Solving Multi-Manned Assembly Line Balancing under Resource and Capacity Constraints in the Biogas Industry

Mohieddine-El Hadi TERKI-HASSAINE, Taha ARBAOUI, Sylvain TOUCHARD, Tao WANG

Abstract— **The balancing of big-sized products can be addressed within the multi-manned assembly line balancing problem (MALBP) when more than one worker is assigned to each workstation and task parallelization is possible. In this work, we introduce a new variant of MALBP motivated by a real industrial case in biogas treatment device assembly: sharing mobile equipment across workstations to reduce total costs, compared to traditional binary equipment assignment variants. We propose a mathematical model to represent this problem and solve it for the case study involving 28 tasks with a total processing time of 65 hours and a cycle time of 21 hours. The results show that a single piece of equipment can be shared between two workstations, avoiding the need for a second unit.**

Keywords: Multi-manned, Assembly Line Balancing, Equipment Sharing, Walking Worker, Biogas Case Study.

I. INTRODUCTION

The assembly line balancing problem (ALBP) involves distributing tasks across workstations while respecting their precedence relationships. The multi-manned ALBP (MALBP) extends ALBP by allowing multiple workers to work simultaneously on different tasks within the same workstation, which is particularly relevant for large-sized products. This problem was first studied by Dimitriadis [1], who proposed a heuristic to minimize the number of workstations and consequently reduce the total line length. Later, Becker and Scholl [2] developed a branch-and-bound method for the MALBP with variable workplaces, where the product is divided into several mounting positions to decrease the risk of interference between workers. Fattahi et al [3] formulated the problem as a mixed-integer programming model and proposed an ant colony optimization method to obtain efficient solutions. However, only a few studies have addressed the MALBP while considering the assignment of equipment.

Human and material resources are critical in assembly systems because of their high costs and limited availability. The assignment process of equipment may involve selecting equipment from a predefined set of alternatives or grouping tasks that require the same tool within a single workstation to reduce investment costs.

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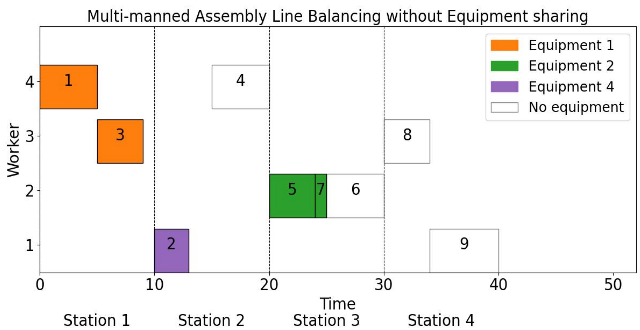
Hashemi-Petroodi et al (2022) [4], Hashemi-Petroodi et al (2023) [5] and Hashemi-Petroodi et al (2025) [6] examined equipment assignment within workstations in mixed-model, walking-worker assembly lines, aiming to minimize both worker and equipment costs. Elyasi et al [7] further extended this research by introducing stochastic task times. Similarly, Sahin and Kellegoz [8] analyzed equipment assignment from an investment perspective, where the number of required pieces of equipment in a workstation corresponds to the number of overlapping tasks using the same tool. Chen et al [9] studied a resource-constrained version of the problem in which each task is assigned to exactly one resource from a predefined set, applying a minimum deviation approach to simultaneously minimize the number of workers, workstations, and resources. Ferrari et al [10] explored equipment allocation considering mounting positions, while Pilati et al [11] investigated the MALBP with equipment sharing, incorporating both mounting-position and task synchronization constraints.

Despite these contributions, existing studies assume that a piece of equipment item is limited to a single workstation and cannot be shared across workstations. To the best of our knowledge, this study is the first to consider mobile equipment in a multi-manned assembly line. In this context, equipment can be used across multiple workstations while considering the movement time required between workstations, which is feasible when the cycle time is sufficiently long to allow the equipment to be relocated. In addition, this work also considers walking workers, where workers are allowed to move between workstations to perform different tasks.

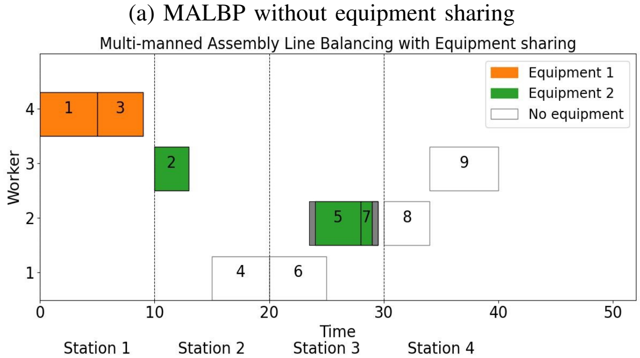
To better illustrate the problem, we refer to the example adapted from [12], presented in Table I. The original precedence data did not include equipment compatibility; this information was extended and added in the present example. The cycle time is set to 10 hours, and the time required to move an equipment item between adjacent workstations is assumed to be 30 minutes. The objective is to minimize the total number of workstations, workers, and equipment units. We can solve the MALBP with walking workers under two scenarios: (i) equipment cannot be shared between workstations and (ii) equipment sharing is allowed. The optimal solution for both cases is shown in Figure 1. In both configurations, the number of workstations and workers is four. However, the number of equipment units differs: three in the non-sharing case (Figure 1a) and two when sharing

TABLE I: Extended data of [12]

Task	Successors	Duration	Equipment compatibility
1	2, 3	5	1, 3
2	4	3	2, 4
3	4	4	1, 2, 3, 4
4	5, 6, 7	5	-
5	8	4	2, 4
6	9	5	-
7	9	1	2, 4
8	9	4	-
9	-	6	-



(a) MALBP without equipment sharing



(b) MALBP considering equipment sharing

Fig. 1: MALBP Solutions With and Without Equipment Sharing Across Workstations

mobile equipment is allowed (Figure 1b). The setup time for the movement of the equipment is represented in the gray rectangle before task 5 and after task 7 in Figure 1b. This new formulation makes the problem more complex by introducing new decision variables and new assignment and scheduling constraints, particularly for large-scale problems.

II. PROBLEM FORMULATION

The problem is motivated by a real case study of the assembly process of a biogas treatment system. The system is large enough to allow several workers to operate simultaneously on the same workpiece, forming what is known as a multi-manned assembly system. In this case study, assignment restrictions such as task grouping and incompatibility constraints where certain tasks cannot share the same workstation are also considered, representing specific assignment restrictions.

In addition, certain tasks require specific equipment, for example a material handling equipment for tasks 1, 2, and 3. To minimize investment costs, such equipment can be shared among different workstations as explained in section I. A setup time is considered for moving the equipment between two workstations, proportional to the distance between them. For instance, the start time of task 3 at workstation 3 must occur after the finish time of task 1 at workstation 1, plus two units of setup time, since the equipment has to be moved across a distance of two workstations. In addition, the setup time required to return the equipment to the workstation of the first task is also taken into account.

Furthermore, walking workers are allowed to move between workstations to reduce the total number of workers required. The overall objective is to minimize total costs, including annual labor costs, workstation setup costs, and equipment investment costs. Following the classification proposed by Boysen et al [13], the problem can be described as follows: $[link, inc, \Delta t_{dir} | pwork, res(01) | Co]$. Here, *link* represents task grouping, *inc* denotes task incompatibility, Δt_{dir} represents setup time due to equipment movement, *pwork* indicates parallel work on the same product, *res(01)* either assign equipment to a workstation or do not assign the equipment to reduce investment costs, and *Co* represents the objective of minimizing total costs.

III. MATHEMATICAL MODEL

The mathematical models proposed by Michels et al [14], Sahin and Kellegoz [15] and Yilmaz and Yilmaz [16] are here extended and modified to model the MALBP with equipment sharing and walking workers. The cycle time is fixed and calculated depending on the industry forecasts and total working hours.

Sets and Indices

- *T:* Set of tasks.
- *S:* Set of workstations.
- *W:* Set of workers.
- *E:* Set of pieces of equipment.
- *P:* Set of precedence relations between two tasks (i, j).
- *Link:* Set of task pairs that must be co-located (i, j).
- *Inc:* Set of incompatible task pairs (i, j).

Parameters

- D_i: Duration of task $i \in T$.
- *CT:* Cycle time.
- b_{ie}: 1 if equipment $e \in E$ can be used for task $i \in T$.
- *WCost:* Annual salary per worker.
- *SCost:* Cost per workstation.
- *ECost:* Investment cost per equipment usage.
- *BigM:* Large constant for linearization.
- *SetupT:* Setup time for the movement of the equipment between two adjacent workstations, this value is then multiplied by the distance between the workstations.

Decision Variables

- $X_{i,s}$: 1 if task $i \in T$ is assigned to workstation $s \in S$, 0 otherwise.
- $Y_{w,s}$: 1 if worker $w \in W$ is assigned to workstation $s \in S$, 0 otherwise.
- $W_{i,w}$: 1 if task $i \in T$ is performed by worker $w \in W$, 0 otherwise.
- Z_s: 1 if workstation $s \in S$ is used, 0 otherwise.
- $F_{j,i}$: 1 if task $i \in T$ is followed by task $j \in T$, 0 otherwise.
- ST_i: Start time $\in \mathsf{R}^+$ of task $i \in T$.
- $Eq_{e,i,s}$: 1 if equipment $e \in E$ is used for task $i \in T$ at workstation $s \in S$, 0 otherwise.
- RS_e: 1 if equipment $e \in E$ is used, 0 otherwise.
- Q_w: 1 if worker $w \in W$ is assigned, 0 otherwise.
- $V_{i,j}$: 1 if the relative completion time of task $i \in T$ (computed within its workstation's cycle, i.e., in the interval $\{0, CT\}$) is less than relative starting time of task $j \in T$, 0 otherwise.
- $A_{i,j,e}$: 1 if task $i \in T$ uses equipment $e \in E$ before task $j \in T$ in relative workstation time (relative completion time of task $i \in T \leq$ relative starting time of task $j \in T$), 0 otherwise.

Objective Function

$$\min\ Z = \sum_{w \in W} \text{WCost} \cdot Q_w + \sum_{s \in S} \text{SCost} \cdot Z_s + \sum_{e \in E} \text{ECost} \cdot RS_e \quad (1)$$

Constraints

Assignment of Task, Worker, Workstation

$$\sum_{s \in S} X_{i,s} = 1 \quad \forall i \in T \quad (2)$$

$$\sum_{w \in W} W_{i,w} = 1 \quad \forall i \in T \quad (3)$$

$$Y_{w,s} \geq X_{i,s} + W_{i,w} - 1 \quad \forall i \in T,\ s \in S,\ w \in W \quad (4)$$

$$\sum_{s \in S} s \cdot X_{i,s} \leq \sum_{s \in S} s \cdot X_{j,s} \quad \forall (i,j) \in P \quad (5)$$

$$F_{j,i} = 1 \quad \forall (i,j) \in P \quad (6)$$

$$F_{j,i} \geq X_{j,s} - \sum_{\substack{k \in S \\ k \geq s}} X_{i,k} \quad \forall i \neq j \in T,\ s \in S \quad (7)$$

$$F_{i,j} + F_{j,i} \geq X_{i,s} + X_{j,s} + W_{i,w} + W_{j,w} - 3 \quad \forall i \neq j \in T,\ s \in S,\ w \in W \quad (8)$$

Task Scheduling

$$ST_i \geq (s-1) \cdot CT - BigM \cdot (1 - X_{i,s}) \quad \forall i \in T,\ s \in S \quad (9)$$

$$ST_i + D_i \leq CT \cdot \sum_{s \in S} s \cdot X_{i,s} \quad \forall i \in T \quad (10)$$

$$ST_i + D_i \leq CT \cdot \sum_{s \in S} Z_s \quad \forall i \in T \quad (11)$$

$$ST_j \geq ST_i + D_i - BigM \cdot (1 - F_{j,i}) \quad \forall i \neq j \in T \quad (12)$$

Symmetry Break Constraints

$$Z_s \geq X_{i,s} \quad \forall i \in T,\ s \in S \quad (13)$$

$$Z_s \geq Y_{w,s} \quad \forall w \in W,\ s \in S \quad (14)$$

$$Z_s \leq Z_{s-1} \quad \forall s \in S \setminus \{1\} \quad (15)$$

$$Y_{w,s} \leq \sum_{k \in S} Y_{w-1,k} \quad \forall s \in S,\ w \in W \setminus \{1\} \quad (16)$$

$$Z_s \leq \sum_{i \in T} X_{i,s} \quad \forall s \in S \quad (17)$$

Link and Incompatibility Constraints

$$\sum_{s \in S} s \cdot X_{i,s} = \sum_{s \in S} s \cdot X_{j,s} \quad \forall (i,j) \in Link \quad (18)$$

$$X_{i,s} + X_{j,s} \leq 1 \quad \forall (i,j) \in Inc,\ s \in S \quad (19)$$

Walking Workers Constraints

$$Y_{w,s} \leq Q_w \quad \forall w \in W,\ s \in S \quad (20)$$

$$\sum_{i \in T} W_{i,w} \cdot D_i \leq CT \quad \forall w \in W \quad (21)$$

$$ST_j + D_j - CT \cdot (\sum_{k \in S} k \cdot X_{j,k} - \sum_{s \in S} s \cdot X_{i,s}) \leq ST_i + BigM \cdot (2 - W_{i,w} - W_{j,w} + V_{i,j}) \quad \forall i \neq j \in T,\ w \in W \quad (22)$$

$$ST_i + D_i - CT \cdot (\sum_{s \in S} s \cdot X_{i,s} - \sum_{k \in S} k \cdot X_{j,k}) \leq ST_j + BigM \cdot (3 - W_{i,w} - W_{j,w} - V_{i,j}) \quad \forall i \neq j \in T,\ w \in W \quad (23)$$

Equipment Assignment

$$BigM \cdot \sum_{e \in E} Eq_{e,i,s} \cdot b_{ie} \geq X_{i,s} \cdot \sum_{e \in E} b_{ie} \quad \forall i \in T,\ s \in S \quad (24)$$

$$Eq_{e1,i,s} + Eq_{e2,j,s} \leq 1 \quad \forall i \neq j \in T,\ e1 \neq e2 \in E,\ s \in S \quad (25)$$

$$Eq_{e,i,s} \leq RS_e \quad \forall i \in T,\ e \in E,\ s \in S \quad (26)$$

Equipment Scheduling

$$ST_i + D_i + SetupT \cdot (k \cdot X_{j,k} - s \cdot X_{i,s}) \leq ST_j + CT \cdot (s \cdot X_{i,s} - k \cdot X_{j,k}) + BigM \cdot (5 - X_{i,s} - X_{j,k} - Eq_{e,i,s} - Eq_{e,j,k} + F_{i,j} - A_{i,j,e}) \quad \forall e \in E,\ s,k \in S,\ i \neq j \in T \quad (27)$$

$$ST_i + D_i + SetupT \cdot (s \cdot X_{i,s} - k \cdot X_{j,k}) \leq ST_j + CT \cdot (s \cdot X_{i,s} - k \cdot X_{j,k}) + BigM \cdot (6 - X_{i,s} - X_{j,k} - Eq_{e,i,s} - Eq_{e,j,k} - F_{i,j} - A_{i,j,e}) \quad \forall e \in E,\ s,k \in S,\ i \neq j \in T \quad (28)$$

$$ST_j + D_j + SetupT \cdot (k \cdot X_{j,k} - s \cdot X_{i,s}) \leq ST_i + CT \cdot (k \cdot X_{j,k} - s \cdot X_{i,s}) + BigM \cdot (4 - X_{i,s} - X_{j,k} - Eq_{e,i,s} - Eq_{e,j,k} + F_{i,j} + A_{i,j,e}) \quad \forall e \in E,\ s,k \in S,\ i \neq j \in T \quad (29)$$

$$ST_j + D_j + SetupT \cdot (s \cdot X_{i,s} - k \cdot X_{j,k}) \leq$$
$$ST_i + CT \cdot (k \cdot X_{j,k} - s \cdot X_{i,s}) + BigM \cdot (5$$
$$-X_{i,s} - X_{j,k} - Eq_{e,i,s} - Eq_{e,j,k} - F_{i,j} + A_{i,j,e})$$
$$\forall e \in E,\ s,k \in S,\ i \neq j \in T \quad (30)$$

Linearization and Logic Variables

$$A_{i,j,e} + A_{j,i,e} \leq \sum_{s \in S} Eq_{e,i,s} \forall e \in E,\ i \neq j \in T \quad (31)$$
$$A_{i,j,e} + A_{j,i,e} \leq \sum_{s \in S} Eq_{e,j,s} \forall e \in E,\ i \neq j \in T \quad (32)$$
$$A_{i,j,e} + A_{j,i,e} \geq \sum_{s \in S} Eq_{e,i,s} + \sum_{s \in S} Eq_{e,j,s} - 1$$
$$\forall e \in E,\ i \neq j \in T \quad (33)$$
$$\sum_{e \in E} A_{i,i,e} = 0 \qquad \forall i \in T \quad (34)$$
$$\sum_{e \in E} \sum_{s \in S} Eq_{e,i,s} \leq 1 \qquad \forall i \in T \quad (35)$$
$$\sum_{e \in E} \sum_{s \in S} Eq_{e,i,s} \leq \sum_{e \in E} b_{ie} \qquad \forall i \in T \quad (36)$$

Equipment Moving Constraints

$$ST_j + D_j - CT \cdot (\sum_{s \in S} s \cdot X_{j,s}) \leq$$
$$-SetupT \cdot (\sum_{k \in S} k \cdot X_{i,k} - \sum_{s \in S} s \cdot X_{j,s})$$
$$+BigM \cdot (3 - \sum_{s \in S} Eq_{e,j,s} - \sum_{k \in S} Eq_{e,i,k}$$
$$+F_{j,i} - A_{i,j,e} + \sum_{\substack{t \in T \\ t \neq i}} A_{t,i,e})$$
$$\forall e \in E,\ i \neq j \in T,\ b_{ie} = b_{je} = 1 \quad (37)$$
$$ST_j + D_j - CT \cdot (\sum_{s \in S} s \cdot X_{j,s}) \leq$$
$$-SetupT \cdot (\sum_{s \in S} s \cdot X_{j,s} - \sum_{k \in S} k \cdot X_{i,k})$$
$$+BigM \cdot (4 - \sum_{s \in S} Eq_{e,j,s} - \sum_{k \in S} Eq_{e,i,k}$$
$$-F_{j,i} - A_{i,j,e} + \sum_{\substack{t \in T \\ t \neq i}} A_{t,i,e})$$
$$\forall e \in E,\ i \neq j \in T,\ b_{ie} = b_{je} = 1 \quad (38)$$

Constraints (2)–(17) are slightly adapted from [14], while Constraints (18)–(19) are added from [16]. Constraints (20)–(23) are modified from [15]. Constraints (24)–(38) are developed for the equipment-scheduling component of the problem. The objective function (1) minimizes the costs of workers, workstations, and equipment usage. Constraints (2) and (3) ensure that each task is assigned to exactly one workstation and performed by exactly one worker, respectively. Constraint (4) links task, workstation, and worker assignments. Constraint (5) enforces precedence relations at the workstation level. Constraint (6) encodes these precedence relations in the binary variable $F_{i,j}$, while Constraint (7) activates the corresponding ordering when task j is assigned to a later workstation than task i. A similar logic applies to workers: if two tasks are assigned to the same worker within the same workstation, one must precede the other, as imposed by Constraint (8).

Constraints (9) and (10) define the start and finish-time intervals of each task within its workstation, and Constraint (11) ensures that the final task is completed within the last open workstation. If task i precedes task j ($F_{j,i}$=1), Constraint (12) enforces that the finish time of i is earlier than the start time of j. Symmetry breaks are included in Constraints (13)–(17) to avoid visiting symmetric solutions.

Constraints (18) and (19) impose task co-location and incompatibility constraints, respectively.

Constraint (20) links variables Q_w and $Y_{w,s}$. The workload of each worker must be below the cycle time, as expressed in Constraint (21). Constraints (22) and (23) ensure the scheduling of tasks assigned to the same worker across different workstations. If the relative finishing time of task i is less than the relative start time of task j ($V_{i,j}$=1) for the worker performing both tasks, then Constraint (23) is active. Otherwise, Constraint (22) is active.

The equipment assignment and scheduling constraints are given in Constraints (24)–(30). Constraint (24) ensures that equipment is assigned to a task only from its feasible equipment set. Constraint (25) ensures that at most one piece of equipment is assigned to a workstation. Constraint (26) links variable $Eq_{e,i,s}$ to RS_e.

Constraints (27)–(30) prevent overlapping usage of the same equipment by different tasks. Here, the relative start and finish times of a task refers to its start and finish within its assigned workstation, independently of its global position on the line (i.e., $0 \leq$ relative start time $\leq CT - D_t$). For example, if task i is assigned to workstation s and task j to workstation $k > s$ (with $F_{i,j} = 0$), and the relative finishing time of task i is less than the relative start time of task j, then $A_{i,j,e} = 1$ if both tasks use the same equipment e and Constraint (27) becomes active. In this case, the setup time required to move the equipment from workstation s to workstation k is added to the finish time of task i, ensuring that the absolute start time of task j respects equipment availability. The remaining configurations are illustrated in Figure 2.

Constraints (31)–(33) provide the linearized formulation of the non-linear relation $A_{i,j,e} + A_{j,i,e} = \left(\sum_{s \in S} Eq_{e,i,s}\right)\left(\sum_{k \in S} Eq_{e,j,k}\right)$, which defines the precedence relation between tasks i and j when both require equipment e. Constraint (34) ensures that no task precedes itself. Constraint (35) guarantees that each task can be assigned to at most one piece of equipment. Constraint (36) prevents infeasible assignments by restricting each task to its feasible equipment set.

Constraints (37)–(38) ensure that the setup time is properly considered when moving equipment from the workstation of the last task in the current cycle to the workstation of the first task in the next cycle. Consider a task j at workstation

s that shares equipment e with task i and starts after task i in relative workstation time ($A_{i,j,e} = 1$). For task i to be the first task using equipment e in the cycle, no other task may precede it, which is enforced by $\sum_{t \in T} A_{t,i,e} = 0$. When task j is assigned to a workstation with a higher index than task i, constraint (38) becomes active; otherwise, constraint (37) is applied.

IV. COMPUTATIONAL STUDY

In this section, we present the data used in the case study of the assembly of a biogas treatment system. The cycle time was determined based on the company's Sales and Operations Planning (S&OP). Table II summarizes the 28 tasks involved in this assembly process, including task relationships, durations and equipment compatibility. Tasks 2, 3, 4 and 5 must be done on the same workstation and without other tasks. This information defines the Inc and Link sets. Additionally, task 26 is incompatible with all tasks. The annual demand for the system is 73 units, while the available production capacity is 1540 hours per year. The resulting cycle time is therefore:

$$CT = \frac{1540}{73} \approx 21 \text{ hours.}$$

The cost parameters considered in the model are an average annual worker salary of 32,000€, a workstation setup cost of 4,000€, and an equipment investment cost of 10,000€. The mathematical model was solved using the Gurobi 13.0.0 solver on a computer equipped with an Intel Core i7-13700H (2.40 GHz) and 32 GB of RAM.

Discussion

After 26 seconds of computation, the model returns an optimal total cost of 154,000€, corresponding to a configuration with four workers, four workstations, and one piece of equipment. The detailed assignment is provided in Figure 3. In this solution, we observe that workers frequently move between workstations. It would be interesting to solve a sub-problem for worker-to-task assignment, given the tasks already allocated to workstations, in order to minimize worker movements and improve ergonomic conditions. Alternatively, once the task-to-workstation assignments and task starting times are known, this step can also be handled manually by the planner based on practical considerations. Allowing equipment to be shared across workstations reduces investment costs, since a single mobile unit can replace two fixed units assigned separately to different workstations.

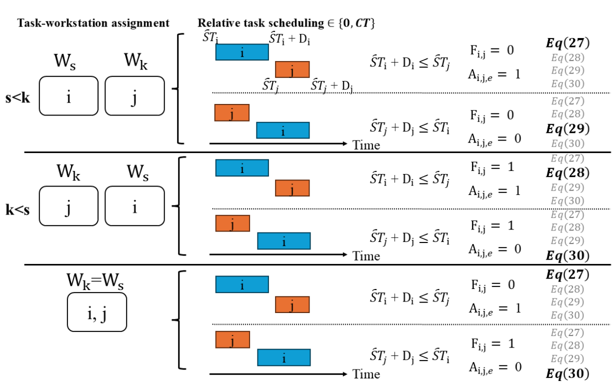


Fig. 2: Example illustrating the scheduling of tasks that share the same equipment

TABLE II: Data of biogas treatment system

Task	Successors	Duration (h)	Equipment
Task 1	8, 10, 6, 9, 20	1	1, 2
Task 2	13	2	1, 2
Task 3	14	2	1, 2
Task 4	11	0.5	1, 2
Task 5	12	1.5	1, 2
Task 6	14, 7	3	1, 2
Task 7	17	1	1, 2
Task 8	11, 12	2.5	1, 2
Task 9	13, 14, 16	45/60	1, 2
Task 10	12, 14, 13	45/60	1, 2
Task 11	13	0.5	-
Task 12	18, 19	1	1, 2
Task 13	15	2	1, 2
Task 14	15	2	1, 2
Task 15	18, 19	3	-
Task 16	18, 19	1	-
Task 17	18, 19	1	-
Task 18	25	2	-
Task 19	24	2	-
Task 20	21, 22, 23, 24	1.5	-
Task 21	25	1.5	-
Task 22	25	2	-
Task 23	25	2.5	-
Task 24	25	1.5	-
Task 25	26, 27	1.5	-
Task 26	28	21	-
Task 27	28	1	-
Task 28	-	4	-

Task 26 lasts 21 hours because it is carried out by a subcontractor in the middle of the assembly process. Fortunately, the cycle time matches this duration. Further research should examine alternative configurations for cases where the cycle time is shorter than 21 hours, for example by introducing parallel workstations or parallel assembly lines.

Sharing equipment between workstations appears to be an effective way to reduce investment costs, but it also requires close attention to equipment availability and maintainability. A failure of the shared unit could disrupt multiple workstations, meaning that preventive maintenance becomes essential to avoid undesirable situations.

The biogas treatment system is a heavy product. During the assembly process, the system remains stationary until it is finished. We assumed that this problem can be modeled as a MALBP, where workstations are considered as occupied spaces by the product. In this context, four workstations indicate that four products are simultaneously processed at different assembly phases. Workers move between products after each cycle time. An alternative modeling approach for this problem could also be investigated.

This problem formulation is complex due to the combination of interdependent subproblems: task scheduling, equipment scheduling, worker scheduling, and the assignment of tasks, workers, and equipment. The proposed mathematical model provides an exact solution for small datasets. The computational validation is limited to a single real-

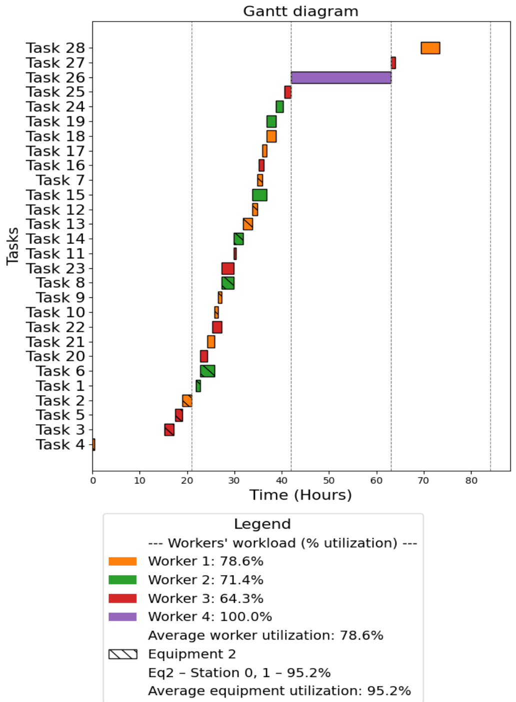


Fig. 3: Result of the case study

world instance and additional data will be provided by the industrial partner. Future work should therefore include additional benchmark instances and larger problem sizes to better understand the model's performance under varying conditions. Furthermore, as the numbers of tasks, workers, and equipment units increase, the size and complexity of the optimization model grow significantly, it would be necessary to develop exact or heuristic solution methods.

V. CONCLUSION

In this work, we introduce a new problem in multi-manned assembly systems, motivated by a real industrial case in biogas treatment: sharing mobile equipment across workstations to reduce total costs. This requires careful task scheduling to ensure that the equipment is never needed by two tasks simultaneously. Walking workers and assignment restrictions are also considered. We consider a setup time for moving the equipment between workstations, which is multiplied by the distance between them. Additionally, we include the time required to return the equipment to the first task in the next cycle.

We propose a mathematical model to represent this problem and solve it for the case study using Gurobi 13.0. After 26 seconds of computation, for 28 tasks with a total processing time of 65 hours and a cycle time of 21 hours, the results show that a single piece of equipment can be shared between two workstations, avoiding the need for a second unit. The solution requires a total of four workstations and four workers.

Future research will include testing the model on larger data sets from the literature, developing heuristic or meta-heuristic approaches to solve the problem within reasonable time for large sized data sets. Additionally, alternative modeling approaches that better fit the specific characteristics of the biogas treatment system will be investigated.

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