Adaptive Sliding Mode Control for FOPDT Systems via Online Gradient Descent Tuning

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***Abstract*— First-order-plus-dead-time (FOPDT) models are widely used to represent industrial processes with dominant lag and transport delay, but dead time and operating-point variations make robust controller tuning difficult. This paper proposes an online-tuned sliding-mode tracking controller for FOPDT-based process that reduces reliance on fixed, model-dependent tuning rules. The time delay is represented by a first-order lag approximation, yielding a second-order input–output model used to define an integral sliding variable. An equivalent (continuous) control law is obtained by enforcing invariance on the sliding manifold, while a boundary-layer switching term mitigates chattering. To automate tuning, we propose a gradient-based update of key sliding-mode parameters using output-sensitivity dynamics. Benchmark results on three representative processes show that the proposed method improves tracking performance (lower ISE) relative to (i) a fixed-parameter SMC tuned from analytical equations and (ii) an SMC tuned via a multi-objective approach, while maintaining comparable control effort (ISCO).**

I. Introduction

First-order-plus-dead-time (FOPDT) models remain a central abstraction in industrial process control because many thermal, fluid, and chemical units exhibit a dominant lag with an effective transport delay (e.g., due to mass transport, sensing and actuation latencies, or cascaded dynamics). As a result, common control workflows reduce higher-order dynamics to first or second-order time-delay forms to support controller synthesis and tuning, including widely adopted model-reduction and tuning rules that explicitly target FOPDT with delay [1], [2]. In parallel, dead-time itself is known to significantly constrain achievable closed-loop performance and robustness, motivating a large body of compensation and design techniques for time-delay processes [3], [4], [5], [6].

Sliding-mode control (SMC) is an attractive alternative for process regulation when robustness to matched uncertainties and disturbances is a primary requirement. By enforcing motion on a designed sliding manifold, SMC can provide invariance properties and strong disturbance rejection, albeit at the cost of discontinuous control action and potential chattering in practical implementations. Foundational developments and modern perspectives on sliding-mode design, including continuous approximations and implementation considerations, are well established [7], [8], [9], [10].

Recent contributions have shown that SMC can be systematically specialized to process-industry classes by leveraging low-order time-delay models. In particular, SMC structures based on FOPDT approximations have been proposed for regulating chemical processes, yielding fixed-structure controllers whose parameters can be expressed in terms of the identified process gain, time constant, and dead time [11]. More generally, there is growing interest in deriving analytical tuning equations for SMC parameters (including for processes approximable by the FOPDT class), so that the controller can be tuned from a compact set of model parameters and desired performance specifications [12].

However, model-based tuning rules—whether for PID-like designs or SMC—inherit a practical limitation: they presume that a low-order model (and especially the dead time) is sufficiently accurate and remains valid over the operating envelope. In industrial settings, effective FOPDT parameters can drift with operating point, product changes, fouling, actuator nonlinearities/saturation, and unmodeled dynamics, while dead-time mismatch is particularly detrimental to performance and robustness [2], [3], [4]. For SMC, inaccurate model information may lead to conservative (overly large) switching gains, increased control activity, and amplified chattering, or—conversely—insufficient robustness when uncertainty bounds are underestimated [7], [8], [10].

These issues motivate online parameter tuning mechanisms that can adjust the switching gain and boundary-layer parameters in real time, reducing the reliance on fixed (and potentially outdated) model-based settings. Adaptive and online-tuned SMC variants have demonstrated that suitable update laws can maintain robustness while mitigating excessive gains and chattering [13], [14], [15].

In this work, we propose an online tuning strategy for an SMC designed from an FOPDT-based process approximation. The paper has three main contributions: (i) the dead time is approximated by a first-order lag to obtain a simple second-order input–output model for the design of an integral sliding-mode tracking controller. This approximation is used as a control-oriented model and is mainly suitable when the delay is not dominant compared with the process time constant; (ii) a gradient-descent adaptation law is derived to update the switching gain, the boundary-layer thickness, and the integral sliding weight using output-sensitivity dynamics, avoiding numerical differentiation of the measured output;

*The authors gratefully acknowledge Escuela Politécnica Nacional for supporting the presentation of this work.

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and (iii) the proposed controller is evaluated on three benchmark processes using the test protocol reported in [12], and its performance is compared with fixed-parameter and multi-objective tuned SMC designs using ISE and ISCO indices.

II. METHODS

The overall architecture of the proposed online-tuned sliding-mode controller (SMC) is shown in Fig. 1. The controller design and the associated online update laws are derived in the following subsections.

A. Sliding-mode control design for an FOPDT process

We consider a first-order-plus-dead-time (FOPDT) process:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{Ke^{-t_0 s}}{\tau s + 1}, \tag{1}$$

where $K \neq 0$ is the steady-state gain, $\tau > 0$ is the time constant, and $t_0 \geq 0$ is the dead time.

a) Delay rational approximation: To obtain an input–output ordinary differential equation suitable for sliding-surface design, the dead time is approximated by a first-order lag:

$$e^{-t_0 s} \approx \frac{1}{t_0 s + 1}, \tag{2}$$

This simple control-oriented approximation keeps the design model low order; for larger delay-to-time-constant ratios, higher-order approximations such as Padé approximations may be more appropriate. The resulting approximation is:

$$\frac{Y(s)}{U(s)} \approx \frac{K}{(\tau s + 1)(t_0 s + 1)}. \tag{3}$$

The corresponding time-domain model is:

$$\tau t_0 \ddot{y}(t) + (\tau + t_0)\dot{y}(t) + y(t) = Ku(t). \tag{4}$$

b) Sliding variable: Let $r(t)$ be the reference and define the tracking error and its integral:

$$e(t) = r(t) - y(t), \qquad i_e(t) = \int_0^t e(\sigma)\, d\sigma. \tag{5}$$

For the second-order model (4), we choose:

$$s(t) = \dot{e}(t) + \lambda_1 e(t) + \lambda_0 i_e(t), \qquad \lambda_0 > 0,\ \lambda_1 > 0. \tag{6}$$

Assuming a constant reference ($\dot{r} = \ddot{r} = 0$), the sliding-surface derivative is:

$$\dot{s}(t) = \ddot{e}(t) + \lambda_1 \dot{e}(t) + \lambda_0 e(t) = -\ddot{y}(t) - \lambda_1 \dot{y}(t) + \lambda_0 e(t). \tag{7}$$

c) Equivalent (continuous) control: The equivalent control is obtained by enforcing $\dot{s}(t) = 0$ on the sliding manifold, which gives

$$\ddot{y}(t) = -\lambda_1 \dot{y}(t) + \lambda_0 e(t). \tag{8}$$

Substituting (8) into (4) yields

$$u_c(t) = \frac{1}{K}\Big((\tau + t_0 - \tau t_0 \lambda_1)\dot{y}(t) + y(t) + \tau t_0 \lambda_0 e(t)\Big). \tag{9}$$

To avoid differentiating the measured output, we select:

$$\lambda_1 = \frac{\tau + t_0}{\tau t_0}, \tag{10}$$

which is well-defined for $t_0 > 0$, cancels the $\dot{y}(t)$ term in (9) and results in:

$$u_c(t) = \frac{1}{K}\left(y(t) + \tau t_0 \lambda_0 e(t)\right). \tag{11}$$

d) Switching (boundary-layer) term: The total control input is:

$$u(t) = u_c(t) + u_d(t), \tag{12}$$

where the switching term is implemented using a sigmoid function to reduce chattering:

$$u_d(t) = K_d \frac{s(t)}{|s(t)| + \delta}, \qquad K_d > 0,\ \delta > 0. \tag{13}$$

e) Lyapunov reaching condition: Consider the Lyapunov candidate:

$$V(t) = \frac{1}{2}s^2(t). \tag{14}$$

Its derivative is $\dot{V}(t) = s(t)\dot{s}(t)$. The role of the switching term u_d is to enforce the reaching condition:

$$s(t)\dot{s}(t) < 0 \quad \text{for} \quad s(t) \neq 0, \tag{15}$$

so that $V(t)$ decreases and trajectories are driven toward the sliding manifold $s(t) = 0$. In the ideal discontinuous case, choosing:

$$u_d(t) = K_d \operatorname{sgn}(s(t)), \tag{16}$$

ensures finite-time reaching under matched bounded uncertainties and disturbances, provided that K_d is selected sufficiently large to dominate the uncertainty bound. In this work, we employ the boundary-layer implementation (13), which approximates (16) while reducing high-frequency switching at the expense of a small neighborhood around $s = 0$, whose size is influenced by δ.

B. Gradient-descent-based online tuning via output sensitivity dynamics

The controller (12)–(13) depends on the tunable parameter vector

$$\theta = \begin{bmatrix} K_d & \delta & \lambda_0 \end{bmatrix}^\top. \tag{17}$$

We minimize the instantaneous quadratic cost:

$$J(t) = \frac{1}{2}e^2(t), \tag{18}$$

whose gradient is:

$$\frac{\partial J}{\partial \theta} = e\frac{\partial e}{\partial \theta} = -e\frac{\partial y}{\partial \theta} = -e\, y_\theta, \tag{19}$$

where $y_\theta := \partial y / \partial \theta$. Thus, the continuous-time gradient update is

$$\dot{\theta}(t) = -\eta \frac{\partial J}{\partial \theta} = \eta\, e(t)\, y_\theta(t), \tag{20}$$

with learning rate $\eta > 0$ (distinct learning rates may be used for each parameter).

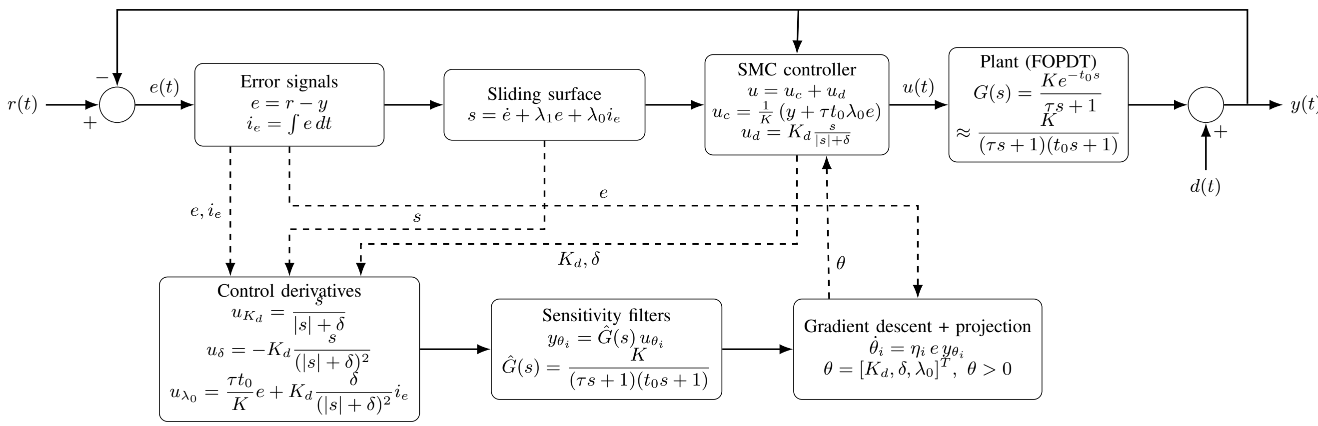

Fig. 1. SMC for an FOPDT process with online gradient-descent tuning using output sensitivity dynamics.

a) Sensitivity dynamics: To compute $y_{\theta_i} = \partial y/\partial \theta_i$ online, we differentiate the input–output model (4) with respect to each controller parameter $\theta_i \in \{K_d, \delta, \lambda_0\}$. Since K, τ, and t_0 do not depend on θ, the sensitivity dynamics satisfy:

$$\tau t_0 \ddot{y}_{\theta_i}(t) + (\tau + t_0)\dot{y}_{\theta_i}(t) + y_{\theta_i}(t) = K\, u_{\theta_i}(t), \tag{21}$$

where $u_{\theta_i} := \partial u/\partial \theta_i$. Equivalently,

$$Y_{\theta_i}(s) = \frac{K}{(\tau s + 1)(t_0 s + 1)}\, U_{\theta_i}(s). \tag{22}$$

In practice, (22) is implemented as stable second-order filters driven by u_{θ_i} to generate y_{θ_i}.

b) Derivatives of the control input: From (13), the partial derivatives needed to drive (21) are:

$$u_{K_d}(t) = \frac{\partial u}{\partial K_d} = \frac{s(t)}{|s(t)| + \delta}, \tag{23}$$

$$u_\delta(t) = \frac{\partial u}{\partial \delta} = -K_d \frac{s(t)}{(|s(t)| + \delta)^2} \tag{24}$$

For λ_0, using (11) and (6),

$$\frac{\partial u_c}{\partial \lambda_0} = \frac{\tau t_0}{K} e(t), \qquad \frac{\partial u_d}{\partial s} = K_d \frac{\delta}{(|s| + \delta)^2}, \qquad \frac{\partial s}{\partial \lambda_0} = i_e(t), \tag{25}$$

which yields:

$$u_{\lambda_0}(t) = \frac{\partial u}{\partial \lambda_0} = \frac{\tau t_0}{K} e(t) + K_d \frac{\delta}{(|s(t)| + \delta)^2}\, i_e(t). \tag{26}$$

c) Discrete-time implementation: With sampling period T_s, the Euler update is:

$$\theta_{i,k+1} = \theta_{i,k} + T_s\, \eta_i\, e_k\, y_{\theta_i,k}, \qquad \theta_i \in \{K_d, \delta, \lambda_0\}. \tag{27}$$

Finally, the parameters are projected onto admissible intervals to preserve $K_d > 0$, $\delta > 0$, and $\lambda_0 > 0$, avoid numerical issues, and keep the online tuning within predefined operating limits.

III. RESULTS

A. Benchmark processes and test protocol

To enable comparison with prior SMC tuning work, the three FOPDT benchmark plants were taken from the simulation examples reported in [12]. Specifically, the aforementioned work presented: (i) a high-order linear system with dominant delay ($G_1(s)$), (ii) a Linear system with inverse response reduced to an FOPDT ($G_2(s)$), and (iii) a nonlinear mixing tank whose operating-point variability is summarized by an average FOPDT model ($G_3(s)$). The corresponding models and their FOPDT approximations are:

$$G_1(s) = \frac{1}{(s+1)\,(0.5s+1)\,(0.25s+1)\,(0.125s+1)}\, e^{-10s}, \tag{28}$$

$$G_1(s) \approx \frac{1}{1.29s + 1}\, e^{-10.7s}, \tag{29}$$

$$G_2(s) = -0.4\left(\frac{s - 0.5}{(s+1)\,(3s+1)\,(s+0.2)}\right), \tag{30}$$

$$G_2(s) \approx \frac{1}{6.51s + 1}\, e^{-4.82s}, \tag{31}$$

$$G_3(s) \approx \frac{-0.858}{0.743s + 1}\, e^{-3.808s}. \tag{32}$$

All simulations used a constant setpoint (step reference) and an additive step disturbance applied at $t_d = 200$ s with amplitude 0.15 (same protocol across G_1–G_3). The horizon was 0–400 s. We compare: (1) baseline SMC (fixed parameters), (2) SMC tuned by multi-objective equations (SMC-MO) from [12], and (3) the proposed online-tuned method (SMC-GD).

B. Evaluation metrics

Performance is reported using the Integral of Squared Error (ISE) and the Integral of Squared Control Output (ISCO):

$$\text{ISE} = \int_0^T e^2(t)\, dt, \tag{33}$$

$$\text{ISCO} = \int_0^T u^2(t)\, dt. \tag{34}$$

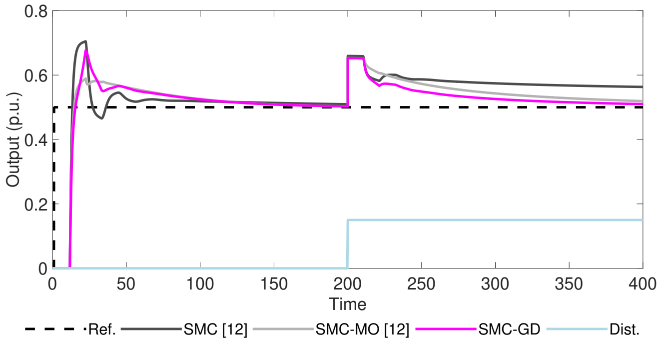

Fig. 2. Closed-loop response for $G_1(s)$ under step reference and additive step disturbance at $t_d = 200$ s (amplitude 0.15).

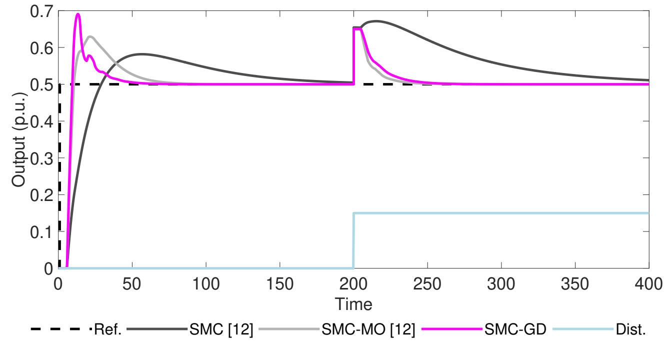

Fig. 3. Closed-loop response for $G_2(s)$ under step reference and additive step disturbance at $t_d = 200$ s (amplitude 0.15).

Note that the online update law minimizes the instantaneous quadratic error, while ISE/ISCO are used only for offline evaluation over the full horizon.

C. Quantitative comparison

Tables I–II summarize tracking accuracy and control effort. Across all three plants, SMC-GD achieves the lowest ISE, indicating improved tracking and disturbance rejection. For $G_2(s)$, the ISE reduction is most pronounced, consistent with the fact that G_2 originates from an inverse-response high-order model reduced to an FOPDT, where fixed-parameter tuning is more sensitive to mismatch [12].

Regarding control activity, SMC-GD reduces ISCO relative to the baseline SMC in all cases. Compared to SMC-MO, SMC-GD yields comparable effort (slightly higher ISCO in G_2 and G_3), suggesting that the online adaptation prioritizes error reduction while maintaining bounded control energy.

D. Transient responses and disturbance rejection

Figures 2–4 show the output trajectories for the three benchmark plants. In all cases, SMC-GD exhibits (i) tighter tracking before the disturbance and (ii) faster recovery with smaller deviation after $t_d = 200$ s, consistent with the ISE improvements. The boundary-layer implementation prevents unbounded switching activity, which aligns with the bounded ISCO values in Table II.

TABLE I

ISE COMPARISON OVER 0–400 S.

Controller	$G_1(s)$	$G_2(s)$	$G_3(s)$
SMC	4.582	4.367	1.811
SMC-MO [12]	3.986	2.025	1.534
SMC-GD (this work)	**3.742**	**1.922**	**1.502**

TABLE II

ISCO COMPARISON OVER 0–400 S.

Controller	$G_1(s)$	$G_2(s)$	$G_3(s)$
SMC	91.51	90.72	114.4
SMC-MO [12]	87.74	81.53	107.7
SMC-GD (this work)	**85.06**	82.08	108.3

E. Online parameter evolution

To verify the adaptive behavior, Fig. 5 reports the time evolution of the tuned parameters $\theta(t) = [K_d(t), \delta(t), \lambda_0(t)]^\top$ for SMC-GD. Two consistent behaviors are observed across plants: (1) an initial transient where parameters move away from their initial values to reduce tracking error, and (2) a visible adjustment around $t_d = 200$ s in response to the disturbance. The initial value of the parameters was set equal to that reported by [12].

IV. CONCLUSIONS

This paper presented an adaptive sliding-mode control strategy for first-order-plus-dead-time (FOPDT) processes in which key controller parameters are tuned online via gradient descent. Starting from an FOPDT model, the dead time was approximated by a first-order lag, yielding a second-order input–output representation used to define an integral sliding variable. An equivalent (continuous) control law was derived by enforcing invariance on the sliding manifold, and a boundary-layer switching term was employed to mitigate chattering while preserving the reaching behavior described by a standard Lyapunov argument.

To reduce reliance on fixed tuning rules that assume accurate and time-invariant process parameters, an online update law was introduced for the switching gain, boundary-layer thickness, and integral sliding weight. The proposed tuning mechanism uses output sensitivity dynamics: analytical control derivatives are filtered through the same stable

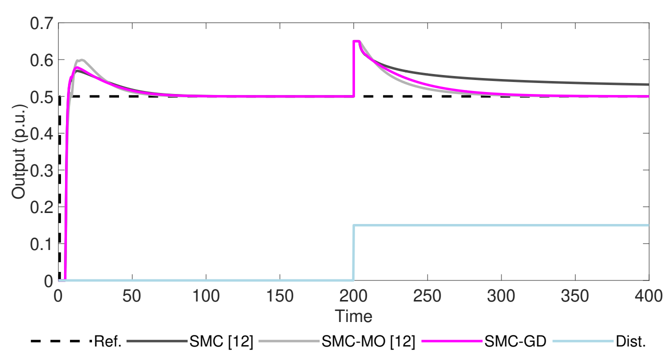

Fig. 4. Closed-loop response for $G_3(s)$ under step reference and additive step disturbance at $t_d = 200$ s (amplitude 0.15).

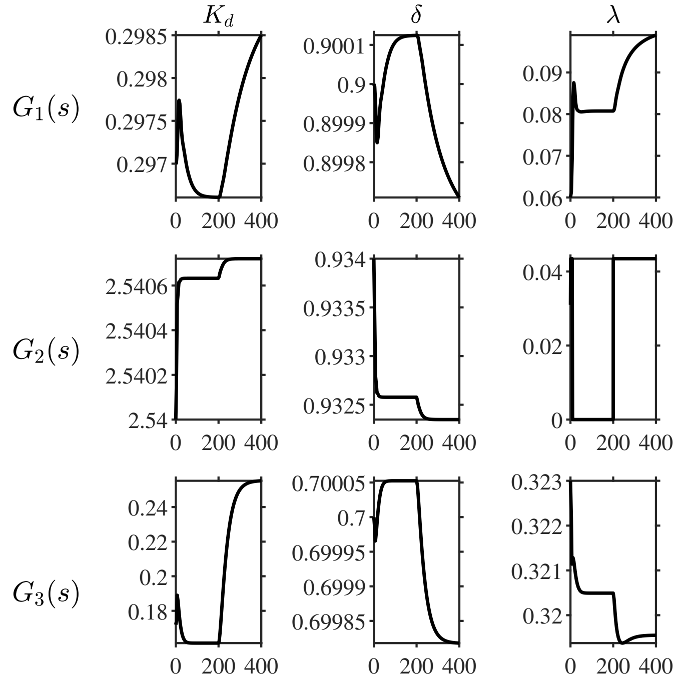

Fig. 5. Time evolution of SMC-GD parameters $K_d(t)$, $\delta(t)$, and $\lambda_0(t)$. The disturbance is applied at $t_d = 200$ s.

second-order dynamics as the plant approximation to obtain sensitivities required by the gradient update. This yields an implementable adaptation scheme that does not require numerical differentiation of the plant output and can be realized with standard transfer-function blocks.

Simulation results on three benchmark FOPDT plants taken from the literature show that the proposed SMC-GD consistently improves tracking accuracy, achieving the lowest ISE in all cases compared with a fixed-parameter SMC and an SMC tuned via multi-objective equations. The method also reduces control energy relative to the baseline SMC and maintains control effort comparable to the multi-objective tuned design, indicating a favorable trade-off between performance and actuation demand. Parameter trajectories further confirm that the gains adapt in response to transients and the applied disturbance while remaining bounded through projection.

Future work will focus on experimental validation on real process hardware, systematic selection of learning rates and projection bounds, and extending the approach to accommodate stronger dead-time mismatch and actuator nonlinearities (e.g., saturation and rate limits). In addition, incorporating robustness guarantees for the coupled plant–adaptation dynamics and extending the method to higher-order or multivariable time-delay processes are promising directions.

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