subsequent stability assessment, yielding:

$$u = \hat{J}^{-1}(\dot{x}_d + \kappa e), \tag{6}$$

where $\hat{J}(q) \in \mathbb{R}^{n\times n}$ is the estimated Jacobian matrix. The prescribed performance gain matrix, $\kappa \in \mathbb{R}^{n\times n}$ is a tunable, error dependent, positive definite diagonal matrix where its i-th diagonal element κ_i is designed as:

$$\kappa_i = k_i\left(1 + \tan^2\left(\frac{\pi}{2}\frac{e_i^2}{\Delta_i^2}\right)\right) \tag{7}$$

with e_i being the i-th component of the tracking error vector, k_i is a control gain, and Δ_i is the corresponding constant performance boundary for the i-th component for all $i = 1, \cdots, n$. Utilizing Property 1 and replacing the actual kinematic parameters with their estimates, the estimated Jacobian times the control input can be expressed as:

$$\hat{J}(q)\,u = J(q)\,u\,|_{\phi=\hat{\phi}} = W(q,u)\,\hat{\phi}, \tag{8}$$

where $\hat{\phi} : [0,\infty) \to \mathbb{R}^p$ denotes the vector of estimated uncertain parameters and W is the regressor matrix introduced in (3). Adding and subtracting the term $\hat{J}u$ in (5), followed by substituting the control law (6) into (5), results in the following closed-loop error dynamics:

$$\dot{e} = -\kappa e - \tilde{J}u, \tag{9}$$

with $\tilde{J}(q) \triangleq J - \hat{J} \in \mathbb{R}^{n\times n}$ being the Jacobian estimation error which satisfies the following relation:

$$\tilde{J}u = W(q,u)\,\tilde{\phi}. \tag{10}$$

In (10), $\tilde{\phi}(t) \in \mathbb{R}^p$ is the parameter estimation error vector, defined as the mismatch between the true uncertain parameter vector and its estimation:

$$\tilde{\phi} \triangleq \phi - \hat{\phi}. \tag{11}$$

The adaptive update law for the parameter estimates $\hat{\phi}$ is designed as[1]:

$$\dot{\hat{\phi}} = -\Gamma W^T \kappa e, \tag{12}$$

where $\Gamma \in \mathbb{R}^{p\times p}$ is a constant, positive definite, diagonal adaptation gain matrix. By utilizing the relationship $\dot{\tilde{\phi}} = -\dot{\hat{\phi}}$ in (12) the parameter estimation error dynamics in become:

$$\dot{\tilde{\phi}} = \Gamma W^T \kappa e. \tag{13}$$

IV. Stability Analysis

Theorem 1: The controller proposed in (6), combined with the simplified adaptation law designed below, guarantees the uniform boundedness of the composite error vector $z(t) \triangleq \begin{bmatrix} e^T & \tilde{\phi}^T \end{bmatrix}^T$ and all closed-loop signals. Furthermore, the task-space tracking error e is guaranteed to converge to zero asymptotically in the sense that $\lim_{t\to\infty} e(t) = 0$.

Proof: Consider the candidate Lyapunov function V as:

$$V \triangleq \sum_{i=1}^{n} \frac{\Delta_i^2 k_i}{\pi} \tan\left(\frac{\pi}{2}\frac{e_i^2}{\Delta_i^2}\right) + \frac{1}{2}\tilde{\phi}^T \Gamma^{-1} \tilde{\phi}. \tag{14}$$

The time derivative of V is calculated as:

$$\dot{V} = \sum_{i=1}^{n} k_i e_i \dot{e}_i \left(1 + \tan^2\left(\frac{\pi}{2}\frac{e_i^2}{\Delta_i^2}\right)\right) + \tilde{\phi}^T \Gamma^{-1} \dot{\tilde{\phi}}. \tag{15}$$

where, in view of the diagonal structure of κ designed in (7), the first term can be reformulated as

$$\sum_{i=1}^{n} k_i e_i \dot{e}_i \left(1 + \tan^2\left(\frac{\pi}{2}\frac{e_i^2}{\Delta_i^2}\right)\right) = e^T \kappa \dot{e}. \tag{16}$$

Substituting the closed-loop error dynamics $\dot{e} = -\kappa e - \tilde{J}u$ from (9) and the parameter estimation error dynamics $\dot{\tilde{\phi}}$ in (13) into $\dot{V}$ in (15) yields:

$$\dot{V} = e^T \kappa(-\kappa e - \tilde{J}u) + \tilde{\phi}^T \Gamma^{-1}(\Gamma W^T \kappa e). \tag{17}$$

Utilizing the parameterization error relation $\tilde{J}u = W\tilde{\phi}$ from (10) and substituting this into (17) results in the expression:

$$\dot{V} = -e^T \kappa^2 e - e^T \kappa W \tilde{\phi} + \tilde{\phi}^T W^T \kappa e \tag{18}$$

and since the final two terms cancel each other, we reach:

$$\dot{V} = -e^T \kappa^2 e. \tag{19}$$

Since, from (7), the minimum eigenvalue of κ is $\min_{i=1,\cdots,n}\{k_i\}$, from (19), we obtain:

$$\dot{V} \leq -\min_{i=1,\cdots,n}\{k_i\}\|e\|^2. \tag{20}$$

Since $V \geq 0$ and $\dot{V} \leq 0$, uniform boundedness of V is proven, and consequently, the uniform boundedness of e and $\tilde{\phi}$, and thus z, are established. Moreover, integrating $\dot{V}$ over time yields

$$\int_0^\infty \|e(t)\|^2 \, dt < \infty, \tag{21}$$

which implies that $e(t) \in \mathcal{L}_2$. From the closed-loop error dynamics, it can be shown that $\dot{e}(t)$ is bounded, *i.e.*, $\dot{e}(t) \in \mathcal{L}_\infty$. Therefore, the tracking error $e(t)$ is uniformly continuous. By invoking Barbalat's Lemma [18], it follows that

$$\lim_{t\to\infty} e(t) = 0, \tag{22}$$

which establishes the asymptotic convergence of the task-space tracking error. ■

V. Numerical Simulations

To demonstrate the effectiveness of the proposed control strategy, a numerical simulation is conducted on a planar, two-link revolute joint robot arm. The kinematic controller, expressed in (6), is implemented in conjunction with the adaptive update law specified in (12). The primary objective is to verify the trajectory tracking capability of the control system under uncertain kinematic parameters, while enforcing a predefined prescribed performance on the tracking error in the sense that $\Delta_i \geq e_i \geq -\Delta_i$ is satisfied for all i.

The robot arm utilized for the simulation study is a two-link planar manipulator with the forward kinematic map of, $f(q) \in \mathbb{R}^2$, that relates the joint angles $q = [q_1, q_2]^T$ to the end-effector Cartesian position as:

$$f(q) = \begin{bmatrix} L_1 c_1 + L_2 c_{12} \\ L_1 s_1 + L_2 s_{12} \end{bmatrix}, \tag{23}$$

where L_1, $L_2 \in \mathbb{R}$ are the unknown link lengths, and the standard abbreviations $c_1 \triangleq \cos(q_1)$, $s_1 \triangleq \sin(q_1)$, $c_{12} \triangleq \cos(q_1+q_2)$, and $s_{12} \triangleq \sin(q_1+q_2)$ are used. For the numerical simulation, the true link lengths are set to $L_1 = 0.6$ m and $L_2 = 0.4$ m. The Jacobian matrix, $J = \partial f(q)/\partial q \in \mathbb{R}^{2\times 2}$, is derived as:

$$J = \begin{bmatrix} -L_1 s_1 - L_2 s_{12} & -L_2 s_{12} \\ L_1 c_1 + L_2 c_{12} & L_2 c_{12} \end{bmatrix}. \tag{24}$$

For the implementation of the adaptive control law, the Jacobian term multiplied by the control input, $J(q)u$, is reformulated into the linearly parameterized form:

$$J(q)u = W(q,u)\phi, \tag{25}$$

where $\phi = [L_1, L_2]^T \in \mathbb{R}^2$ is the unknown parameter vector. The corresponding regression matrix, $W(q,u) \in \mathbb{R}^{2\times 2}$, is obtained as:

$$W(q,u) = \begin{bmatrix} -s_1 u_1 & -s_{12}(u_1+u_2) \\ c_1 u_1 & c_{12}(u_1+u_2) \end{bmatrix} \tag{26}$$

where $u = [u_1, u_2]^T$.

The desired task space trajectory was selected as:

$$x_d = \begin{bmatrix} 0.45 + 0.1\,(1-\exp(-t))\cos(\pi t) \\ 0.45 + 0.1\,(1-\exp(-t))\sin(\pi t) \end{bmatrix} \text{ m.} \tag{27}$$

[1]The proposed adaptive controller in (6) requires the estimated Jacobian matrix $\hat{J}$ to be invertible. Therefore, the adaptive update law should ideally be complemented with a projection algorithm to maintain the parameter estimates within known bounds and prevent potential singularities.

The numerical simulation was initialized with specific conditions to demonstrate the controller's performance. The robot's initial joint configuration was set to $q(0) = [0.135,\ \ 1.82]^T$ rad, which resulted in the initial end-effector position of $x(0) = [0.4446,\ \ 0.4516]^T$ m. The initial estimate for the uncertain kinematic parameter vector was conservatively chosen as $\hat{\phi}(0) = [0.2,\ \ 0.2]^T$. Controller parameters were configured as $k_1 = 1$, $k_2 = 1$ and $\Gamma = \mathrm{diag}\{30,\ \ 30\}$.

Two different simulations were run to highlight the performance benefits of the proposed prescribed performance control approach. In both simulations, the entries of the error vector are required to reside in $\Delta_1 \geq |e_1|$ and $\Delta_2 \geq |e_2|$ with $\Delta_1 = 0.02$m and $\Delta_2 = 0.01$m. The simulation outcome depicted in Figure 1 corresponds to a baseline scenario where the PPC mechanism is turned off by selecting the control gain matrix as a constant matrix $\kappa = \mathrm{diag}\{0.825, 0.825\}$ instead of the error-dependent gain designed in (7). An examination of the tracking error displayed in Figure 1 reveals poor transient response in the sense that both entries of the error vector exceeded their respective limits. In contrast, the subsequent Figures 2–5 illustrate the results obtained when the control gain κ_e is selected as designed in (7). As shown in Figure 2, the tracking error's evolution confirms that the performance rigorously adheres to the predefined boundaries established by the prescribed performance function. This adherence demonstrates that the tracking error remains strictly within the pre-specified performance envelopes, validating the effectiveness of the PPC-based adaptive kinematic controller in ensuring guaranteed tracking quality. In Figure 3, the xy plot of the time evolutions of the end-effector position and the desired trajectory are plotted. In figure 4, parameter estimates are presented. Figure 5 illustrates the designed control input u which confirms that the control signal remains smooth throughout the entire tracking maneuver, satisfying a crucial requirement for practical implementation.

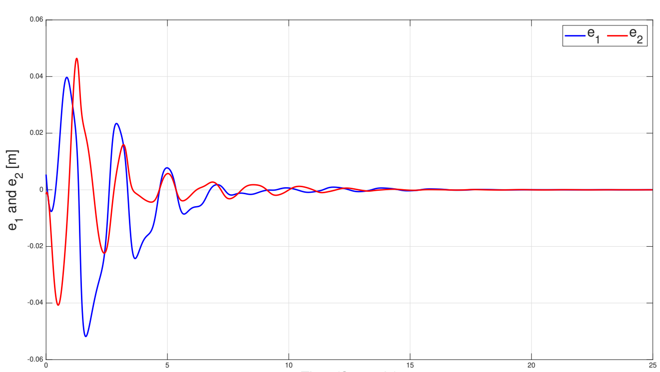

Fig. 1. Cartesian space tracking error for constant κ

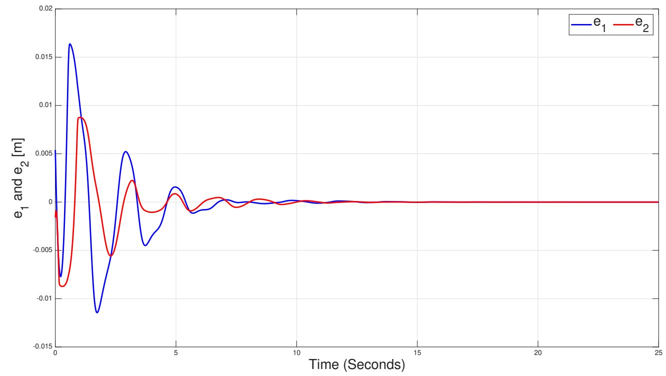

Fig. 2. Cartesian space tracking error for κ in (7)

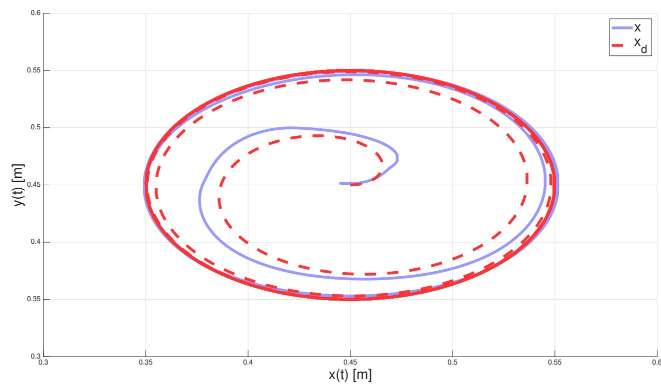

Fig. 3. End-effector trajectory tracking performance

VI. Conclusions

This paper proposed an adaptive kinematic control scheme for tasks-space trajectory tracking of robot manipulators in the presence of kinematic parametric uncertainties. By incorporating the prescribed performance control framework, strict guarantees on both transient and steady-state task-space tracking performance were achieved. Specifically, the developed controller constrains the Cartesian space tracking error within a predefined envelope, while an adaptive update law estimates unknown kinematic parameters online. Lyapunov-based analysis established uniform boundedness of all closed-

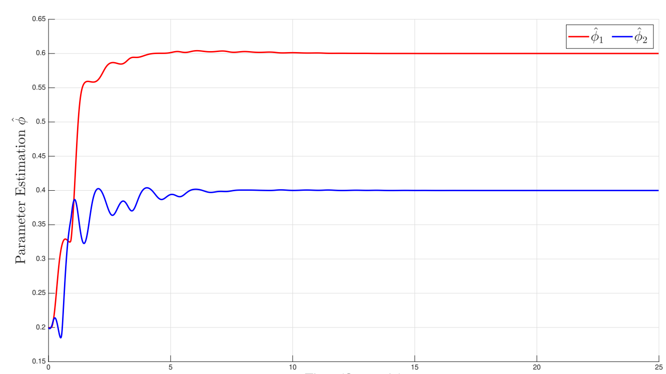

Fig. 4. Parameter estimation $\hat{\phi}$

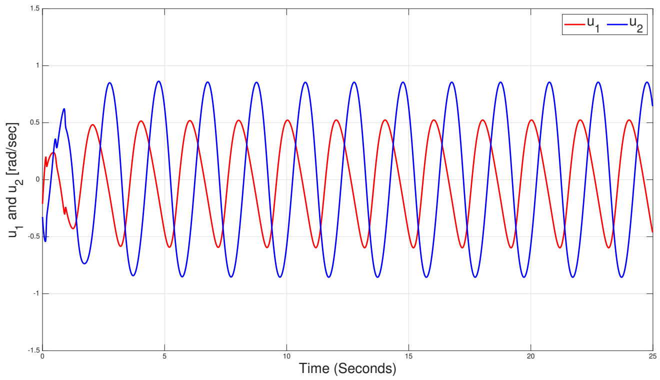

Fig. 5. Control input $u(t)$

loop signals and asymptotic convergence of the tracking error. Numerical simulations confirmed the effectiveness of the proposed approach in satisfying the prescribed performance bounds despite parametric uncertainties. Future work will focus on extending the proposed control framework to redundant robot manipulators and implementing the strategy on physical robotic platforms for experimental validation.

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