Long-term Experimental investigation of Active Magnetic Levitation in Damped Oscillatory Mode

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***Abstract*— This elaboration presents an experimental investigation of an active magnetic levitation (AML) system operating in a damped oscillatory mode. The system was investigated in 3 hour scenarios to observe its behavior. The system was modeled as a third-order nonlinear system. A stabilizing controller was designed to ensure asymptotic stabilization of the levitated sphere over a wide range of its displacements, thereby guaranteeing specific dynamic properties. As the coil heats up as a result of the flowing current, its resistance increases. The integral control action was activated periodically during the object's stabilization phase to reduce the steady-state error. The proposed control scheme allows the system to be identified during continuous operation. The observed changes in frequency and damping indicate that the parameters describing the electromagnetic actuator are non-stationary.**

I. INTRODUCTION

Several research papers are devoted to Active Magnetic Levitation (AML) control problems for stabilization and trajectory tracking: PID [1, 2], state feedback [3], robust [4, 5], feedback linearization [6], fuzzy logic [7, 8], neural networks [9, 10, 11], gain scheduling [12], time-optimal [13], sliding mode [14], ADRC [15, 16], reinforcement learning [17], fractional order [18], MPC [19, 20]. The AML phenomena are used in many practical applications: MagLev train, magnetic bearings, flywheels, heatpumps, LVAD pumps, self bearing drives, XY planar motion systems. Usually, the 2^{nd} order or 3^{rd} order nonlinear stationary differential equations are applied as a mathematical representation of the 1 DOF AML system. Typically, the short term experiments are realized to demonstrate the performance of the designed controller. The present research is an extension of a long-term experiment, the objective of which is to demonstrate the behavior of the system when it is operating in levitation mode.

A. Motivation

The main motivation for conducting the research was to investigate the dynamics of a closed-loop levitation system that operates over an extended period of time, in order to demonstrate the quality of the control. To achieve this objective, a stabilizing controller was designed to ensure that the levitation system operates in a damped oscillation mode. The system was subjected to periodic excitation in order to enable the analysis of it's dynamics. Once the system had stabilized, the changes in position and current intensity were plotted based on the average values calculated from the 2 seconds prior to the commencement of excitation (see Fig. 1). The absence of an integral term in the controller's formula resulted in a steady-state error, which increased with time. The current intensity also rose, as the controller endeavored to maintain the set-point. These changes were attributable to the heating of the electromagnetic actuator (coil and core). The question of the effect of heating on the dynamics of the levitation system operating under the control of a state-based controller was raised.

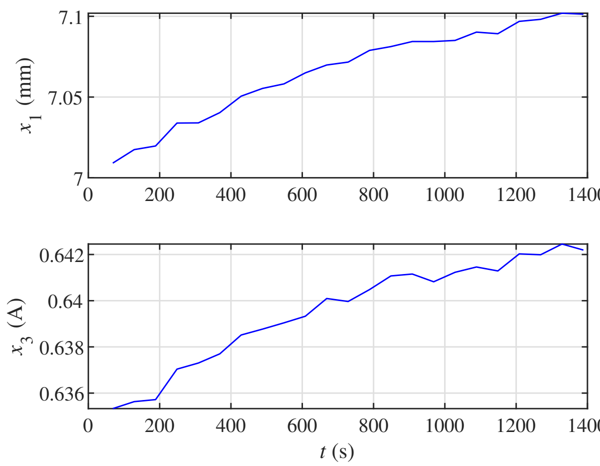

Fig. 1. Observed drift in displacement and coil current

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II. ACTIVE MAGNETIC LEVITATION

The active magnetic levitation system enables the contactless suspension of an object within an electromagnetic field regulated by a feedback control system. The system exhibits strong nonlinearity resulting from the interaction between the electromagnetic field and the levitated body. The electromagnetic force is modeled as a nonlinear function depending on the air gap between the object and the electromagnet pole face, as well as on the coil current. The inductance of the coil varies with the position of the levitated object, which introduces additional coupling between the electrical and mechanical subsystems. The circuit dynamics is taken into account when the levitation system is driven by the voltage applied to the coil terminals. Figure 2 shows the active magnetic levitation system MLS1EM and the characteristics of the position sensor, the electromagnetic force, and the inductance.

The third-order nonlinear model of a voltage-controlled

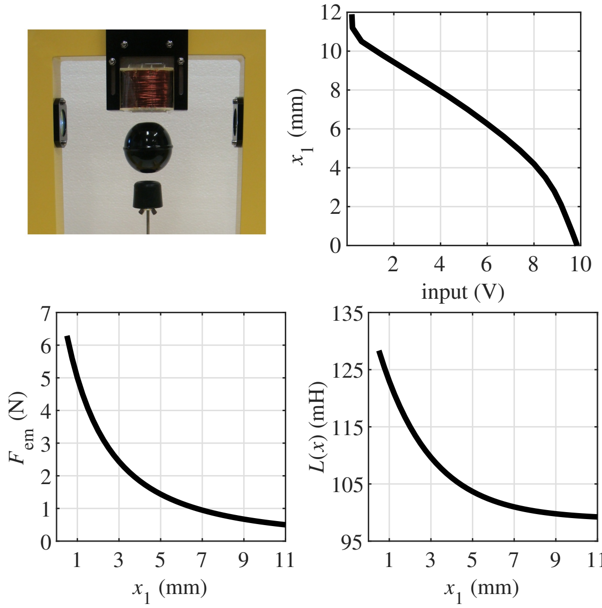

Fig. 2. MLS1EM with a ferromagnetic sphere at levitation stage, main characteristics of the system: distance sensor, force and coil inductance vs sphere position

active magnetic levitation system is represented by the following set of equations:

$$\begin{cases} \dot{x_1} &= x_2 \\ \dot{x_2} &= -\frac{F_{em}(x_1,x_3)}{m} + g \\ \dot{x_3} &= -\frac{1}{L(x_1)}\frac{dL(x_1)}{dx_1}x_2x_3 - \frac{R}{L(x_1)}x_3 + \frac{k_u u}{L(x_1)} \end{cases} \tag{1}$$

Based on the conducted identification, the following form of the function (see Fig. 2) is proposed to describe the electromagnetic force

$$F_{em}(x_1, x_3) = (p_{1F}e^{-p_{2F}x_1} + p_{3F}e^{-p_{4F}x_1})x_3^2 \tag{2}$$

The identified coil inductance, which depends on the air gap between the electromagnet and the levitated object (see Fig. 2), is modeled by the following function:

$$L(x_1) = L_{inf} + L_0 e^{-L_b x_1} \tag{3}$$

In the case of the electromagnetic force and coil inductance, the function parameters were obtained using the nonlinear least-squares optimization method based on experimental data. The complete set of parameters within the mathematical model is summarized in Table I.

The mathematical model of an AML system, which is a commonly applied model, is nonlinear, steady-state, structurally unstable, and requires control to maintain the ferromagnetic object in a specified position. The following research questions are hereby posed:

- Is the active magnetic levitation system stationary or not?
- How does the state controller perform over the long term?

TABLE I
IDENTIFIED PARAMETERS OF MLS1EM SYSTEM WITH PAC CONTROLLER

parameter	value	unit
m	0.039545	[kg]
L_{inf}	0.0988	[H]
L_0	0.0361	[H]
L_b	400	[1/m]
R	4.07	[Ω]
k_u	1.21	[]
p_{1F}	5.4103	$[N]$
p_{2F}	615.6099	$[1/m]$
p_{3F}	2.8668	$[N]$
p_{4F}	162.6617	$[1/m]$
g	9.81	$[kg{\cdot}m/s^2]$
x_{1min}	0.0	[mm]
x_{1max}	11.0	[mm]

- How to compensate for the steady-state error and realize research on the closed-loop dynamics guaranteed by the designed controller?

III. METHODS

In order to identify any changes in the dynamics of a closed-loop system, an analysis of changes in the state variables is required. A 30-seconds task (T_{task}) has been designed, in which four phases can be distinguished (see Fig. 3).

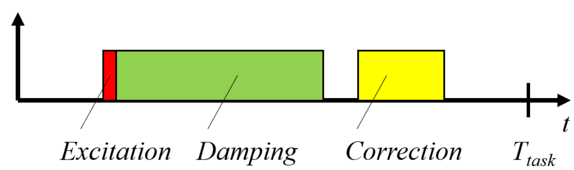

Fig. 3. Tasks executed during real-time control

It is assumed that the object is levitating (Standstill Phase) and is then excited by a dedicated control signal, which allows the system state to be changed – altering the current in the actuator winding and, consequently, changing the position and velocity of the levitating object. Excitation is achieved by modifying the setpoint control value through the addition of a pulse with an amplitude of 0.5V lasting 2 milliseconds (Excitation Phase). The configured control system constitutes a state-feedback controller whose task is to bring the system to a steady-state condition in which the current intensity assumes a set value and the object remains stationary at a specified levitation level. The transient state is characterized by damped oscillations (Damping Phase). Owing to the fact that the system is nonlinear, it is expected that for certain states the dynamics will be nonlinear, whilst as the system approaches the equilibrium point, it will exhibit characteristics similar to a linearized system, in particular a damped oscillator with constant spring and damping coefficients. Based on the observed drift in position and current intensity, a correction phase was proposed, employing an integral controller activated periodically during the stabilization phase of the object. Such a modification of the control value should bring the system into the vicinity of the setpoint.

A. State feedback controller

Linearization of the nonlinear system (1) at the equilibrium point $x_0 = col\{x_{10}, 0, x_{30}\}$ with fixed control u_0 was achieved. The requested values of conjugate eigenvalues λ_1 and λ_2 were calculated to meet the proposed stiffness and damping. Due to the presence of the electrical subsystem, the third eigenvalue was located outside the dominant region [21] choosing $\lambda_3 = 7 * Re(\lambda_1)$. The pole-placement method was used to obtain a state-feedback controller gains K_1, K_2, and K_3. The controller formula is in the following form:

$$u = u_0 + K_1 e_{x_1} + K_2 e_{x_2} + K_3 e_{x_3} \tag{4}$$

B. Triggered correction of steady-state error

The steady-state control correction was implemented via an event triggered every 30 seconds. During this 5-second event, the position error of the levitating object was calculated and the correction value was determined by integrating this error with a gain $K_{corr} = 100$;

$$u_{corr}(k) = u_{corr}(k-1) + K_{corr} * (t(k) - t(k-1)) * e_{x_1}(k-1) \tag{5}$$

C. Control with scheduled gain K_3

It was hypothesized that, as a result of long-term operation and therefore heating of the electromagnet, the main cause was a change in resistance, which in turn led to a change in u_0. The nonlinear model was linearized for various values of R, and the state feedback was recalculated to maintain the desired eigenvalues. A linear relationship between the controller's gain K_3 and u_0 was obtained and the control equation was subsequently modified to the following form:

$$u = u_0 + K_1 e_{x_1} + K_2 e_{x_2} + K_3(u_0) e_{x_3} \tag{6}$$

Experiment E2 was repeated to observe the effects of feedback that took into account changes in steady-state control.

D. Selected analysis

The system's response is characterized by a settling time, which defines the moment at which the trajectory enters the desired region around the equilibrium state. For the experiments performed, the region was restricted to 50 micrometers. The settling time was determined from the moment the excitation pulse was turned off. The levitation system, with its designed control scheme, should exhibit dynamics similar to those of a second-order damped oscillatory system, particularly in the region of small deviations from the setpoint. In this case, the motion of the levitating object should follow the equation. Based on data for the initial (I) and final (F) phases, nonlinear least-squares optimization of the function (7) was performed to obtain the damping and frequency values. The approximation goodness is indicated by the R^2 and $RMSE$ factors.

$$y(t) = Ae^{-\delta t} sin(2\pi f t + \phi) + of \tag{7}$$

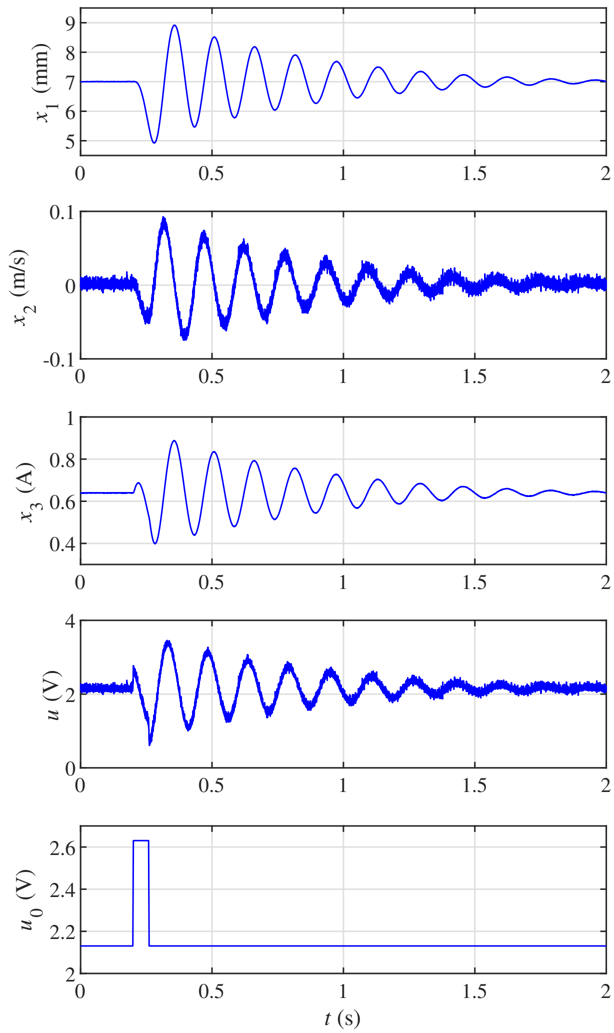

Fig. 4. System states and control at the beginning of the experiment E1 corresponding to operation from 95.06s

IV. Results and Discussion

The AML system was subjected to a rigorous three-hour investigation, employing the proposed method in two experiments E1 and E2. In the first experiment, the control formula (4) was utilized, whereas in the second experiment the formula was used (6). Figure 6 shows the u_{corr} signal from the entire experiment. The correction of the control signal u_0 ensured the maintenance of the set levitation level and the recording of damped oscillations on a periodic basis.

Figures 4 and 5 illustrate the Standstill, Excitation, and Damping phases at the start and end of the experiment named E1. The correct cyclical nature of the events is evident. It can be seen that the steady-state control reference level u_0 has changed. A comparative analysis of the data in Figures 8 and 10 allows an assessment of the changes in the dynamics of the levitation system. The optimization of the 600 milliseconds preceding the settling time was performed to determine the damping factor and the oscillation frequency. The results are summarized in Table II. The approximation

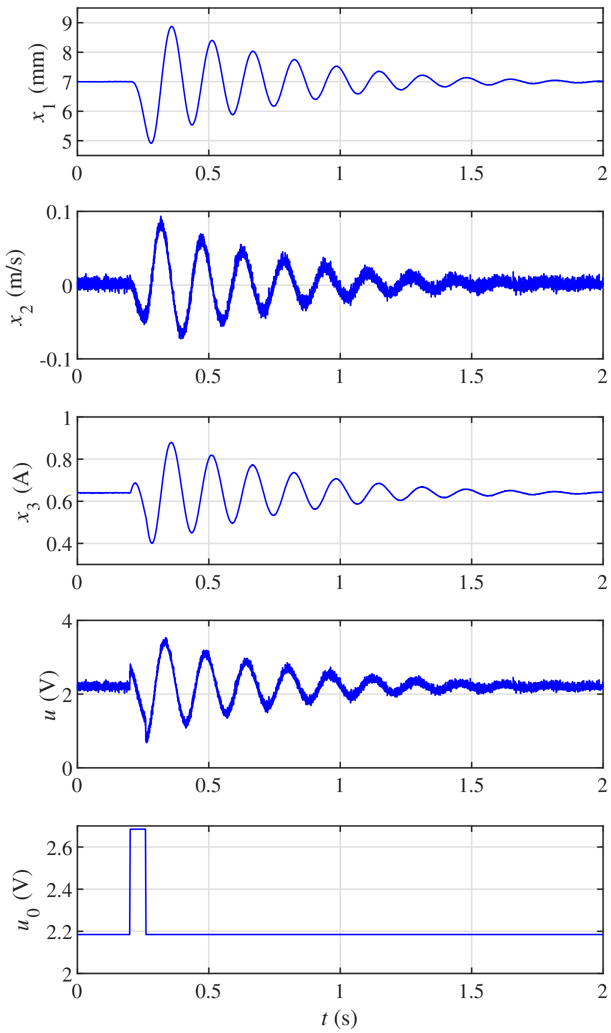

Fig. 5. System states and control at the end of the experiment E1 corresponding to operation from 10775.06s

and envelopes are presented in Figures 9 and 11.

TABLE II

SUMMARY OF SETTLING TIME t_r AND DAMPING δ AND FREQUENCY f PARAMETERS OF FUNCTION (7) AND THEIR OPTIMIZATION QUALITY FACTORS R^2 AND $RMSE$

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Exp.	Ph.	t_r	δ	f	R^2	RMSE*10^{-6}
E1	I	1.7136	2.739	6.0053	0.9982	4.8468
E1	F	1.4916	2.830	6.0055	0.9984	5.1880
E2	I	1.8530	2.611	6.0867	0.9980	4.3693
E2	F	2.0790	2.261	6.0657	0.9987	3.0791

In experiment E1, it was observed that the settling time of the final stage was shorter than in the initial stage. The damping was higher at the end of the experiment E1, indicating the direct consequence of the actuator's heating. In the second experiment, E2, an increase in settling time was observed, attributable to reduced damping resulting from a control modification that was closely linked to a change in

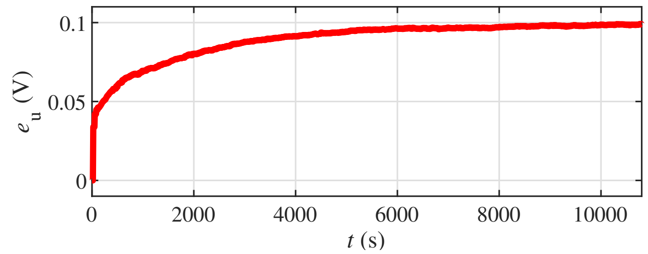

Fig. 6. Control correction during the whole experiment

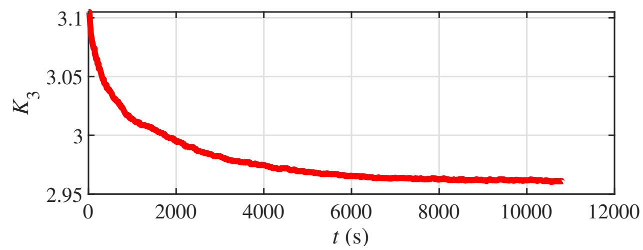

Fig. 7. Modified control gain K_3 on the basis of updated u_0 in experiment E2

coil resistance.

The experiments conducted demonstrated a change in the properties of the electromagnetic actuator due to heating and the need to revise the electromagnetic force model to account for heat exchange between the source (coil) and the surroundings. In order to maintain the desired dynamics, the controller gains should depend on the observed or measured temperature of the actuator.

V. CONCLUSIONS

The experimental studies conducted, revealed that the active magnetic levitation system was non-stationary. As the coil heated up due to the flowing current, its resistance increased. Consequently, heat from the coil dissipated into the surroundings, thereby heating the electromagnet core, and altering its magnetic properties. This ultimately resulted in a change in the electromagnetic force which in turn caused an alternation in the dynamics of the closed-loop system, consequent to a rise in the temperature of the electromagnetic actuator. Although a controller with fixed settings stabilized the system, it did not ensure the desired dynamics throughout the experiment. In particular, the stiffness and damping properties of the magnetic suspension changed. Moreover, a state-based controller without an integrator failed to maintain the desired state of the levitation system. However, when fitted with even periodic steady-state control correction, it is capable of stabilizing the system at the desired state. Due to the time constant of the thermal processes, the proposed error correction method is sufficient to reduce the steady-state error. The proposed state controller and cyclic control correction enabled identification of the levitation system during continuous operation. The observed variations in frequency and damping confirmed that the parameters describing the electromagnetic actuator were non-stationary.

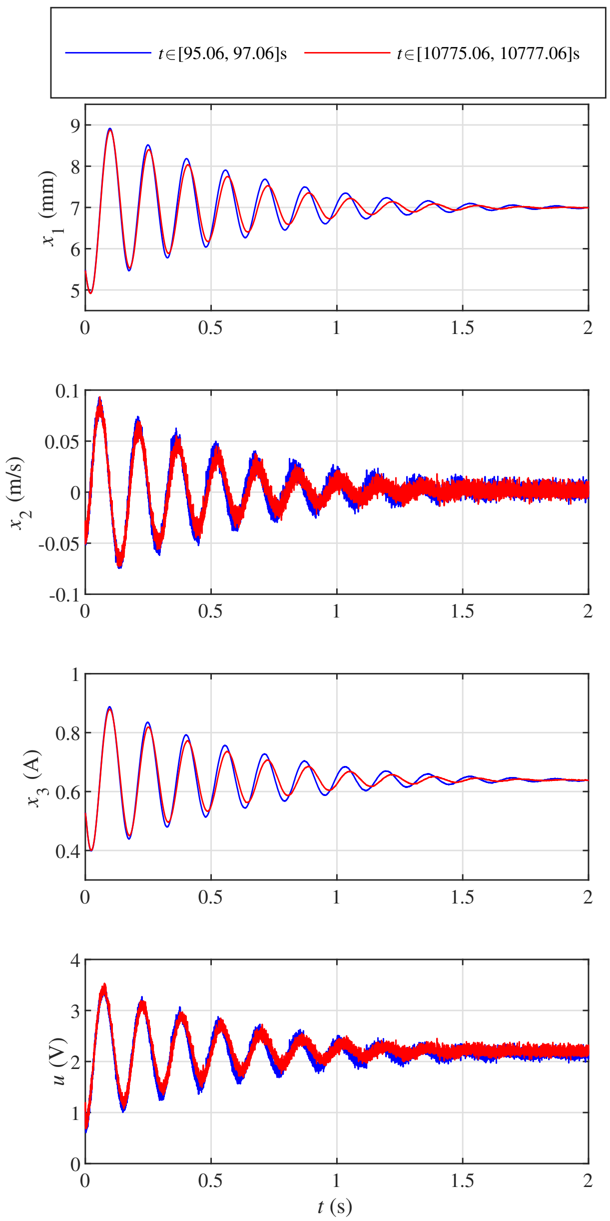

Fig. 8. Comparison of the system response and control in experiment E1

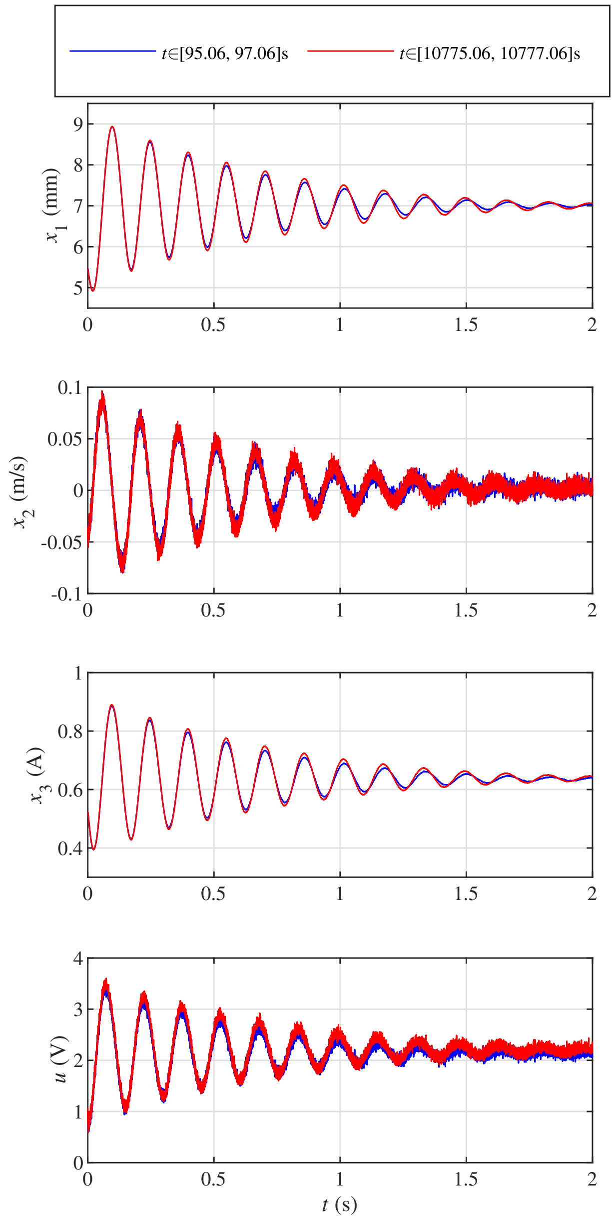

Fig. 10. Comparison of the system response and control formula 6 in experiment E2

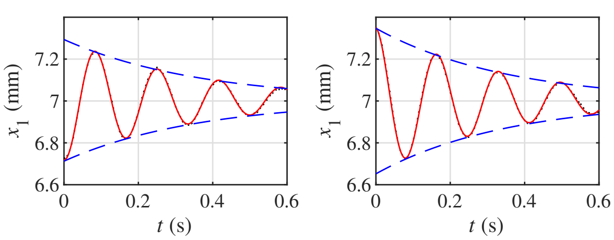

Fig. 9. Approximation of damped oscillations at initial and final stage of the experiment E1

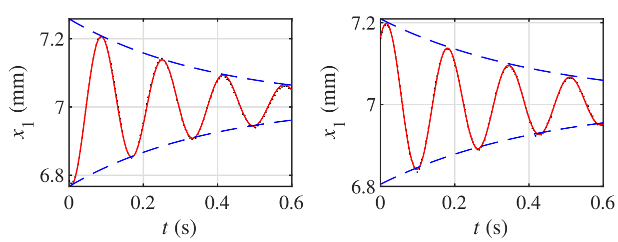

Fig. 11. Approximation of damped oscillations at initial and final stage of the experiment E2

To ensure constant dynamics, it is necessary to modify the model. The model should be extended to account for changes in resistance as a function of temperature and changes in force characteristics as a function of the heating of the electromagnet. This is of particular importance when using the linearizing feedback method. Furthermore, in order to ensure constant dynamics, it is necessary to design a controller that is robust to temperature changes or takes such changes into account.

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