Robust MIMO Over-the-Air Computation for Distributed Estimation

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***Abstract*—As industrial networks transition toward the Industry 5.0 paradigm, the integration of sensing, communication, and computation into a unified, task-oriented framework becomes essential for supporting resilient digital twins and human-robot collaboration. Over-the-Air Computation (AirComp) has emerged as a vital technology for low-latency data aggregation in industrial IoT. However, the inherent reliance of AirComp on the arithmetic mean makes it critically vulnerable to outliers generated by sensor failures or Byzantine behavior. This paper proposes a robust MIMO-AirComp framework developed around the Geometric Median. By leveraging the spatial dimensions provided by MIMO architectures, we adapt the iterative Weiszfeld algorithm for simultaneous analog transmission over the wireless multiple-access channel. We demonstrate that by transmitting weighted measurement vectors, the fusion center obtains a resilient global estimate even when nearly half the network is compromised. Our methodology includes a detailed analysis of the pre-processing requirements at the sensors and the beamforming strategies at the receiver. Simulation results confirm that our robust estimator provides a significant increase in resilience against data-poisoning phenomena compared to state-of-the-art AirComp techniques, with negligible overhead in benign conditions.**

***Index Terms*—Over-the-Air Computation, MIMO, Distributed Estimation, Geometric Median, Robustness, Industry 5.0.**

I. Introduction

Future wireless interfaces and Internet-of-Things (IoT) networks, particularly those envisioned within the 6G paradigm, are expected to integrate sensing, communication, and computation into a unified framework capable of delivering wideband, low-latency, and scalable services. A critical shift in this direction is the transition from traditional data-oriented transmission toward task-oriented communication. In this new paradigm, the objective of the communication system is no longer merely the reliable delivery of raw data, but the efficient support of downstream tasks such as inference, learning, or distributed state estimation in industrial scenarios.

Among the most relevant task-oriented paradigms is Over-the-Air Computation (AirComp) [1]–[4], a technique that exploits the superposition property of a wireless Multiple-Access Channel (W-MAC) to compute functions of distributed data directly in the air. AirComp has been successfully employed in various scenarios, ranging from High-Mobility Multimodal (HMM) sensing to federated learning, where it drastically reduces latency by enabling simultaneous transmissions and analog function computation. Extending AirComp from Single-Input Single-Output (SISO) to Multiple-Input Multiple-Output (MIMO) systems, as shown in [5], further increases scalability and robustness by leveraging spatial multiplexing and beamforming for concurrent multiple functions computation.

In distributed sensing networks, the design of robust and accurate state estimation mechanisms is of paramount importance, especially when operating in the presence of noise, fading, or even malicious malfunctioning nodes. Traditional AirComp approaches rely on the arithmetic mean as an aggregation function [6]–[8]. While the naive mean is trivial to compute via AirComp and is Minimum-Variance Unbiased Estimator (MVUE) under purely Gaussian noise, it is highly sensitive to outliers and adversarial perturbations. To overcome this limitation, this work investigates a robust alternative: the geometric median.

The problem of robust AirComp aggregation has been extensively studied in the federated learning community, particularly in the presence of Byzantine attacks [9], [10]. In the context of AirComp, Byzantine nodes are defined as malicious entities that exploit the superposition property of the wireless channel to intentionally inject distortion to the aggregated signal. Since multiple transmissions are simultaneously combined in the air, even a small number of malicious users may significantly bias the computed function and degrade the accuracy of the global model or estimated statistic. The geometric median has emerged as a robust aggregation rule capable of mitigating the influence of extreme outliers injections while remaining compatible with the additive structure of AirComp. This property, originally exploited in gradients aggregation [11], [12] for distributed edge learning, is particularly attractive also for distributed estimation where outliers arise naturally due to severe noise, deep fading, and especially from external malicious intent. It was demonstrated that when the number of Byzantine nodes satisfies $B < \frac{K}{2}$ (where K is the number of edge nodes), distributed stochastic gradient descent with geometric median aggregation rule linearly converges to a neighborhood of the optimal solution [11], [12]. Motivated by these robustness guarantees, validated in distributed learning scenarios, in this paper we apply and validate the geometric median as the aggregation mechanism in the context of distributed state estimation, for which the state-of-the-art literature [6] has considered only aritmetic mean.

The primary contributions of this paper are as follows:

- We propose a complete MIMO-AirComp architecture for robust distributed state estimation, where each sensor performs local preprocessing to transmit an encoded vector through a MIMO W-MAC extending [6].
- We study the implementation of an over-the-air geometric median estimator, formulated through a smoothed version of the Weiszfeld algorithm proposed in [11]. This iterative update is reformulated to match the additive structure of the MAC, allowing for robust estimation directly in the wireless medium.
- We provide an extensive performance evaluation through MATLAB simulations, analyzing the system's convergence behavior and resilience against intentional or unintentional data poisoning: sign-flip attacks by Byzantine nodes or higher variance by malfunctioning nodes.

The remainder of this paper is organized as follows: Section II introduces the theoretical foundations of AirComp and MIMO architectures. Section III describes the mathematical framework for distributed estimation. Section IV presents the proposed robust aggregation method based on the geometric median. Section V evaluates the framework through simulation results, and Section VI concludes the paper.

II. Over-the-Air Computation Model

In traditional Wireless Sensor Networks (WSNs), the standard approach is to separate communication from computation. Orthogonal Multiple Access (OMA) schemes, such as Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA), are typically employed to prevent inter-node interference. This allows the fusion center (FC) to decode individual measurements s_i from each sensor $i \in \{1, \dots, K\}$ before computing a desired function $f(s_1, \dots, s_K)$. However, as the number of nodes K increases, OMA becomes a bottleneck due to the linear scaling of latency and high synchronization overhead.

AirComp represents a paradigm shift toward "compute-when-communicate." It exploits the natural superposition property of the W-MAC. If K sensors transmit signals d_i simultaneously over the same resource block, the received signal $\boldsymbol{g}$ at the FC is a coherent sum:

$$g = \sqrt{\eta} \sum_{i=1}^{K} h_i b_i d_i + n, \tag{1}$$

where $\sqrt{\eta}$ is a magnitude alignment factor, h_i models the fading channel, b_i is the equalization factor, d_i is the pre-processed measure and n is the additive white Gaussian noise (AWGN). By performing pre-processing at the sensors (e.g., adapt the measurement in order to achieve a non-linear function through a MAC) and post-processing at the FC (e.g., recover the non linearity), the system can compute targeted functions in a single time slot, regardless of K.

A. Nomographic Function Processing

The scope of AirComp is not limited to simple addition. According to the nomographic representation theorem [13], a broad class of multivariate functions can be decomposed into a sum of univariate pre-processing functions $\phi_i(\cdot)$ and a single post-processing function $\psi(\cdot)$. In an ideal MAC we have:

$$f(s_1, \dots, s_K) = \psi\left(\sum_{i=1}^{K} d_i\right). \tag{2}$$

In this framework, each sensor performs a local transformation $d_i = \phi_i(s_i)$ on its measurement before transmission. The FC then applies ψ to the aggregated signal to recover the target function. This decomposition allows AirComp to compute various statistical measures, including arithmetic means, variances, and more complex non-linear iterative estimators.

B. MIMO-AirComp and Spatial Diversity

Extending AirComp to MIMO systems provides significant advantages in terms of reliability and throughput. Unlike SISO systems that rely purely on scalar power control, MIMO-AirComp leverages spatial degrees of freedom to mitigate fading and noise.

In a MIMO setup, let $\boldsymbol{H}_i \in \mathbb{C}^{N_r \times N_t}$ be the channel matrix between sensor i and the FC. Each sensor applies a precoding matrix $\boldsymbol{B}_i$ to its data vector $\boldsymbol{s}_i$, and the FC applies a receive beamforming matrix $\boldsymbol{A}$. Where $\boldsymbol{s}_i = \boldsymbol{C}_i \mathbf{x} + \boldsymbol{v}_i$ with $\boldsymbol{x} \in \mathbb{R}^{L_x}$ being the state of interest, $\boldsymbol{v}_k \in \mathcal{N}(0, \sigma_{ns}^2 \boldsymbol{I})$ the measurement noise, and $\boldsymbol{C}_i \in \mathbb{R}^{L_s \times L_x}$ is the observations matrix. It has been assumed that each sensor acquire from the environment a data which is constant over the discrete time iterations. The estimated function $\hat{\boldsymbol{g}}$ is expressed as:

$$\hat{\boldsymbol{g}} = \boldsymbol{A}\left(\sum_{i=1}^{K} \boldsymbol{H}_i \boldsymbol{B}_i \boldsymbol{d}_i + \boldsymbol{n}\right), \tag{3}$$

where $\boldsymbol{n} \in \mathcal{CN}(0, \sigma_{nc}^2 \mathbf{I})$ is the thermal AWGN at the FC. The goal of the joint transceiver design is typically to minimize the Mean Squared Error (MSE) between the desired aggregate $\boldsymbol{g}$ and the estimate recovered after receive beamforming $\hat{\boldsymbol{g}}$:

$$\text{MSE} = \mathbb{E}\left[\|\hat{\boldsymbol{g}} - \boldsymbol{g}\|^2\right]. \tag{4}$$

This spatial processing allows for the parallel computation of vector-valued functions, which is essential for the high-dimensional state estimation tasks explored in this work.

III. Application to Distributed Estimation

We consider a distributed sensing network consisting of K wireless nodes and a single FC arranged in a centralized star topology. Each node $i \in \{1, \dots, K\}$ generates a set of L_s heterogeneous sensing measurements, denoted by the vector $\mathbf{s}_k = [s_{i,1}, \dots, s_{i,L_s}]^T \in \mathbb{R}^{L_s}$. To exploit spatial diversity, each node is equipped with N_t transmit antennas, while the FC is equipped with N_r receive antennas.

The objective of the system is to compute L_f target functions of the aggregate data $D = \{\mathbf{s}_i\}_{i=1}^{K}$ such that:

$$f(D) : \mathbb{R}^{K \times L_s} \to \mathbb{R}^{L_f}. \tag{5}$$

Following the nomographic representation, any real-valued multivariate function can be decomposed into a finite sum of univariate pre-processing functions $\phi_i(\cdot)$ and a post-processing function $\psi(\cdot)$. In this state estimation context, the nodes transmit their processed signals simultaneously to harness the superposition property of the wireless medium.

To ensure that the signals from different nodes combine coherently at the FC, the transmit precoding matrices $\boldsymbol{B}_i \in \mathbb{C}^{N_t \times L_d}$ and the receive beamforming matrix $\boldsymbol{A} \in \mathbb{C}^{N_r \times L_d}$ must be jointly optimized. In our framework, we employ zero-forcing equalization to suppress channel distortion. The transmit power is subject to individual constraints P_{max} at each sensor, reflecting the limitations of battery-powered IoT devices.

The goal of the optimization is to find $\boldsymbol{A}$ and $\{\boldsymbol{B}_i\}_{i=1}^{K}$ that minimize the MSE of the estimate (4). This process transforms the fading MIMO W-MAC into a set of parallel Gaussian channels suitable for computation.

A critical requirement for successful AirComp is symbol-level synchronization among all K nodes. Therefore, we assume that the nodes are synchronized in both time and frequency, allowing their analog waveforms to overlap precisely at the FC's antenna array. This alignment is essential to prevent phase offsets from destructive interference, which would otherwise introduce significant bias into the estimated state. A solution to achieve a shared clock over the air is discussed in [14].

IV. Proposed Robust Approach

Despite the efficiency of standard MIMO-AirComp, the reliance on the arithmetic mean makes the system inherently fragile under unintentional or intentional data poisoning. In real-world IoT deployments, sensor malfunctions due to hardware degradation can lead to abnormally inaccurate measurements. More critically, Byzantine nodes may deliberately transmit corrupted values to bias the global estimate. To address these vulnerabilities, we propose a robust aggregation framework based on the geometric median.

A. Geometric Median as a Robust Aggregation Rule

The geometric median is a spatial estimator that minimizes the sum of Euclidean distances to the data samples, effectively attenuating the influence of outliers. For a set of local estimates $\{\boldsymbol{s}_i\}_{i=1}^{K}$, the geometric median $\boldsymbol{z}^*$ is defined as:

$$\boldsymbol{z}^* = \arg\min_{\boldsymbol{z}\in\mathbb{R}^{L_x}} \sum_{i=1}^{K} \|\boldsymbol{z} - \boldsymbol{s}_i\|_2, \tag{6}$$

where $\|\cdot\|_2$ denotes the Euclidean norm. Unlike the arithmetic mean, the geometric median has a breakdown point of 50%, meaning it can maintain estimation stability even if nearly half of the sensors are compromised.

To solve this optimization problem, we employ the iterative Weiszfeld algorithm [15]:

$$\boldsymbol{z}^{(k+1)} = \frac{\sum_{i=1}^{K} \beta_i^{(k)} \boldsymbol{s}_i}{\sum_{i=1}^{K} \beta_i^{(k)}}, \quad \beta_i^{(k)} = \frac{1}{\|\boldsymbol{z}^{(k)} - \boldsymbol{s}_i\|_2}. \tag{7}$$

This iterative scheme converges to the geometric median provided that no iterate coincides exactly with one of the s_i. In practice, extreme fading or noise spikes may cause $\|z^{(k)} - s_i\|$ to become very small, which may lead to numerical instability.

To ensure numerical stability and prevent division by zero when an iterate coincides with a measurement, we adopt a smoothed variant of the Weiszfeld algorithm [9]:

$$\beta_i^{(k)} = \frac{1}{\max(\nu, \|\boldsymbol{z}^{(k)} - \boldsymbol{s}_i\|_2)}, \tag{8}$$

where ν is a small smoothing parameter.

B. Integrating the Geometric Median into MIMO-AirComp

The primary challenge is to execute these iterative updates over-the-air without decoding individual sensor readings. We reformulate the Weiszfeld update into an additive structure compatible with the W-MAC.

In each iteration k, the FC broadcasts the current estimate $\boldsymbol{z}^{(k)}$ to all nodes. Each node i then computes its local weight $\beta_i^{(k)}$ based on its private measurement $\boldsymbol{s}_i$. To reconstruct the update at the FC, nodes transmit an augmented symbol vector $\boldsymbol{d}_i \in \mathbb{R}^{L_x+1}$:

$$\boldsymbol{d}_i^{(k)} = [\beta_i^{(k)} \boldsymbol{s}_i, \beta_i^{(k)}]^T. \tag{9}$$

This allows the FC to recover the numerator and the denominator of the Weiszfeld update simultaneously through the superposition property of the MIMO channel. Specifically, the receive beamformer at the FC yields an aggregate vector

$$\hat{\boldsymbol{d}}^{(k)} \approx \sum_{i=1}^{K} \boldsymbol{d}_i^{(k)},$$

from which the next iterate is computed as

$$\boldsymbol{z}^{(k+1)} = \frac{\hat{\boldsymbol{d}}^{(k)}(1 : L_x)}{\hat{\boldsymbol{d}}^{(k)}(L_x + 1)}. \tag{10}$$

This process repeats until convergence.

A key architectural implication of the proposed robust aggregation framework concerns the dimensionality of the transmitted symbol. Specifically, the Weiszfeld-based geometric median requires each sensor to convey not only its locally estimated state vector but also the associated Weiszfeld weight β_i. This scalar quantity plays a critical role in the iterative refinement process and must be computed over-the-air as we have shown. Consequently, one additional transmission degree of freedom must be allocated for the weight component β_i.

In practical terms, this means that each sensor must be equipped with one extra transmit antenna compared to the naive averaging scenario. While the naive mean requires transmitting only L_x state-related components, the integration of the geometric median in the MIMO AirComp framework requires $L_x + 1$ components. The extra dimension is therefore dedicated exclusively to conveying β_i, ensuring that the FC receives not only the aggregated weighted measurements but also the aggregated weights themselves, thus enabling the reconstruction of the Weiszfeld ratio. Other than the augmented hardware complexity, another related concern to take into account is that the required extra antenna won't be used for spatial diversity.

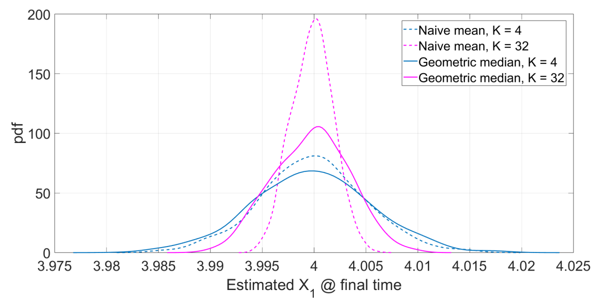

Fig. 1. MIMO AirComp-based estimated state $\hat{x}$ precision and accuracy assessment through empirical PDF with different numbers of nodes $K = 4, 32$.

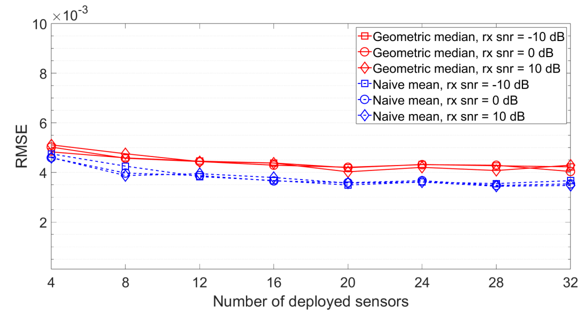

Fig. 2. MIMO AirComp-based estimated state $\hat{x}$ distortion assessment through RMSE with a span from 4 to 32 of K and with different values of rx $\text{snr}_{\text{dB}} = -10, 0, 10$.

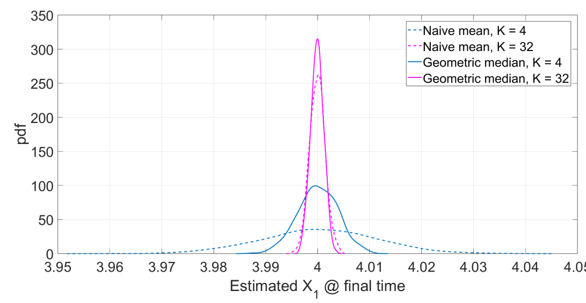

Fig. 3. MIMO AirComp-based estimated state $\hat{x}$ precision and accuracy assessment through empirical PDF with different numbers of nodes $K = 4, 32$ and with $M = 2$ malfunctioning nodes.

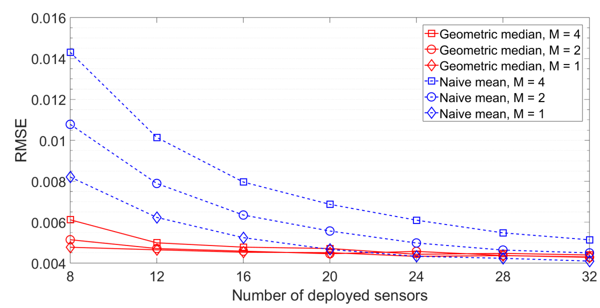

Fig. 4. MIMO AirComp-based estimated state $\hat{x}$ distortion assessment through RMSE with a span from 8 to 32 of K and with different number of malfunctioning nodes $M = 4, 2, 1$.

V. Simulations and Results

This section evaluates the performance of the proposed robust MIMO-AirComp framework through extensive MATLAB simulations. We compare the resilient geometric median estimator against the standard naive mean under various network conditions and adversarial scenarios.

A. Simulation Setup

The simulation parameters are summarized in Table I. We consider a distributed sensing network where the true state is set at $x = 4$ for monodimensional sensing. Multidimensional symbol vectors (e.g. $\boldsymbol{x} = [20, 10]^T$) were tested and were consistent with the results from the monodimensional case. The system employs Nakagami fading to model the wireless channel, with joint beamforming optimization performed at each symbol interval. Performance is quantified using three primary metrics: the temporal evolution of the estimated state over $T = 100$ iterations, the empirical Probability Density Function (PDF), and the Root Mean Squared Error (RMSE). Both the PDF and RMSE are averaged over $N = 1000$ Monte Carlo trials. The temporal evolution plots are not presented here for the sake of brevity.

TABLE I
Simulation Parameters

Parameter	Value
Monte Carlo Simulations (N)	1000
Time iterations (T)	100
Node Transmit Power (P_0)	10 mW
Antennas (N_t, N_r)	4, 4
Measurement Noise variance (σ^2_{ns})	10^{-4} to 10^{-2}
Signal to noise ratio (snr_{dB})	-10, 0, 10
Malfunctioning Nodes (M) / Byzantine Nodes (B)	1 to 4
Smooth factor (ν)	$\sqrt{\sigma_{ns}}$

B. Performance in Benign Environments

The following results highlight the performance comparison between the geometric median and the naive mean in absence of data poisoning. In these scenario, as shown in Fig. 1 and Fig. 2, both the naive mean and the geometric median successfully converge toward the true state. As expected, the naive mean serves as a performance upper bound under purely Gaussian noise, being the MVUE. However, results indicate that the geometric median provides a highly accurate approximation of the mean, with nearly identical convergence speed and only a marginal increase in variance. Furthermore, increasing the number of sensors K from 4 to 32 significantly reduces fluctuations for both estimators due to the $\approx 1/K$ noise variance reduction factor inherent in AirComp with respect to a sequential acquisition of data such as TDMA.

C. Resilience to Malfunctioning Nodes

We model unintentional sensor failure as a subset of nodes M experiencing a sensing noise variance 100 times higher than the nominal value ($\sigma^2_{ns,malf} = 100\sigma^2_{ns}$). Under these conditions, as shown in Fig. 3 and Fig. 4, the naive mean exhibits high dispersion and bias as it lacks a mechanism to down-

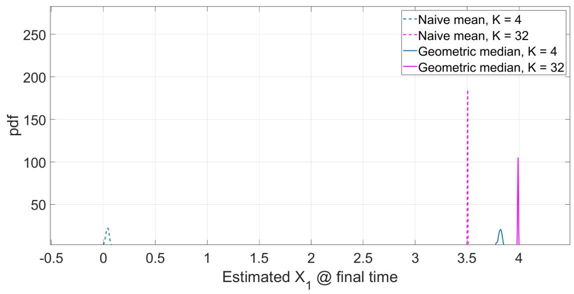

Fig. 5. MIMO AirComp-based estimated state $\hat{x}$ precision and accuracy assessment through empirical PDF with different numbers of nodes $K = 4, 32$ and with $B = 2$ malicious nodes performing Byzantine attacks.

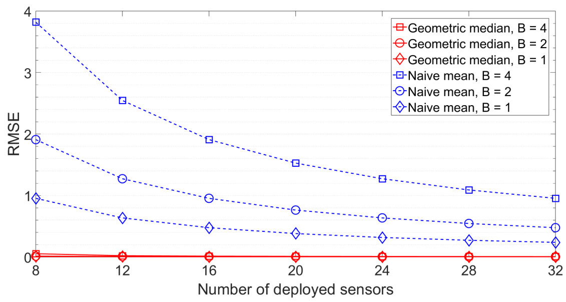

Fig. 6. MIMO AirComp-based estimated state $\hat{x}$ distortion assessment through RMSE with a span from 8 to 32 of K and with different numbers of malicious nodes $B = 4, 2, 1$ performing Byzantine attacks.

weight the high-variance outliers. In contrast, the geometric median effectively ignores these "heavy-tailed" measurements, maintaining a sharp empirical PDF centered at the true state and an RMSE significantly lower than that of the canonical naive mean AirComp approach.

D. Byzantine Resilience (Sign-Flip Attack)

The robustness of the framework is most evident under Byzantine attacks, where a subset of nodes B are malicious and flips the sign of their measurements ($y_\ell = -y_i$). From Fig. 5 and Fig. 6 the following observations can be done.

- The arithmetic average is heavily pulled toward the malicious values, leading to a permanent estimation bias and high RMSE.
- Even with multiple malicious nodes (up to $B < K/2$), the geometric median remains stable. The RMSE results demonstrate that the proposed robust MIMO-AirComp architecture preserves tracking accuracy, effectively "filtering" the sign-flipped data at the physical layer through the iterative Weiszfeld process.

The results confirm that while standard AirComp is optimal for ideal Gaussian settings, the proposed robust approach is essential for reliable distributed estimation in realistic or adversarial IoT environments.

VI. Conclusions

In nominal operation, with no data poisoning, the over-the-air geometric median converges with accuracy comparable to the over-the-air naive mean, demonstrating the correctness of the approach and the absence of unnecessary performance penalties. In contrast, when data poisoning is introduced, the over-the-air naive mean exhibits clear bias growth. The over-the-air geometric median accuracy is remarkable especially when Byzantine nodes attempt a sign flip attack attenuating the influence of corrupted data and preserving reliability. The over-the-air naive mean is still a preferable choice when there is a guarantee that there are no outliers, also the required pre-processing task for the sensors is lighter. Future extensions include the decentralization of AirComp-based robust estimation to peer-to-peer or clusterbased topologies.

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