An MPC-Based Framework for Enhanced Analysis and Mitigation of the Bullwhip Effect in Supply Chains

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***Abstract*— The Bullwhip Effect (BE) in supply chains is a well-known phenomenon whereby small variations in customer demand generate progressively larger fluctuations in orders and inventory levels upstream. It is traditionally quantified as the amplification of demand variance from the end customer to the supplier. However, this variance-based definition captures only the magnitude of amplification and neglects an equally critical aspect: the frequency of fluctuations. Given that high costs are typically associated with variations in order quantities, ignoring this aspect may lead to inefficient inventory replenishment policies. The twofold aim of this work is to provide a novel characterization of the BE that explicitly accounts for this aspect and to embed it within a Model Predictive Control (MPC) framework capable of balancing conflicting requirements, namely maximizing demand fulfillment, reducing overstocking, and mitigating the BE.**

I. Introduction

The Bullwhip Effect (BE) remains a major barrier to achieving efficient and resilient supply chain (SC) management [1]. Its consequences are well established: distorted demand signals lead to excessive inventories and holding costs, unstable production schedules, and reduced customer service. These issues are particularly acute in supply chains for perishable goods, where amplified demand fluctuations drive overproduction, accelerated spoilage, and frequent stock-outs, ultimately undermining operational efficiency of the supply chain [2]. In response to these challenges, this study proposes a replenishment policy designed to satisfy the following requirements:

-(R1) maximizing the amount of fulfilled customer while avoiding overstocking,

-(R2) reducing the BE through effective containment of both the amplitude and frequency of order oscillations.

As for (R1), the relevance of Model Predictive Control (MPC) has been extensively acknowledged and documented [3]. This stems from its inherent capability to manage physical constraints and from the receding horizon structure of its control law. The former enables explicit limitations on inventory levels and replenishment orders, while the latter provides the ability to apply real time adjustments to the control actions based on newly observed data. This second feature is particularly useful in SC contexts, where only a time varying forecast of customer demand is available within a limited planning horizon.

As for R2, its fulfillment requires a more comprehensive characterization of the BE. Traditionally, the BE is quantified through the amplification of demand variance along the supply chain (see, e.g., [[4]-[6]). However, such a characterization captures only the magnitude of variability and neglects its temporal structure, in particular the frequency and persistence of order fluctuations. The contribution of this work is to propose a novel time-domain characterization of the BE that accounts for both variance and oscillatory behavior. In view of these considerations and prior contributions ([9]-[11]), the newly developed characterization of the BE is here leveraged and integrated into an optimal MPC framework to develop an inventory replenishment policy that satisfies requirements R1 and R2.

The paper is structured as follows. Section II introduces the SC model and specifies the assumptions regarding customer demand. Section III provides a new characterization of the BE. Section IV formulates the min-max MPC problem, while Section V presents a computationally more efficient least-squares reformulation. Section VI discusses the tuning of MPC parameters. Finally, Sections VII and VIII present the simulation results and the concluding remarks, respectively.

II. SC model

A. Inventory level equation

We consider a periodically reviewed SC consisting of a retailer connected in series with a manufacturer. The manufacturer is represented as a pure time delay. We make the following assumptions:

A1) Inside each review interval $[kT,(k+1)T)$, with $k \in \mathbb{Z}^+$, the retailer carries out the following actions: updating the inventory level, receiving shipments from the manufacturer, fulfilling customer orders, and placing a replenishment order. All these activities are coordinated and occur at the start of the review period.

A2) The manufacturer fully satisfies every non zero replenishment order placed by the retailer, and does so with a delay $L = \ell T$, where $\ell \in \mathbb{Z}^+$.

A3) Products are delivered to the retailer in new condition and deteriorate while stored in inventory.

Under these assumptions, the inventory dynamics over the k-th review period has the following form

$$y(k+1) = \rho\left(y(k) + u(k-L) - h(k)\right), \quad y(0) = 0, \qquad (1)$$

where:

-$y(k+1)$ is the non deteriorated inventory level at the end of the k-th review period;

-$u(k-L)$ is the replenishment order issued at time $(k-L)$;

-$h(k)$ is the fulfilled part of the customer demand $d(k)$. It is

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given by

$$h(k) \stackrel{\triangle}{=} \min\{y(k)+u(k-L), d(k)\} = d(k) - z(k), \quad (2)$$

where $z(k) \in [0,\, d(k)]$ denotes unmet demand.
-$\rho \in (0,1)$ represents the known decay factor accounting for inventory deterioration.

B. Assumptions on Future Customer Demand

As frequently observed in practical applications, customer demand exhibits highly volatile and unpredictable behavior which makes it extremely challenging to obtain accurate forecasts using conventional mathematical models [8]. For this reason, and in line with the principles of robust control, the demand forecast adopted for implementing the proposed MPC relies solely on the following very intuitive assumption:
A4) At each time k, the future demand over a prediction horizon

$$H_{p,k} \triangleq \{k+1, \dots, k+M\}$$

is assumed to be unknown but bounded, and belongs to a compact uncertainty set $\mathscr{D}_k$. This set is defined by two known trajectories, $d^-(k+j)$ and $d^+(k+j)$, such that

$$d^-(k+j) \le d(k+j) \le d^+(k+j), \quad j = 1, \dots, M.$$

This representation allows the demand to exhibit arbitrary temporal variations, including rapid oscillations, while remaining consistent with known operational limits. As a result, it captures a wide range of realistic demand patterns without requiring probabilistic assumptions. To support prediction within the MPC framework, the demand sequence is decomposed as

$$d(k+j) = \hat{d}(k+j|k) + \delta d(k+j|k),$$

where $\hat{d}(k+j|k)$ is the nominal prediction and $\delta d(k+j|k)$ is an unknown but bounded prediction error. The nominal trajectory $\hat{d}(k+j|k)$ is selected as the midpoint of the uncertainty interval, i.e.,

$$\hat{d}(k+j|k) = \frac{d^+(k+j) + d^-(k+j)}{2},$$

which minimizes the worst-case prediction error in the ℓ_2-norm sense over the admissible set.
This modeling choice provides two key advantages. First, it enables a distribution-free treatment of uncertainty, making the approach robust to model mismatch and abrupt demand changes. Second, it leads to a tractable robust MPC formulation, where worst-case performance over $\mathscr{D}_k$ can be systematically addressed.
Finally, the prediction horizon M is intentionally chosen to be relatively short. This reflects the limited reliability of long-term forecasts in highly uncertain environments and ensures that the control policy remains responsive to newly observed demand data.

III. THE BULLWHIP EFFECT

The BE is commonly quantified through demand variance amplification (see, e.g., [4]-[6]). However, this measure fails to capture the full temporal structure of order fluctuations. In particular, the frequency of oscillations, which is closely related to operational costs, is not explicitly addressed. The contribution of this paper is to propose a novel time-domain characterization of the BE that accounts for both the amplitude and the frequency of order variations through finite-difference operators. This formulation is naturally suited to discrete-time systems and directly compatible with MPC design. Figure 1 illustrates the rationale behind the proposed approach. It depicts two discrete-time sequences of replenishment orders generated under two distinct inventory policies. The two sequences are permutations of the same set of samples and therefore share identical mean and variance. Nevertheless, the second policy exhibits significantly more frequent oscillations and directional reversals. When the cost structure penalizes changes in order quantities (e.g., setup, changeover, and administrative costs), such high-frequency fluctuations translate into increased operational expenditures. This example highlights the limitations of traditional variance-based measures of the BE, which fail to capture the temporal frequency and intensity of order adjustments, focusing solely on magnitude-based variability.

To reduce the amplitude of oscillations of a generic replenishment policy $f(k), k \in Z^+$, it is necessary to reduce the modulus of the first difference series $\Delta f(k) = f(k) - f(k-1)$. To also reduce the number of oscillations it is necessary to reduce the turning points of $f(k)$ imposing some constraints on sign changes of $\Delta f(k)$. A sign change in the first difference can only take place if the second difference $\Delta^2 f(k)$ influences the behavior of $\Delta f(k)$. This may happen either because $\Delta^2 f(k)$ becomes large enough to force $\Delta f(k)$ to increase or decrease noticeably, or because $\Delta^2 f(k)$ itself changes sign, thereby reversing the local trend of the first difference. This leads to a useful observation: if the second difference $\Delta^2 f(k)$ is kept uniformly small, then $\Delta f(k)$ can only vary slowly. It cannot cross zero frequently, since each zero-crossing would require a substantial change in $\Delta f(k)$, a change that a small second difference cannot induce. Consequently, over any finite interval, the function $f(k)$ can exhibit only a limited number of turning points.

IV. THE MPC BASED REPLENISHMENT STRATEGY

The uncertainty in future customer demand, represented as interval bounds, naturally gives rise to a min-max formulation of the MPC problem.
For any $k \in \mathbb{Z}^+$, define the control horizon

$$H_{c,k} \triangleq [k, \dots, k+N-1], \quad (3)$$

of length $N \le M - L + 1$, and let

$$U_k \triangleq [u(k \mid k), \dots, u(k+N-1 \mid k)] \quad (4)$$

denote the sequence of predicted replenishment orders to be computed. In accordance with the receding-horizon strategy,

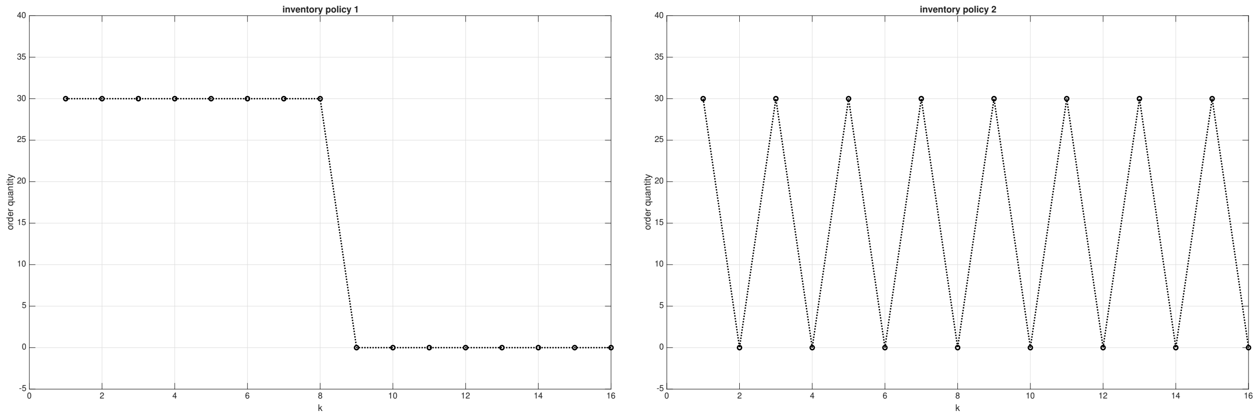

Fig. 1. The order quantity patterns of two different inventory policies

only the first control input is implemented:

$$u(k) = u(k \mid k). \tag{5}$$

The sequence U_k is obtained as the solution to the following robust MPC optimization problem.

$$\min_{U_k} \max_{d(\cdot)\in\mathscr{D}_k} J_k \tag{6}$$

$$\text{subject to}$$

$$u(k+i|k) \in [d^-(k+L+i), d^+(k+L+i)] \tag{7}$$

$$i = 0, \cdots, N-1$$

where:

$$\begin{aligned} J_k &= \sum_{i=0}^{N-1} e^T(k+L+i|k) q_i(k) e(k+L+i|k) \\ &+ (\Delta u(k|k))^T \lambda_1(k) \Delta u(k|k) \\ &+ (\Delta^2 u(k|k))^T \lambda_2(k) \Delta^2 u(k|k) \end{aligned} \tag{8}$$

$$\begin{aligned} e(k+L+i|k) &= (x(k+L+i|k) + u(k+i|k)) \\ &- d^+(k+L+i) \end{aligned} \tag{9}$$

$$\Delta u(k|k) = u(k|k) - u(k-1) \tag{10}$$

$$\Delta^2 u(k|k) = u(k|k) - 2u(k-1) + u(k-2) \tag{11}$$

The cost functional is formulated to simultaneously achieve accurate inventory tracking and regulate the dynamic behavior of the replenishment policy.

The primary objective is to minimize the predicted deviation between the available inventory $x(k+L+i|k)+u(k+i|k)$ and the upper bound demand trajectory $d^+(k+L+i)$, hereby ensuring high levels of demand fulfillment while preventing excessive stock accumulation or waste.

In parallel, the penalties applied to the first order variations $\Delta u(k|k)$ and second order variations $\Delta^2 u(k|k)$ of the order sequence enforce smoothness in the replenishment decisions, effectively limiting abrupt adjustments and suppressing high frequency oscillations. Since such oscillatory and reactive ordering behaviors are major contributors to bullwhip amplification, these regularization terms play a fundamental role in stabilizing the control signal and containing the propagation of variability across the SC.

The hard constraints (7) applied to the predicted replenishment orders $u(k+i|k)$ are chosen so as to ensure that oscillations of $u(k)$ do not exceed the amplitude of the interval containing customer demand.

Through this integrated formulation, the functional supports a replenishment strategy that is not only robust to demand uncertainty but also inherently resilient to the BE.

Equation (1) gives

$$\begin{aligned} y(k+L+i|k) &= \rho^{L+i} y(k) + \sum_{\ell=0}^{L-1} \rho^{L+i-\ell} u(k+\ell-L) \\ &+ \sum_{\ell=0}^{i-1} \rho^{i-\ell} u(k+\ell|k) - \rho^{L+i} h(k) \\ &- \sum_{\ell=1}^{L+i-1} \rho^{L+i-\ell} h(k+\ell|k) \end{aligned} \tag{12}$$

where $h(k+\ell|k) \triangleq \hat{d}(k+\ell|k) + \delta d(k+\ell|k) - z(k+\ell|k)$ is the predicted satisfied demand and $z(k+\ell|k)$ is the predicted unmet demand.

V. Reformulation of the Min-Max MPC Problem

Following established results ([9]-[11]) in robust MPC the original min-max optimization problem can be reformulated as a constrained robust least-squares problem [12] , which enables efficient numerical computation via interior-point algorithms [13]. Referring to ([9]-[11]) for more details, this result is obtained defining extended matrices and vectors (denoted by D_k, e_k, b_k, δb_k, u_k^-, and u_k^+) which allow the cost function to be recast in the equivalent form

$$\min_{U_k} \max_{\|\delta b_k\| \le \xi_k} \left\| (b_k + \delta b_k) - D_k U_k^\top \right\|^2,$$

subject to the element-wise constraints

$$u_k^- \le U_k^\top \le u_k^+.$$

Because the uncertainty term enters the expression affinely and is norm-bounded, the inner maximization admits a closed-form characterization. This permits rewriting the problem in the tractable form

$$\min_{U_k} \left\| b_k - D_k U_k^\top \right\| + \xi_k,$$

while preserving the same box constraints. The scalar ξ_k denotes the maximum Euclidean norm of the perturbation δb_k and is independent of U_k; it quantifies the overall magnitude of the demand uncertainty over the prediction horizon.

The value of ξ_k can be reduced through two complementary mechanisms:

- selecting the nominal predicted demand trajectory $\hat{d}(k+\ell \mid k)$ as the midpoint of the uncertainty interval, thereby minimizing the associated prediction error;
- reducing the predicted unmet demand $z(k+\ell \mid k)$, which corresponds to maximizing the fulfilled demand and minimizing the tracking error appearing in the original cost functional.

This reformulation preserves the robustness of the original min-max problem while significantly improving computational tractability, making it suitable for real-time implementation in SC systems characterized by substantial demand uncertainty.

VI. How to choose the weights of the functional

The weights $q_i(k)$, $i=0,\dots,N-1$, and $\lambda_j(k)$, $j=1,2$, in the cost functional play a central role in balancing two competing objectives: accurate inventory tracking and suppression of undesirable variability in the replenishment signal. According to the guidelines proposed in [14], each weight is chosen as inversely proportional to the square of the maximum admissible variation of the corresponding variable.

A. Tracking error weighting

The coefficients $q_i(k)$ penalize deviations between the available inventory and the reference demand trajectory. They are defined as

$$q_i(k) = \frac{\zeta^i}{(\bar{e}(k+L+i))^2}, \quad i=0,\dots,N-1,$$

where $\zeta \in (0,1)$ is a forgetting factor that progressively reduces the importance of predictions farther in the horizon. The quantity $\bar{e}(k+L+i)$ represents the maximum tolerable tracking error and is defined as

$$\bar{e}(k+L+i) = \varepsilon_{d^+}\, d^+(k+L+i),$$

with $\varepsilon_{d^+} \in (0,1)$ denoting the acceptable relative deviation from the upper demand bound.
This choice provides a clear physical interpretation: small values of ε_{d^+} enforce tight inventory tracking, while larger values allow greater flexibility to reduce control effort.

B. Control variation weighting

The coefficients $\lambda_1(k)$ and $\lambda_2(k)$ penalize, respectively, the first and second differences of the replenishment order signal:

$$\lambda_1(k) = \frac{1}{(\bar{\Delta}u(k))^2}, \qquad \lambda_2(k) = \frac{1}{(\bar{\Delta}^2 u(k))^2}.$$

where $\bar{\Delta}u(k)$ and $\bar{\Delta}^2u(k)$ denote the maximum admissible variations for $\Delta u(k|k)$ and $\Delta^2 u(k|k)$, respectively. They are computed as a given percentage of the maximum amplitude of the intervals where $\Delta u(k|k)$ and $\Delta^2 u(k|k)$ can take values, i.e.: $d^+(k+L) - d^-(k+L) \stackrel{\triangle}{=} \mathscr{A}_k$ and $2\mathscr{A}_k$ respectively, namely

$$\bar{\Delta}u(k) = \varepsilon_{\Delta u}\mathscr{A}_k \quad \bar{\Delta}^2u(k) = \varepsilon_{\Delta^2 u}(2\mathscr{A}_k)$$

These parameters have a direct operational interpretation:

- $\varepsilon_{\Delta u}$ controls the maximum allowable adjustment in order quantities between consecutive periods;
- $\varepsilon_{\Delta^2 u}$ regulates the rate of change of such adjustments, and therefore directly limits the frequency of oscillations.

In particular, smaller values of $\varepsilon_{\Delta^2 u}$ enforce smoother control actions and suppress rapid directional changes, which are known to amplify the BE.

VII. Numerical results

We simulate a periodically reviewed perishable supply chain characterized by a lead time $L=2$ and an initial inventory level $y(0)=0$. The contiguous and overlapping uncertainty sets D_k, $k \in \mathbb{Z}^+$, enclosing the realized demand trajectory are illustrated in Fig 2. The prediction horizon is set to $M=3$. In the MPC-based implementation, the min–max controller operates with a control horizon of length $N = M-L+1 = 2$.

Two simulation scenarios (denoted Sim 1 and Sim 2) are performed to illustrate how different penalization structures in the MPC cost functional influence the dynamic behavior of the replenishment policy and its effectiveness in mitigating the BE.

The parameters determining the weights $q_i(k)$ and $\lambda_i(k)$, $i=1,2$, are reported in Table I.

The two scenarios differ in their choice of the parameters $\varepsilon_{\Delta u}$ and $\varepsilon_{\Delta^2 u}$, which define the allowable amplitudes of the first- and second-order differences in the control sequence, respectively. In Sim 2, both parameters are reduced, prompting the controller to attenuate both amplitude and frequency of oscillations in the order sequence.

As observed in Figure 3, the policy in Sim 2 produces substantially smoother and lower frequency replenishment adjustments compared with Sim 1.

TABLE I

Numerical values of the weight-selection parameters used in the MPC cost functional for simulations 1 and 2

Parameter	**Sim 1**	**Sim 2**
ε_{d^+}	0.01	0.01
ζ	0.36	0.36
$\varepsilon_{\Delta u}$	0.10	0.01
$\varepsilon_{\Delta^2 u}$	0.10	0.01

To quantify service and stock behavior, the following performance indices are adopted:

$$I_{UCD} = \frac{\sum_{k=0}^{N_s} d(k) - h(k)}{\sum_{k=0}^{N_s} d(k)} \times 100$$

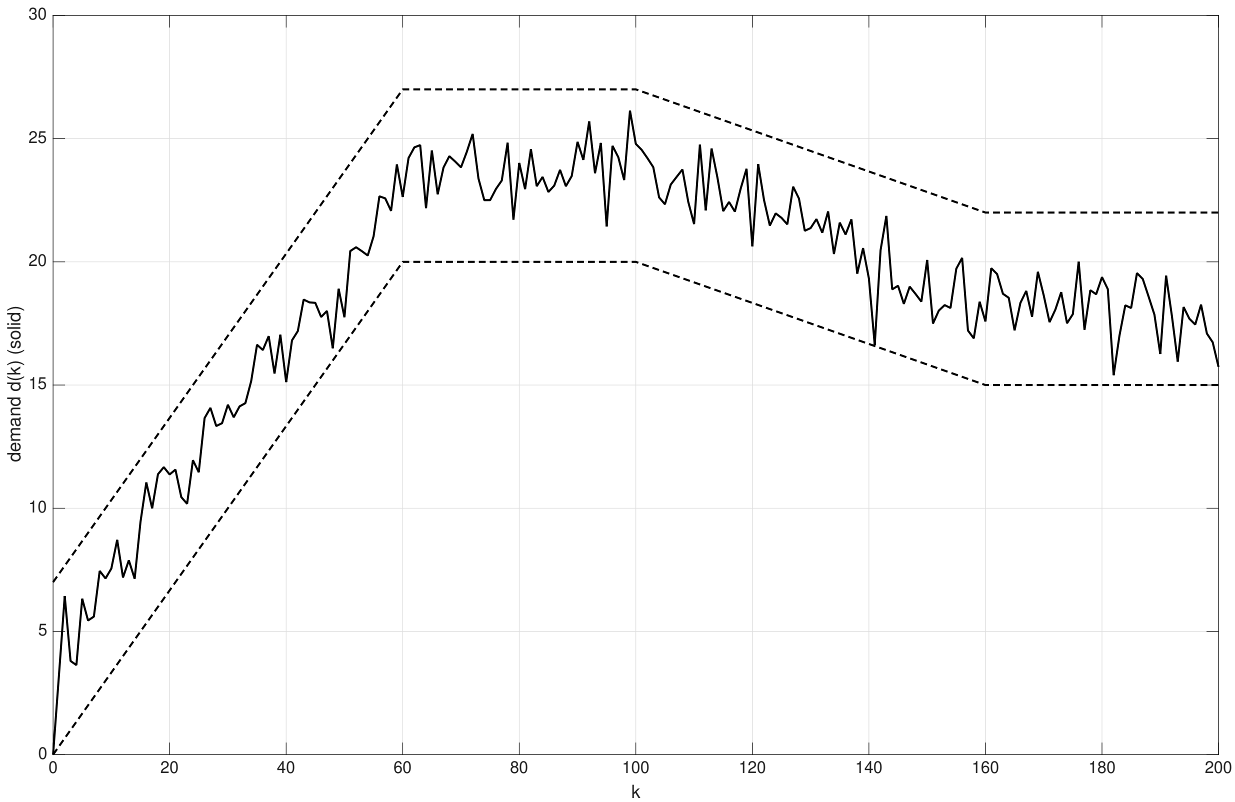

Fig. 2. Actual customer demand trajectory $d(k)$ (solid line) together with the consecutive, contiguous, and overlapping uncertainty sets $\mathscr{D}_k$ (dashed lines) used for the MPC prediction.

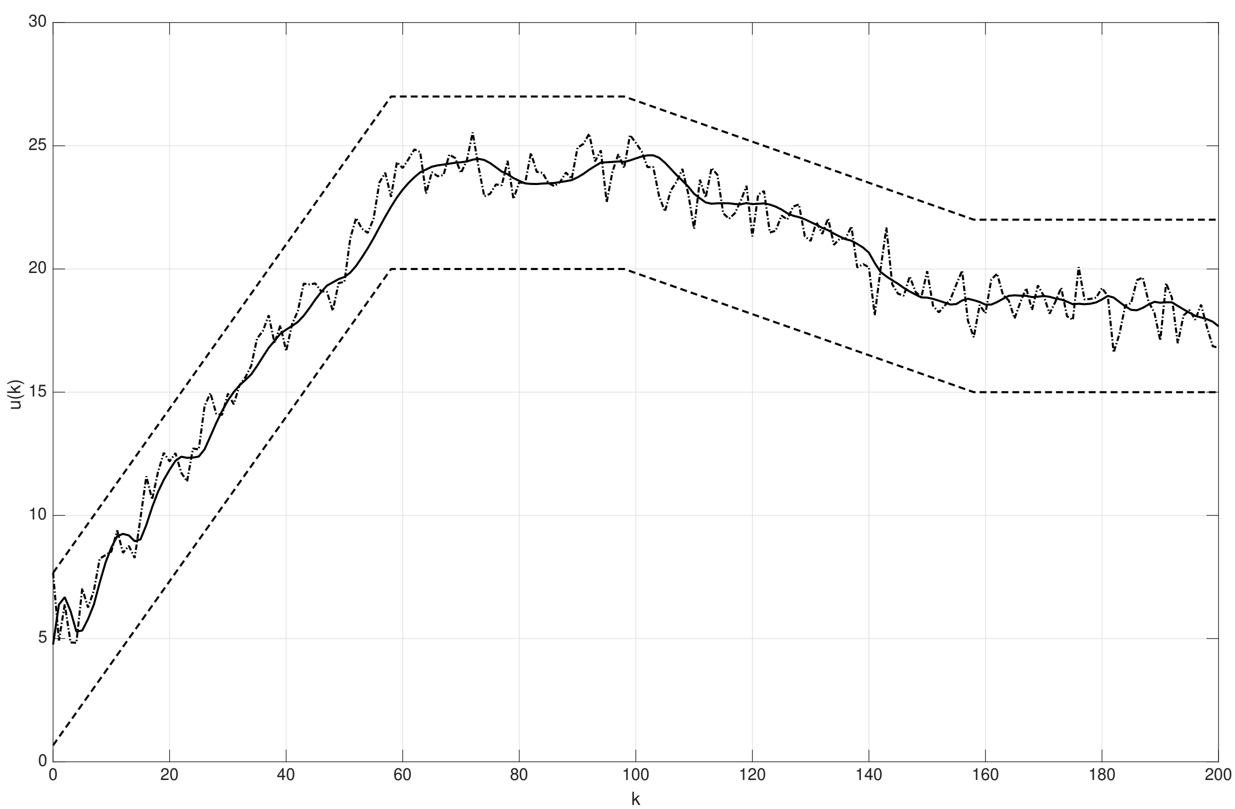

Fig. 3. Replenishment order sequences $u(k)$ generated under Sim1 (dashed dotted line) and Sim2 (solid line)

$$I_{SG} = \sum_{k=0}^{N_s} y(k)$$

where $N_s = 200$ is the length of simulations.

The index I_{UCD} measures the percentage of unmet demand (lower is better) while I_{SG} quantifies cumulative non deteriorated stock. The resulting performance values are reported in Table II.

TABLE II

PERFORMANCE INDICES FOR SIMULATIONS 1 AND 2

Indices	**Sim 1**	**Sim 2**
I_{UCD}	0.01	0.04
I_{SG}	594	564

Table II shows that increased regularity in the replenishment policy does not significantly influence the performance metrics, while substantially reducing the BE.

VIII. CONCLUSIONS

This paper has introduced a new and more rigorous characterization of the BE that explicitly accounts for the frequency of demand-induced oscillations. By recognizing oscillatory dynamics as a critical dimension of SC behavior, the analysis goes beyond traditional variance-based measures and enables a more comprehensive assessment of instability. Building on this conceptual foundation, an MPC-based inventory replenishment policy has been derived to actively limit undesirable large and frequent oscillations while maintaining a high service level in meeting demand. Numerical results substantiate the effectiveness of the proposed approach.

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