

A Minimum-Energy Control Approach for Redundant Mobile Manipulators in Physical Human-Robot Interaction Applications

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Abstract—Research on mobile manipulation systems that physically interact with humans has expanded rapidly in recent years, opening the way to tasks which could not be performed using fixed-base manipulators. Within this context, developing suitable control methodologies is essential since mobile manipulators introduce additional degrees of freedom, making the design of control approaches more challenging and more prone to performance optimization.

This paper proposes a control approach for a mobile manipulator, composed of a mobile base equipped with a robotic arm mounted on the top, with the objective of minimizing the overall kinetic energy stored in the whole-body mobile manipulator in physical human-robot interaction applications. The approach is experimentally tested with reference to a peg-in-hole task, and the results demonstrate that the proposed approach reduces the overall kinetic energy stored in the whole-body robotic system and improves the system performance compared with the benchmark method.

Index terms— Mobile Manipulator, Inverse Differential Kinematics, Physical Human-Robot Interaction.

I. INTRODUCTION

The interest in collaborative robotics applications has significantly increased in the last two decades. This includes applications such as rehabilitation robotics [1], [2], robot-assisted surgery [3], and industrial tasks [4].

The aforementioned applications typically involve the use of fixed-base manipulators to accomplish these tasks. However, certain tasks such as co-transportation [5] and others, which can be physically demanding and affect the operator's health, can gain significant benefit from the use of a mobile manipulator. This concept falls within the area of the so-called human augmentation field, which exploits the use of technologies in conjunction with human capabilities [6]. This makes it possible to overcome human body limitations and to enhance human abilities.

The considered mobile manipulator is shown in Fig. 1(a), while the considered scenario is depicted in Fig. 1(b), showing the mobile robot equipped with a handle. In this scenario,

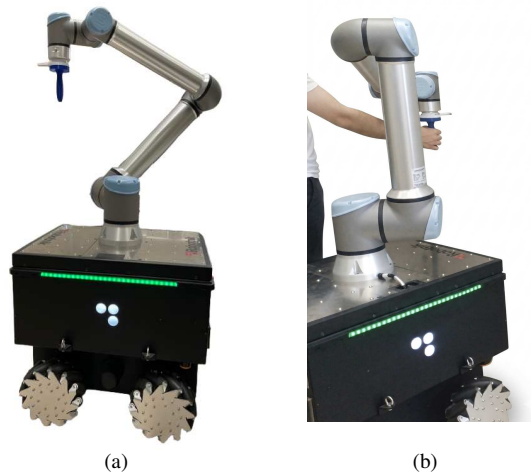


Fig. 1: (a) Considered mobile manipulator and (b) an example of Human/Mobile Manipulator cooperative task.

the human interacts with the end-effector to perform a peg-in-hole task. A classical architecture for human-robot interaction applications is shown in Fig. 2 [7]–[11]. This architecture depicts a human operator interacting with the robot through the following virtual admittance dynamics

$$\mathbf{M}\dot{\mathbf{v}}_d + \mathbf{B}\mathbf{v}_d = \mathbf{f}_h, \quad (1)$$

where $\mathbf{M}, \mathbf{B} \in \mathbb{R}^{(6 \times 6)}$ are the virtual mass and damper matrices, $\mathbf{f}_h \in \mathbb{R}^{(6 \times 1)}$ is the human wrench, and $\mathbf{v}_d \in \mathbb{R}^{(6 \times 1)}$ is the generated desired twist in the operational space of the end-effector. In Fig. 3, the admittance model (1) is represented using the Power-Oriented Graphs (POG) formalism [12], which clearly highlights the energetic port through which the human operator interacts with the virtual admittance model to generate the desired end-effector twist $\mathbf{v}_d$.

By solving the Inverse Differential Kinematics (IDK) in Fig. 2, the desired velocities $\dot{\mathbf{q}}_d$ in the joint space for the mobile base and for the manipulator arm are determined.

In this paper, we propose an approach to solve the IDK problem which enables to minimize the kinetic energy stored in the whole-body robotic system.

The advantages of the proposed control, emerging from the experimental validation, include a smoother user experience compared to the considered benchmark, as well as the expected reduction of the overall kinetic energy stored in the whole-body robotic system. Minimizing the kinetic energy stored in the system also translates into minimizing the movements of the heaviest system component, that is the mobile base weighing 115 kg. In turn, this also allows

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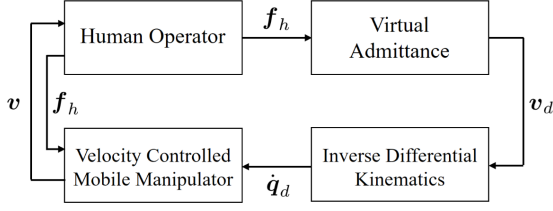


Fig. 2: Architecture for physical interaction between a human operator and a mobile manipulator through a virtual admittance model.

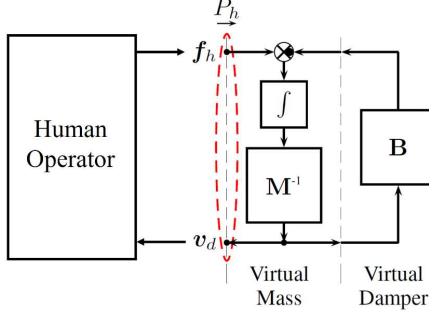


Fig. 3: Power-Oriented Graph (POG) representation of the virtual admittance model, showing the energetic port of the interaction between the human operator and the admittance model to generate the desired end-effector twist v_d .

to enhance the safety of the resulting physical interaction between the human and the mobile manipulator.

The remainder of this paper is organized as follows. The related works are discussed in Sec. II. The proposed control approach is presented in Sec. III, while the experimental setup is described in Sec. IV. The obtained results are presented and discussed in Sec. IV-A, while the conclusions of this work are reported in Sec. V.

II. RELATED WORKS

Most of the related works in the literature employ standard inverse differential kinematics or inverse kinematics approaches for redundant robots to transform velocities from the operational space to the joint space. For instance, in [13] they propose a control approach based on inverse kinematics. Specifically, they apply the FABRIK algorithm [14], originally meant for fixed-base manipulators, to mobile manipulators, with the objective of performing path tracking tasks. This approach allows to drive mobile base and manipulator arm while satisfying additional requirements, such as obstacle avoidance and joint limits avoidance. In [15], the inverse kinematics problem is solved as an optimization problem, to avoid joint limits and perform collision avoidance.

In the control framework illustrated in Fig. 2, the inverse kinematics of the mobile manipulator is not needed. However, as previously mentioned, it is necessary to solve the inverse differential kinematics problem, which consists in solving the following relation with respect to $\dot{q}_d$:

$$v_d = J(q)\dot{q}_d, \quad \text{where} \quad J(q) = [J_a(q_a) \quad J_b] \quad (2)$$

is the Jacobian of the whole system, whereas $J_a \in \mathbb{R}^{6 \times 6}$ and $J_b \in \mathbb{R}^{6 \times 3}$ are the Jacobians of the manipulator arm and of the mobile base, respectively.

The desired end-effector twist in the operational space and the desired joint velocity vector are given by, respectively:

$$v_d = \begin{bmatrix} v_{x_d} \\ v_{y_d} \\ v_{z_d} \\ \omega_{x_d} \\ \omega_{y_d} \\ \omega_{z_d} \end{bmatrix} \quad \text{and} \quad \dot{q}_d = \begin{bmatrix} \dot{q}_{1_d} \\ \vdots \\ \dot{q}_{6_d} \\ v_{bx_d} \\ v_{by_d} \\ \omega_{bz_d} \end{bmatrix}. \quad (3)$$

Note that vector $\dot{q}_d$ is composed of both the desired arm joint velocities $\dot{q}_{a_d}$ and the desired virtual joint velocities $\dot{q}_{b_d}$ of the mobile base, which must be transformed and applied to the base's wheels. Since an omnidirectional base is considered, these virtual velocities correspond to the linear velocities v_{bx_d} , v_{by_d} along the x , y directions and to the angular velocity ω_{bz_d} about the z axis.

In [16], the Inverse Differential Kinematics (IDK) is solved by formulating a Quadratic Programming (QP) problem, minimizing a joint velocities quadratic cost that includes a manipulability index of the robot arm. The optimization is performed by including the constraint indicated in Eq. (2), so as to guarantee an admissible solution of the IDK and further constraints in terms of joint velocity and position limits. It is important to highlight that the method they propose is intended for general-purpose applications; however, due to the computational time required for solving the QP problem (which, as reported in the paper, takes at least 4.8 seconds) it is not suitable for real-time scenarios such as the one described in the architecture of Fig. 2, where the reference twist is generated through human-robot interaction and must be translated in real time into corresponding joint velocities. For this reason, in all works involving physical interaction between a mobile manipulator and a human operator, the inverse differential kinematics problem is typically addressed using a closed-form solution, generally based on the pseudoinverse of the Jacobian matrix $J(q)$ [7]–[9]. For instance, this approach is adopted in [17], where the authors consider a scenario in which a mobile manipulator assists individuals with mobility impairments in walking. In their method, virtual damping is adaptively tuned using fuzzy logic based on the human's intent, and a null-space controller is employed to avoid singular configurations.

The approach proposed in [10], [11], which serves as a benchmark in the experimental evaluation of our method, addresses the IDK problem using a prioritized control technique. The resulting solution consists of two contributions, namely:

$$\dot{q}_d = \dot{q}_{d,p} + \mathcal{N}(J(q))\dot{q}_{d,s}, \quad (4)$$

where $\mathcal{N}(\mathbf{J}(\mathbf{q})) = \mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q})$ is a projector onto the null space of the Jacobian. The term $\dot{\mathbf{q}}_{d,p}$ accomplishes a primary task consisting in solving the damped least-squares problem minimizing the following objective function:

$$\mathcal{L}_p(\dot{\mathbf{q}}_d) = \frac{1}{2} \|\mathbf{v}_d - \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}_d\|_{\mathbf{W}_\alpha}^2 + \|\dot{\mathbf{q}}_d\|_{\mathbf{W}_\beta}^2, \quad (5)$$

with the two terms $\mathbf{W}_\alpha$ and $\mathbf{W}_\beta$ denoting two weight matrices. The term $\dot{\mathbf{q}}_{d,s}$ accomplishes a secondary task consisting, in the considered case, in minimizing the distance from a preferred configuration $\mathbf{q}_{des}$ [10]:

$$\mathcal{L}_s(\mathbf{q}) = \frac{1}{2} \|\mathbf{q}_{des} - \mathbf{q}\|_{\mathbf{G}}^2, \quad (6)$$

with the term $\mathbf{G}$ being a gain matrix. The closed-form solution of (4) with the objective functions (5) and (6) is:

$$\dot{\mathbf{q}}_d = (\mathbf{J}(\mathbf{q})^\top \mathbf{W}_\alpha \mathbf{J}(\mathbf{q}) + \mathbf{W}_\beta)^{-1} \mathbf{J}(\mathbf{q})^\top \mathbf{W}_\alpha \mathbf{v}_d + \mathcal{N}(\mathbf{J}(\mathbf{q})) \mathbf{G}(\mathbf{q}_{des} - \mathbf{q}). \quad (7)$$

III. PROPOSED APPROACH

Reference is made to the control architecture depicted in Fig. 2. The objective of the proposed approach is to solve the IDK problem so as to minimize the kinetic energy stored in the whole-body redundant mobile manipulator, which is composed of the mobile base and the robotic arm, during the execution of the task. In doing so, the movements of the heaviest system component, that is the mobile base, will be minimized, thereby enhancing safety in physical human-robot interaction. The resulting formulation corresponds to a weighted pseudoinverse using the inertia matrices $\mathbf{M}_a(\mathbf{q}_a)$ and $\mathbf{M}_b$ of the arm manipulator and of the mobile base, respectively.

A. Optimization Problem Formulation

Reference is made to the redundant mobile manipulator consisting of a mobile base and a robotic arm shown in Fig. 1. The IDK problem is solved through the following optimization problem.

Optimization Problem 1: Find the optimal desired arm joint velocities $\dot{\mathbf{q}}_{a,d}^*$ and the optimal desired virtual joint velocities $\dot{\mathbf{q}}_{b,d}^*$ such that:

$$\min_{\dot{\mathbf{q}}_{a,d}, \dot{\mathbf{q}}_{b,d}} E_K(\mathbf{q}_a, \dot{\mathbf{q}}_{a,d}, \dot{\mathbf{q}}_{b,d}) \quad (8)$$

where the objective function

$$E_K(\mathbf{q}_a, \dot{\mathbf{q}}_{a,d}, \dot{\mathbf{q}}_{b,d}) = \frac{1}{2} \dot{\mathbf{q}}_{a,d}^\top \mathbf{M}_a(\mathbf{q}_a) \dot{\mathbf{q}}_{a,d} + \frac{1}{2} \dot{\mathbf{q}}_{b,d}^\top \mathbf{M}_b \dot{\mathbf{q}}_{b,d} \quad (9)$$

represents the total kinetic energy of the whole-body robotic system, with $\mathbf{M}_a(\mathbf{q}_a)$ and $\mathbf{M}_b$ being the inertia matrices of the arm manipulator and of the mobile base, respectively, subject to the constraint

$$\mathbf{J}_a(\mathbf{q}_a) \dot{\mathbf{q}}_{a,d} + \mathbf{J}_b \dot{\mathbf{q}}_{b,d} = \mathbf{v}_d. \quad (10)$$

Note that the constraint (10) derives from (2) and (3).

Property 1: The optimal solution $\dot{\mathbf{q}}_d^* = [\dot{\mathbf{q}}_{a,d}^* \quad \dot{\mathbf{q}}_{b,d}^*]^\top$ solving the optimization problem (8) is given by

$$\dot{\mathbf{q}}_d^* = \mathbf{J}_M^\dagger(\mathbf{q}) \mathbf{v}_d, \quad (11)$$

where $\mathbf{v}_d$ is the desired operational-space twist defined in (3), and $\mathbf{q} = [\mathbf{q}_a \quad \mathbf{q}_b]^\top$ is the vector of generalized coordinates. The matrix

$$\mathbf{J}_M^\dagger(\mathbf{q}) = \mathbf{M}(\mathbf{q}_a)^{-1} \mathbf{J}(\mathbf{q})^\top (\mathbf{J}(\mathbf{q}) \mathbf{M}(\mathbf{q}_a)^{-1} \mathbf{J}(\mathbf{q})^\top)^{-1} \quad (12)$$

is the dynamically consistent Jacobian pseudoinverse [18], with the whole-body inertia matrix $\mathbf{M}(\mathbf{q}_a)$ defined as

$$\mathbf{M}(\mathbf{q}_a) = \begin{bmatrix} \mathbf{M}_a(\mathbf{q}_a) & 0 \\ 0 & \mathbf{M}_b \end{bmatrix}. \quad (13)$$

The off-diagonal inertial coupling terms between the arm and the mobile base are neglected in (13). This approximation is motivated by the operating conditions of the considered HRI scenario, where the mobile base moves at deliberately low velocities and accelerations for safety reasons. Under these conditions, arm-base dynamic coupling effects are expected to be limited, especially compared with the dominant inertia of the mobile platform. Therefore, the block-diagonal approximation of the whole-body inertia matrix provides a sufficiently accurate model for computing the dynamically consistent pseudoinverse.

Proof. The optimization problem (8) can be equivalently rewritten in the following matrix form:

$$\min_{\dot{\mathbf{q}}_d} E_K(\mathbf{q}_a, \dot{\mathbf{q}}_d), \quad \text{subject to} \quad \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}_d = \mathbf{v}_d, \quad (14)$$

where the total kinetic energy $E_K(\mathbf{q}_a, \dot{\mathbf{q}}_d)$ is expressed by the following matrix quadratic function:

$$E_K(\mathbf{q}_a, \dot{\mathbf{q}}_d) = \frac{1}{2} \dot{\mathbf{q}}_d^\top \mathbf{M}(\mathbf{q}_a) \dot{\mathbf{q}}_d,$$

with the inertia matrix $\mathbf{M}(\mathbf{q}_a)$ being defined in (13). The Lagrangian of (14) is the following:

$$\mathcal{L} = \frac{1}{2} \dot{\mathbf{q}}_d^\top \mathbf{M}(\mathbf{q}_a) \dot{\mathbf{q}}_d + \boldsymbol{\lambda}^\top (\mathbf{v}_d - \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}_d),$$

which yields the following first-order optimality (KKT) conditions [19]:

$$\mathbf{M}(\mathbf{q}_a) \dot{\mathbf{q}}_d + \mathbf{J}(\mathbf{q})^\top \boldsymbol{\lambda} = 0, \quad (15)$$

$$\mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}_d = \mathbf{v}_d. \quad (16)$$

Solving (15) and (16) for $\dot{\mathbf{q}}_d$ yields the optimal solution $\dot{\mathbf{q}}_d^*$ in (11). $\square$

From $\dot{\mathbf{q}}_d^*$, the optimal arm joint and base velocities $\dot{\mathbf{q}}_{a,d}^*$ and $\dot{\mathbf{q}}_{b,d}^*$ can be extracted recalling the definition of vector $\dot{\mathbf{q}}_d$ in (3).

The desired velocities $\dot{\mathbf{q}}_d$ in the joint space used in the control architecture of Fig. 2 are then given by:

$$\dot{\mathbf{q}}_d = \dot{\mathbf{q}}_d^* + \mathcal{N}(\mathbf{J}(\mathbf{q})) \dot{\mathbf{q}}_{d,s}, \quad (17)$$

where $\mathcal{N}(\mathbf{J}(\mathbf{q})) = \mathbf{I} - \mathbf{J}_M^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q})$ is a projector onto the null space of the Jacobian and where

$$\dot{\mathbf{q}}_{d,s} = \mathbf{G}(\mathbf{q}_{des} - \mathbf{q}). \quad (18)$$

The purpose of the term (18) is to ensure the end-effector does not deviate excessively from the preferred configuration, which is designed to ensure an ergonomic user configuration.

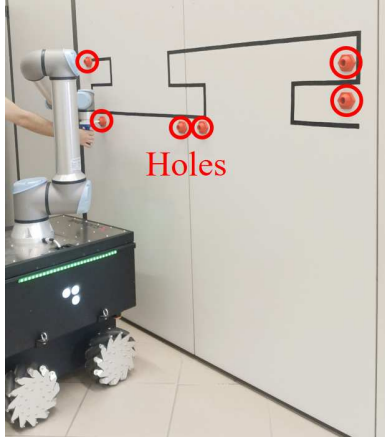


Fig. 4: Experimental test scenario, consisting in performing a peg-in-hole task with respect to the holes encircled in red.

IV. EXPERIMENTAL RESULTS

The employed mobile manipulator is shown in Fig. 1, which is composed of the UR10e cobot and of the Robotnik RB-KAIROS+ omnidirectional mobile base. The force sensor used is an FTE-AXIA80. The end-effector of the arm consists in the peg-and-handle system shown in Fig. 1.

Twenty-seven participants took part in the study. The users were asked to perform the task depicted in Fig. 4, consisting in inserting the peg into a set of holes to perform a peg-in-hole task. The task was executed using the following three different control approaches: (a) the locomotion mode in [10], [11]; (b) the switch mode in [10], [11], where in the latter the user is allowed to switch between manipulation and locomotion operating modes whenever desired; (c) the proposed minimum-energy approach, i.e. the control architecture of Fig. 2 solving the IDK problem through **Optimization Problem 1** in Sec. III-A, where the inverse of matrix $\mathbf{M}(\mathbf{q}_a)$ in (13) has been computed using the `inv` function of the submodule `linalg` of the Python library NumPy. The virtual inertia matrix $\mathbf{M}$ and the virtual friction matrix $\mathbf{B}$ in (1) were set to $\mathbf{M} = \text{diag}(4, \dots, 4)$ and $\mathbf{B} = \text{diag}(75, \dots, 75)$, while the gain matrix $\mathbf{G}$, in (7) and (18) for the locomotion and for the minimum-energy approaches, respectively, was set to $\mathbf{G} = \text{diag}(1, 1, 1, 1, 1, 1, 0, 0, 0)$. The acquired quantities include the norm of the velocity vector $\|\mathbf{v}(kT)\|$, the norm of the human force vector $\|\mathbf{f}_h(kT)\|$ (with the rotational components held constant due to the nature of the task, which requires the peg to remain orthogonal to the wall), and the total stored kinetic energy $E_K(kT)$, as defined in (9), evaluated at discrete time instants $t = kT$. From these, the average energy $\bar{E}$, the average human force $\bar{F}_h$ and the average execution velocity $\bar{v}$ over the execution time T_f have been computed as follows:

$$\begin{aligned} \bar{E} &= \frac{1}{N} \sum_{k=1}^N E_K(kT), & \bar{F}_h &= \frac{1}{N} \sum_{k=1}^N \|\mathbf{f}_h(kT)\|, \\ \bar{v} &= \frac{1}{N} \sum_{k=1}^N \|\mathbf{v}(kT)\|, & \text{where } N &= T_f/T. \end{aligned} \quad (19)$$

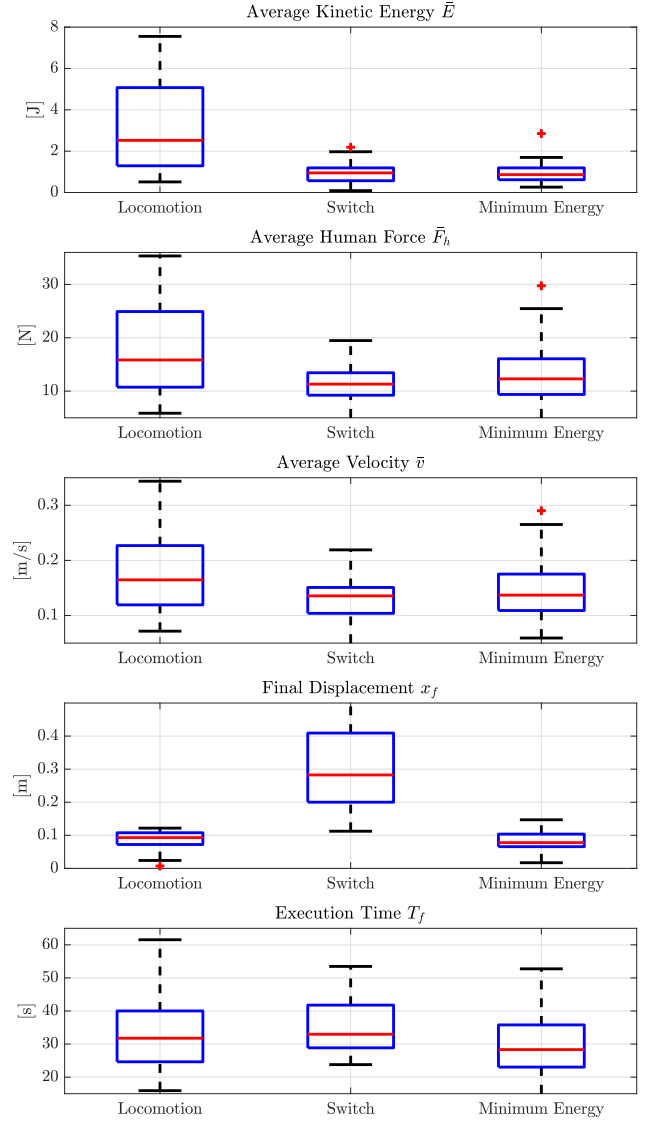


Fig. 5: Average energy $\bar{E}$, average human force $\bar{F}_h$, average execution velocity $\bar{v}$, final displacement x_f and execution time T_f for the executions of the 27 users.

Let $\mathbf{p}_{des} = \text{FK}(\mathbf{q}_{des})$ be the desired position in the operational space, where “FK” is the forward kinematics operator and $\mathbf{q}_{des}$ is the reference position in the joint space introduced in (7) and (18), and let $\mathbf{p} = \text{FK}(\mathbf{q})$. The final displacement x_f is given by:

$$x_f = \|\mathbf{p}(T_f) - \mathbf{p}_{des}\|. \quad (20)$$

The average energy $\bar{E}$, the average human force $\bar{F}_h$, the average execution velocity $\bar{v}$, the final displacement x_f , and the execution time T_f have been acquired for each execution of the 27 different users for each of the three aforementioned control approaches: locomotion (a), switch (b) and the proposed minimum-energy approach (c).

The experimental results are shown in Fig. 5, Fig. 6, Fig. 7 as well as in Table I, Table II and Table III, and are presented and discussed in the following section.

$\bar{E}$ [J]	$\bar{F}_h$ [N]	$\bar{v}$ [m/s]	x_f [m]	T_f [s]
2.52	15.84	0.16	0.09	31.77

TABLE I: Metrics median values out of the 27 users executions of the peg-in-hole task using the locomotion approach.

$\bar{E}$ [J]	$\bar{F}_h$ [N]	$\bar{v}$ [m/s]	x_f [m]	T_f [s]
0.95	11.31	0.14	0.28	32.97

TABLE II: Metrics median values out of the 27 users executions of the peg-in-hole task using the switch approach.

$\bar{E}$ [J]	$\bar{F}_h$ [N]	$\bar{v}$ [m/s]	x_f [m]	T_f [s]
0.86	12.30	0.14	0.08	28.33

TABLE III: Metrics median values out of the 27 users executions of peg-in-hole task using minimum energy approach.

A. Results Presentation and Discussion

The boxplots of Fig. 5 show the execution results of the 27 different users using the three considered control approaches, while the median values are reported in Table I, Table II and Table III. A Friedman test was conducted and, subsequently, a Wilcoxon signed-rank test followed by Holm-Bonferroni correction, to assess the statistical significance of the results of the different metrics between each pair of the three control approaches (a), (b) and (c) in Sec. IV. The results are shown in Table IV.

By comparing the median results in Table I, Table II and Table III, the following conclusions can be drawn. The average human force $\bar{F}_h$ indicates that the locomotion approach requires the highest human effort due to the extensive use of the mobile base. In contrast, the switch and minimum-energy approaches reduce the effort by approximately 29% and 22%, respectively. This is consistent with the corresponding differences in average execution speed $\bar{v}$ and with the mobile base being the heaviest system component. The mobile base usage is predominant in the locomotion approach, user-dependent in the switch approach (leading to higher cognitive load and slightly longer execution time), and automatically minimized in the minimum-energy approach through **Optimization Problem 1** in Sec. III-A.

The average total kinetic energy $\bar{E}$ is approximately 62% and 66% lower in the switch and minimum-energy approaches, respectively, than in the locomotion approach. The switch and minimum-energy approaches achieve similar energy levels (as confirmed by the statistical non-significance of the corresponding metrics in Table IV); however, the former relies on user intervention to limit mobile base usage, whereas the latter handles this trade-off automatically, resulting in a smoother energy profile. Moreover, in the switch approach the whole-body stored kinetic energy strongly depends on user experience in properly determining when to switch, while in the minimum-energy approach it is consistently regulated by **Optimization Problem 1**.

The absence of switching operations in locomotion and minimum-energy approaches also improves execution time, as shown by the metric T_f . In this case, the minimum-energy

TABLE IV: Statistical significance analysis of the results reported in Fig. 5.

Metric Comparison		p_{corr}	Significance
$\bar{E}$	Locomotion vs Switch	1.682×10^{-5}	Significant
$\bar{E}$	Locomotion vs Minimum Energy	1.682×10^{-5}	Significant
$\bar{E}$	Switch vs Minimum Energy	0.337	Not significant
$\bar{F}_h$	Locomotion vs Switch	9.410×10^{-4}	Significant
$\bar{F}_h$	Locomotion vs Minimum Energy	0.0153	Significant
$\bar{F}_h$	Switch vs Minimum Energy	0.0609	Not significant
$\bar{v}$	Locomotion vs Switch	0.0172	Significant
$\bar{v}$	Locomotion vs Minimum Energy	0.0977	Not significant
$\bar{v}$	Switch vs Minimum Energy	0.0977	Not significant
x_f	Locomotion vs Switch	1.682×10^{-5}	Significant
x_f	Locomotion vs Minimum Energy	0.414	Not significant
x_f	Switch vs Minimum Energy	1.682×10^{-5}	Significant
T_f	Locomotion vs Switch	0.0285	Significant
T_f	Locomotion vs Minimum Energy	0.0285	Significant
T_f	Switch vs Minimum Energy	4.412×10^{-4}	Significant

approach resulted to be the best performing one.

Regarding the final end-effector displacement x_f , the locomotion and minimum-energy approaches achieve comparable accuracy, with the latter performing slightly better. This expected result, confirmed by the statistical non-significance of the corresponding metric in Table IV, is due to the fact that both exploit secondary tasks in the Jacobian null space for this purpose. In the switch approach, instead, the final displacement depends on user's action during manipulation, which may lead to suboptimal configurations.

Fig. 6 and Fig. 7 show the norm $\|v(kT)\|$ of the end-effector velocity and the total stored kinetic energy $E_K(kT)$ in (9) during task execution for the user most frequently associated with the median values in Fig. 5 and Tables I, II, and III. From Fig. 6, it can be observed that the locomotion approach exhibits more pronounced velocity fluctuations compared to the other approaches, resulting in a jagged and less stable motion profile. Since $\|v(kT)\|$ is proportional to the momentum, this indicates more frequent and abrupt movements, leading to reduced user comfort and safety. At the same time, the switch approach shows similarly high peaks when the locomotion mode is activated, lower peaks when manipulation mode is activated. The minimum-energy approach represents the best compromise, as the use of the mobile base is automatically limited to strictly necessary movements in order to reduce the kinetic energy stored in the mobile base representing the heaviest system component, and the user does not need to handle the switching between different operating modes.

Fig. 7 shows the time evolution of the total kinetic energy. In the locomotion approach, high energy peaks are observed due to large movements of the mobile base. In the switch approach, the stored energy is low during manipulation (i.e., when the manipulator arm is used) and increases significantly during locomotion phases (i.e., when the mobile base is used). Finally, the minimum-energy approach exhibits the smoothest profile with lower peak values, resulting in reduced overall kinetic energy stored in the whole-body system, thus enhancing human safety during the interaction and giving a smoother user experience.

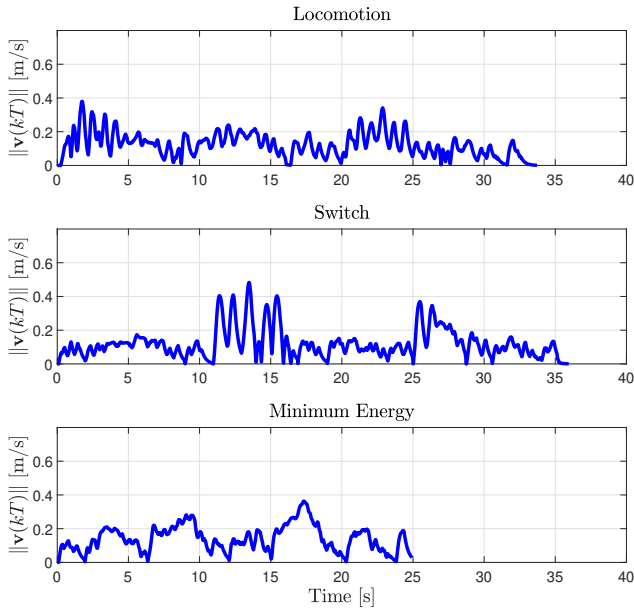


Fig. 6: End-effector velocity during task execution by the user most frequently associated with median values in Fig. 5.

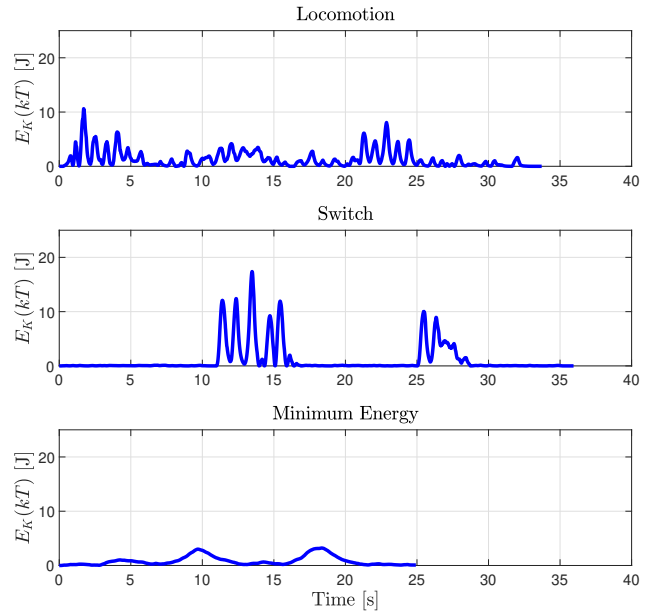


Fig. 7: Total kinetic energy during task execution by the user most frequently associated with median values in Fig. 5.

V. CONCLUSIONS

This paper addressed the control of a mobile manipulator for human–robot interaction, aiming to minimize the overall kinetic energy of the system. The approach was detailed and experimentally validated on a peg-in-hole task. Results show that the proposed minimum-energy method achieves the best overall performance across the selected metrics, leading to improved physical human–robot interaction. Future work will focus on task-dependent tuning of inertia-based weights to accommodate a wider range of tasks, and on enhancing user perception through haptic feedback to improve situational awareness for collision avoidance.

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