The Extended Nullspace-based Fault Detection and Isolation Filter

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***Abstract*— A novel architecture for the design and implementation of fault detection and isolation (FDI) filters for nonlinear systems is proposed. The approach uses an extended Kalman filter (EKF) to provide input decoupling from the residual via state estimation in a nonlinear setting. The subsequent adaptive nullspace-based residual generator fuses the EKF's estimation error to residuals which decouple disturbances and isolate faults. Therein, the nullspace-based design strategy offers a systematic and intuitive approach for designing the required fusion filter. The proposed separation between state estimation and residual generation yields a flexible and scalable framework that can be readily adapted to changing requirements with minimal redesign effort. The effectiveness of the proposed methodology is demonstrated through an actuator FDI problem applied on a small unmanned aircraft.**

I. INTRODUCTION

Nonlinear system dynamics present a challenge for conventional fault diagnosis techniques. A widely used strategy for fault detection is to linearize the nonlinear dynamics around an operating point and apply linear synthesis methods to design fault detection filters adapted to the resulting linear time-invariant (LTI) model, see [1], [2], [3], [4]. The filter's performance, however, degrades when the linear approximation cannot accurately capture the system's nonlineartities or the system strongly deviates from the specified operating point. To address systems with nonlinearities and varying system dynamics, the nonlinear fault diagnosis problem has to be addressed directly using nonlinear techniques. Beside signal-based approaches, model-based methods continue to play an important role in fault diagnosis because of their ability to provide strong guarantees on robustness and diagnostic reliability. Among the available techniques for handling system nonlinearities, observer-based approaches are well-established. These methods employ an observer to reconstruct the system states and outputs and use the output estimation error for fault detection. For nonlinear systems, extended Kalman filters (EKF) provide a natural extension of their linear counterparts and have been widely used for fault detection [1]. A practical example is provided for failure detection in aircraft control systems in [5], where faults and unknown parameters are modeled as states for the optimization. Such an inclusion as state not only requires adequate fault models but makes the tuning of the weighting matrices more cumbersome. In [6], actuator fault isolation for an unmanned aerial vehicle is achieved by running multiple EKFs, each modeling a specific fault component as additional state. Such multi-model observer schemes are common in observer-based fault isolation but lead to a growing number of observers with the number of considered faults. The explicit decoupling of unknown disturbances from the residual is captured by the concept of unknown-input observers. While originally developed for linear systems, this concept has been extended by [2] to a limited class of nonlinear systems under restrictive structural assumptions.

Beside observer-based approaches, a powerful framework for the synthesis of least-order fault detection filters is given by the nullspace-method. Developed for LTI systems, the approach has been extended in [7] and applied in [8] to approximate solutions to the class of linear parameter-varying systems (LPV). In this setting, symbolic computations are performed to design a linear fault detection filter that itself is parameter-varying. Its solution is, however, constraint by the complexity of the underlying problem, as the additionally parameters shrink the available nullspace. Similar, symbolic solutions have been developed in [9] for linear time-varying systems (LTV), where a LTV filter is designed based on the a priori knowledge of the systems explicit time dependent trajectory. The solution eliminates the need to store large sets of LTI filter typically required in a gridded approach. All the aforementioned extensions of the nullspace-method for LTI systems have another common issue, namely the requirement of storing and scheduling of the control and measurements variables' linearization points, which is imposed by their linear nature.

To overcome these issues, this paper extends the nullspace-based filter design idea to provide an efficient and elegant method to design fault detection filters for nonlinear systems. The approach combines state and output estimation via a EKF (i.e., without fault state) with real-time linearization and filter synthesis techniques, to generate residuals that are independent on the systems operating point. This extended nullspace-based fault detection filter fuses the strength of the EKF technique to cover system nonlinearities with the ones from the nullspace-based design, i.e., a direct computation of a minimal residual generator for fault detection and isolation (FDI) [4]. The approach, however, keeps both designs separated to enable design flexibility. Section IV presents the design of the extended nullspace-based fault detection filter, which represents the solution to the nonlinear fault detection problem introduced in Section III. The fundamental concepts are discussed in Section II and the proposed method is applied to the flight dynamics of a fixed-wing unmanned aerial vehicle (UAV) in Section IV.

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II. FUNDAMENTALS

The fundamental concepts used in this paper are presented for a linear time-invariant (LTI) system of the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_u u(t) + B_d d(t) + B_f f(t) \\ y(t) &= Cx(t) + D_u u(t) + D_d d(t) + D_f f(t)\,, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{m_u}$ the control input vector, $d(t) \in \mathbb{R}^{m_d}$ the disturbance input vector, $f(t) \in \mathbb{R}^{m_f}$ the fault input vector, and $y(t) \in \mathbb{R}^p$ the measurement output vector. Further, $A \in \mathbb{R}^{n\times n}$, $B_u \in \mathbb{R}^{n\times m_u}$, $B_d \in \mathbb{R}^{n\times m_d}$, $B_f \in \mathbb{R}^{n\times m_f}$, $C \in \mathbb{R}^{p\times n}$, $D_u \in \mathbb{R}^{p\times m_u}$, $D_d \in \mathbb{R}^{p\times m_d}$, and $D_f \in \mathbb{R}^{p\times m_f}$ define the state-space matrices of the system. Residual-based fault detection for (1) requires the generation of a residual which is decoupled from control inputs u and disturbances d, while being coupled to any fault f_i of the fault vector f, for $i = 1,...,m_f$. Both the nullspace-based design method [4] and observer-based fault detection [1], [3] provide frameworks to solve this problem.

A. Nullspace-based fault detection filter design

The theoretical concepts of the nullspace-based design method are described in terms of the LTI system (1) expressed in input-output form as

$$y(s) = G_u(s)u(s) + G_d(s)d(s) + G_f(s)f(s), \tag{2}$$

where $y(s)$, $u(s)$, $d(s)$ and $f(s)$ are Laplace-transformed vectors of the system's measurement output vector $y(t)$, control input vector $u(t)$, disturbance input vector $d(t)$, and fault vector $f(t)$. $G_u(s)$, $G_d(s)$ and $G_f(s)$ are the transfer function matrices (TFM) from control inputs, disturbances, and faults to the output respectively. A fault detection filter, also referred to as residual generator, processes the control input and measurement output of the system (2) to generate the scalar residual r that indicates the absence or presence of faults. The input-output form of the filter

$$r(s) = Q(s)\begin{bmatrix} u(s) \\ y(s) \end{bmatrix}, \tag{3}$$

can be transformed to internal form by replacing $y(s)$ in (3) by the right side of (2), resulting in

$$r(s) = R_u(s)u(s) + R_d(s)d(s) + R_f(s)f(s)\,, \tag{4}$$

where

$$[R_u(s)\,|\,R_d(s)\,|\,R_f(s)] := Q(s)G_e(s)\,, \tag{5}$$

with

$$G_e(s) := \left[\begin{array}{c|c|c} G_u(s) & G_d(s) & G_f(s) \\ I & 0 & 0 \end{array}\right]. \tag{6}$$

Thus, the residual in (3) depends via the system output $y(s)$ on all system inputs $u(s)$, $d(s)$ and $f(s)$. For fault detection, the residual needs to be decoupled from disturbances and arbitrary control inputs, i.e., $R_u(s) = 0$ and $R_d(s) = 0$. These decoupling conditions are equivalent to

$$Q(s)G_0(s) = Q(s)\begin{bmatrix} G_u(s) & G_d(s) \\ I_{m_u} & 0 \end{bmatrix} = 0, \tag{7}$$

which implies that $Q(s)$ must be a left annihilator of the TFM $G_0(s)$. The sensitivity of the residual to any fault component f_i, $i = 1,...,m_f$ is covered by the detection condition $R_{f_i}(s) \neq 0$, which is expressed algebraically through the constraint

$$R_{f_i}(s) = Q(s)\begin{bmatrix} G_{f_i}(s) \\ 0 \end{bmatrix} \neq 0\,, \tag{8}$$

where $G_{f_i}(s)$ is the TFM from the i-th fault to the residual. The stronger condition $R_{f_i}(s=0) \geq \varepsilon$ requires constant faults f_i to be transferred to the residual r with a minimum gain $\varepsilon \geq 0$, a property referred to as strong fault detectability.

Thus, the synthesis of a residual generator $Q(s)$ using the nullspace-method involves computing the left nullspace of $G_0(s)$ defined in (7) and checking if the resulting design fulfills the detection condition (8). For implementability on hardware, the designed residual generator must be proper and stable. Numerical tools to perform a least-order filter synthesis together with the design freedom of assigning the filter's poles are available in [4]. Note that the nullspace method can easily be adapted to perform not only fault detection but also strong fault isolation. For this purpose, a set of residuals r_i, $i = 1,...,m_f$ is generated, where each residual is sensitive to only one specific fault component. This is achieved by treating all fault components to be decoupled from r_i as additional disturbances in (7) and repeatedly apply the nullspace method to compute the filter Q_i, generating the i-th residual [4].

B. Observer-based fault detection

In observer-based fault detection approaches, a standard observer estimates the system's output in response to the known control inputs. Thus, the dynamics of a full-order linear observer for the LTI system (1) are given by

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + B_u u(t) + K(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) + D_u u(t)\,, \end{aligned} \tag{9}$$

where $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{y}(t) \in \mathbb{R}^p$ are the state and output estimates and $K \in \mathbb{R}^{n\times p}$ is the observer gain. Residuals for fault detection are generated from the output estimation error $e_y(t) = y(t) - \hat{y}(t)$ according to

$$r(t) = We_y(t)\,, \tag{10}$$

where $W \in \mathbb{R}^{p_r\times p}$ is the weighting that linearly transforms the output estimation error to the residual [1]. To analyze how exactly the external inputs u, d, and f influence the estimation error and therefore the residual, (1) and (9) are combined to yielding the error dynamics

$$\begin{aligned} \dot{e}_x(t) &= A_c e_x(t) + B_{d_c} d(t) + B_{f_c} f(t) \\ e_y(t) &= Ce_x(t) + D_d d(t) + D_f f(t)\,, \end{aligned} \tag{11}$$

where $e_x(t) = x(t) - \hat{x}(t)$ is the state estimation error and $A_c = A - KC$, $B_{d_c} = B_d - KD_d$ and $B_{f_c} = B_f - KD_f$. Equation (11) reveals that the output estimation error e_y is decoupled from the control input u. The influence of disturbances d and faults f, however, is not eliminated. For fault inputs, this characteristic of the observer-based fault detection is not a bug but the feature which enables the detection of faults via the estimation error. Observer-based fault detection methods exploit the design freedom in choosing the matrices K and W in (9) and (10) to achieve disturbance decoupling, for example via eigenstructure assignment [1].

III. The nonlinear FDI problem

With the fundamental fault detection concepts for LTI systems outlined in Section II, a generalized form of the fault detection and isolation problem is introduced next for nonlinear systems. The definition of the nonlinear fault detection and strong isolation problem (NFDIP) relies on the nonlinear system

$$\begin{aligned}\dot{x}(t) &= g(x(t),u(t),d(t),f(t))\\ y(t) &= h(x(t),u(t),d(t),f(t)),\end{aligned} \tag{12}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{m_u}$ the control input vector and $y(t) \in \mathbb{R}^p$ the measurement vector. Denoted by $d(t) \in \mathbb{R}^{m_d}$ and $f(t) \in \mathbb{R}^{m_f}$ are the disturbance and fault input vector, respectively. The nonlinear function $g(\dots)$ describes the system dynamics, $h(\dots)$ the measurement model. In contrast to the previous section, where $x(t)$, $y(t)$ and $u(t)$ represent deviations from an operating point in the linear system (1), the variables in (12) and in the following describe the nonlinear dynamics.

A fault and isolation detection filter for (12) processes the control input u and measurement output y and is of the form

$$\begin{aligned}\dot{x}_r(t) &= g_r(x_r(t),u(t),y(t))\\ r(t) &= h_r(x_r(t),u(t),y(t)),\end{aligned} \tag{13}$$

where $x_r(t) \in \mathbb{R}^{n_r}$ is the fault detection filter's state vector, and $r(t) \in \mathbb{R}^{m_f}$ the residual vector. The nonlinear equations $g_r(\dots)$ and $h_r(\dots)$ define the dynamical behavior of the fault detection filter. Assuming system (12) to be deterministic, i.e., no stochastic inputs are considered explicitly, the NFDIP can be formulated in a general form as follows: Based on the system (12), determine a causal and stable fault detection filter of the form (13) such that:

(i) $r_i(t) = 0$ when $f_i(t) = 0 \, \forall \{u(t), d(t), f_j(t)\}$ for $j \neq i$
(ii) $r_i(t) \neq 0$ when $f_i(t) \neq 0$, for $i = 1,\dots,m_f$
(iii) $r_i(t)$ is asymptotically bounded.

As mentioned in Section II, for fault isolation a residual vector $r \in \mathbb{R}^{m_f}$ is required, where each residual component r_i, $i = 1,...,m_f$, must be sensitive to only one specific fault component f_i, a property which is referred to as strong fault isolation. While (i) requires the decoupling of control inputs, disturbances and faults not to be detected by r_i, condition (ii) ensures that respective faults are detected. Finally, the additional design constraint (iii) requires bounded-input bounded-output stability of the residual.

While the basic requirements $(i)-(iii)$ of the NFDIP are identical to those for fault detection and isolation in linear systems, the main challenge is to design the nonlinear mappings g_r and h_r in (13). A powerful approach in dealing with nonlinearities is to linearize the nonlinear system (12) around the current operating point enabling the application of the linear synthesis techniques for fault detection introduced in Section II. With this approach, the resulting objective is to ensure that conditions $(i)-(iii)$ are satisfied at every operating point.

IV. Solving the nonlinear FDI problem

The architecture of the proposed extended nullspace-based FDI filter that solves the NFDIP is given in Fig. 1. The FDI filter is composed of an EKF for state and output estimation while a residual generator provides the residual vector indicating the absence or presence of faults. The outer part, consisting of the linearization and nullspace method, updates the residual generator each time step to match the current operating point. In each time-step the state estimate $\hat{x}(t)$ of the EKF is used to derive the linear dynamics (11) and subsequently design the corresponding nullspace-based residual generator online so that the conditions $(i)-(iii)$ of the NFDIP are fulfilled. The residual generator is driven by the output estimation error $e_y(t)$, similar to observer-based approaches. Since the output estimation error is already decoupled from the control inputs, the nullspace method only needs to ensure decoupling of disturbances and respective fault components. Details on the different elements of this architecture are discussed in the following.

A. Extended Kalman filter

For the system (12) nonlinear state estimation is performed via an extended Kalman filter. In this work the EKF is used to enable adequate estimates of the system states and outputs in the presence of nonlinearities within the deterministic setting. The design, however, can easily be adapted to optimally consider stochastic effects, by changing its design weights accordingly. The continuous-time dynamics of the EKF according to [10] are given by

$$\begin{aligned}\dot{\hat{x}}(t) &= g(\hat{x}(t),u(t),0,0) + K(t)(y(t)-\hat{y}(t))\\ \hat{y}(t) &= h(\hat{x}(t),u(t),0,0),\end{aligned} \tag{14}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state estimate vector, $\hat{y}(t) \in \mathbb{R}^p$ the output estimate vector, and $K(t) \in \mathbb{R}^{n\times p}$ the Kalman gain. The Kalman gain is derived by solving in each time step

$$\begin{aligned}K(t) &= P(t)C(t)^T R_v^{-1}\\ \dot{P}(t) &= A(t)P(t) + P(t)A(t)^T + Q_w - P(t)C(t)^T R_v^{-1} C(t)P(t),\end{aligned} \tag{15}$$

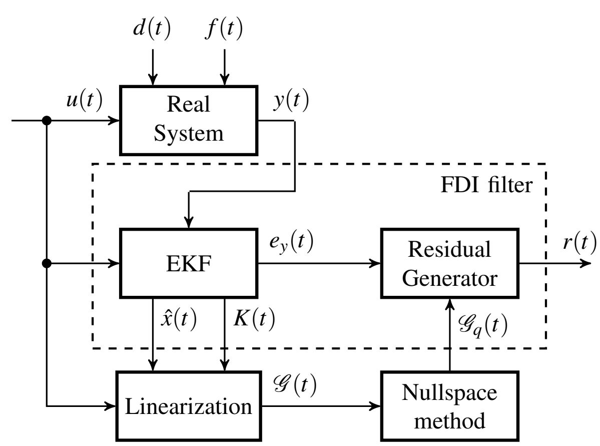

Fig. 1. Extended nullspace-based FDI filter.

where $A(t) \in \mathbb{R}^{n\times n}$ and $C(t) \in \mathbb{R}^{p\times n}$ are the state-space matrices of the system (12) linearized around the state estimate $\hat{x}(t)$ and control input $u(t)$, i.e.,

$$A(t) = \left.\frac{\partial g}{\partial x}\right|_{\hat{x}(t),u(t)}, C(t) = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}(t),u(t)}. \tag{16}$$

In a stochastic setting, the matrices $Q_w \in \mathbb{R}^{n\times n}$ and $R_v \in \mathbb{R}^{p\times p}$ represent the covariances of the process and measurement noise that are required for computing the state estimation error covariance $P(t) \in \mathbb{R}^{n\times n}$. In a deterministic setting they can be interpreted as design parameters, where Q_w^{-1} represents the confidence in the model (12) and R_v^{-1} the confidence in the measurements [11]. Following [11], both Q_w and R_v are assumed to be constant matrices. For initialization the EKF needs to be provided with a first guess about the matrix $P(0)$ and state estimate $\hat{x}(0)$.

For accurate estimation, the state estimation error $e_x(t) = x(t) - \hat{x}(t)$ and thus the output estimation error $e_y(t) = y(t) - \hat{y}(t)$ need to converge to zero. In presence of the unknown inputs d and f, however, the EKF cannot eliminate their influence on the estimation errors, see (11). The conditions under which the perturbed system ensures that the filter remains in a bounded region around the true state, i.e., the EKF is locally stable, are given in [11] and are assumed to hold here. Note that, in the presence of faults, modeling errors and disturbances, the linearization (16) is based on an incorrect operating point, which may affect the Kalman gain (15) and the resulting residual generator. The impact on detection performance and robustness is therefore assessed in simulation, as is standard for fault detection approaches.

B. Linearized design model

The state estimate $\hat{x}(t)$ of the EKF and the Kalman gain $K(t)$ form the basis for the subsequent linearization. To derive the interconnection from external inputs to the estimation error e_y, a linearized model of the plant is connected to a linearized version of the observer. Based on (11) the linear error dynamics are given by

$$\begin{aligned}\dot{e}_x(t) &= A_c(t)e_x(t) + B_{d_c}(t)d(t) + B_{f_c}(t)f(t)\\ e_y(t) &= C(t)e_x(t) + D_d(t)d(t) + D_f(t)f(t),\end{aligned} \tag{17}$$

where $e_x(t) = x(t) - \hat{x}(t)$ and $e_y(t) = y(t) - \hat{y}(t)$ are the state and output estimation errors, respectively. The state-space matrices $A_c(t) = A(t) - K(t)C(t)$, $B_{d_c}(t) = B_d(t) - K(t)D_d(t)$ and $B_{f_c}(t) = B_f(t) - K(t)D_f(t)$ result from (15), (16) and

$$\begin{aligned}B_d(t) &= \left.\frac{\partial g}{\partial d}\right|_{\hat{x}(t),u(t)}, \quad B_f(t) = \left.\frac{\partial g}{\partial f}\right|_{\hat{x}(t),u(t)},\\ D_d(t) &= \left.\frac{\partial h}{\partial d}\right|_{\hat{x}(t),u(t)}, \quad D_f(t) = \left.\frac{\partial h}{\partial f}\right|_{\hat{x}(t),u(t)}.\end{aligned} \tag{18}$$

In contrast to (11), the system (17) becomes linear time-varying due to the time-varying Kalman-gain $K(t)$ in (15) and the time-varying linearization points. Finally, let $\mathscr{G}(t) := \{A_c(t), B_{d_c}(t), B_{f_c}(t), C(t), D_d(t), D_f(t)\}$ define the set of the state-space matrices of the system (17).

C. Nullspace-based residual generator design

A residual generator Q that fuses the estimation error e_y to the required residual vector r can now be designed by applying the nullspace method described in Section II-A to the linear model (17). For a fixed point in time $t = t_0$, the input-output form of the system (17) is given by

$$e_y(s) = G_d(s)d(s) + G_f(s)f(s), \tag{19}$$

where $e_y(s)$ is the Laplace-transformed vector of the output estimation error vector $e_y(t)$, and the TFMs $G_d(s)$ and $G_f(s)$ result from

$$\begin{aligned}G_d(s) &= C(t)(sI - A_c(t))^{-1}B_{d_c}(t) + D_d(t)\\ G_f(s) &= C(t)(sI - A_c(t))^{-1}B_{f_c}(t) + D_f(t).\end{aligned} \tag{20}$$

The task is to design a residual generator for the system (19) that enables a residual r fulfilling the design constraints $(i)-(ii)$ of the NFDIP. The input-output form of the residual generator is given by

$$r(s) = Q(s)e_y(s) = \begin{bmatrix} Q_1(s)\\ \vdots \\ Q_{m_f}(s)\end{bmatrix} e_y(s). \tag{21}$$

The design requirement (i) of the NFDIP is equal to the algebraic expression

$$Q_i(s)G_{0_i}(s) := Q_i(s)\begin{bmatrix} G_d(s) & \widehat{G}_{d_i}(s)\end{bmatrix}, \tag{22}$$

which implies that $Q_i(s)$ mus be a left annihilator of $G_{0_i}(s)$. In (22) those fault components f_j for $j \neq i$ that need to be decoupled from the residual r_i are defined as additional disturbances, whose transfer behavior is given by

$$\widehat{G}_{d_i}(s) = C(t)(sI - A_c(t))^{-1}\widehat{B}_{d_{c_i}}(t) + \widehat{D}_{d_i}(t), \tag{23}$$

with $\widehat{B}_{d_{c_i}}(t) = \widehat{B}_{d_i}(t) - K(t)\widehat{D}_{d_i}(t)$, where $\widehat{B}_{d_i}(t)$ and $\widehat{D}_{d_i}(t)$ are formed from the j-th columns of $B_f(t)$ and $D_f(t)$ for all faults f_j. In (22) the decoupling of control inputs does not need to be considered, as the output estimation error in (19) itself is already decoupled from control inputs. The additional desgin requirement to fulfill (ii) of the NFDIP is

$$R_{f_i}(s) = Q_i(s)G_{f_i}(s) \neq 0. \tag{24}$$

A residual generator that fulfills (22) and (24) is derived by employing the numerical tools provided by [4] to the system (17). These tools ensure local stability and properness of the residual generator and are implemented such that the filter's system matrix A_q, and thus the eigenvalues of the filter, is constant over time. The state-space form of the resulting residual generator is given by

$$\begin{aligned}\dot{x}_q(t) &= A_q x_q(t) + B_q(t)e_y(t)\\ r(t) &= C_q(t)x_q(t) + D_q(t)e_y(t),\end{aligned} \tag{25}$$

where $x_q \in \mathbb{R}^{n_q}$ is the residual generator's state vector, $r \in \mathbb{R}^{m_f}$ the residual vector and $A_q \in \mathbb{R}^{n_q\times n_q}$, $B_q(t) \in \mathbb{R}^{n_q\times p}$, $C_q(t) \in \mathbb{R}^{m_f\times n_q}$ and $D_q(t) \in \mathbb{R}^{m_f\times p}$ the state-space matrices. The matrices form the set $\mathscr{G}_q(t) := \{A_q, B_q(t), C_q(t), D_q(t)\}$ which is time-varying due to the residual generator design being performed online each time step. The time-varying matrices $B_q(t)$, $C_q(t)$ and $D_q(t)$ are uniformly bounded which provides a sufficient condition for the bounded-input bounded-output (BIBO) stability of the residual generator.

D. The extended nullspace-based FDI filter

The dynamics of the extended nullspace-based FDI filter (13) with its structure depicted in Fig. 1 combine the dynamics of the EKF and residual generator. The state vector is given by $x_r(t) = [\hat{x}(t)\, x_q(t)]^T$, and the nonlinear dynamic equations result in

$$\begin{aligned} g_r(x_r(t),u(t),y(t)) &= \begin{pmatrix} g(\hat{x}(t),u(t)) \\ A_q(t)x_q(t) \end{pmatrix} + \begin{pmatrix} K(t) \\ B_q(t) \end{pmatrix} e_y(t) \\ h_r(x_r(t),u(t),y(t)) &= C_q(t)x_q(t) + D_q(t)e_y(t)\,, \end{aligned} \tag{26}$$

with $e_y(t) = y(t) - h(\hat{x}(t),u(t))$. Local stability of the extended nullspace-based FDI filter, and hence fulfilling (*iii*), follows if both the residual generator and the EKF are locally stable, as the former acts purely as post-processing and does not influence the EKF recursion.

The framework combines the strengths of an EKF for state estimation and decoupling of control inputs from the estimation errors in a nonlinear setting with the efficiency and structural transparency of the nullspace-based approach for residual generation. Due to its modularity, changes in the fault detection and isolation problem, such as the inclusion of new faults or additional disturbances, only require modifications in the nullspace-based design, while the nonlinear estimation structure of the EKF remains unchanged. The separation between state estimation and residual design yields a flexible and scalable framework that can be readily adapted to evolving requirements with minimal redesign effort. This modularity of the approach even enables the possibility of replacing the EKF by any other filtering techniques if a different observer based approach is to be used.

For implementation, structural and dimensional inconsistencies in the residual generator due to its continuously reinitialized design must be avoided. For most physical systems this is naturally fulfilled as the nullspace structure is consistent over the whole operating envelope. State consistency of the residual generator can then be ensured by transforming the residual generator into a canonical state space realization such as the controller canonical form [12].

V. Application

The extended nullspace-based fault detection and isolation filter is applied to the longitudinal flight dynamics of a fixed-wing UAV. To evaluate the filter's performance, a flight scenario is simulated in which the aircraft autonomously changes its altitude and speed. The fault detection task deals with the detection and isolation of actuator faults in the presence of wind disturbance. The studied UAV is the Urban Condor X, a research aircraft operated at the Technische Universität Dresden, for which a nonlinear flight simulator inducing linearization capabilities is available, see [13], [14].

A. Urban Condor fault detection and isolation problem

For the considered fault detection task only the longitudinal flight dynamics are considered, which are described by standard nonlinear flight dynamic equations of the form (12), see [15]. The state vector x comprises the translational velocities U and W in the body-frame x_b- and z_b-axes, the pitch angle θ and pitch rate q describing the rotational motion, and the aircraft's position in the earth-fixed frame, given by the altitude h. Control inputs u are the elevator deflection η and thrust T. The considered faults f are the elevator deflection fault η_f and thrust fault T_f. As disturbances d to be decoupled from the residuals, the wind inputs $w_{x,E}$ and $w_{z,E}$, modeled along the x_E- and z_E-axes of the earth-fixed reference frame, are considered. Measurements y from the ADS about the true airspeed V_{TAS} and angle-of-attack α, as well as measurements from the IMU about the pitch angle, pitch rate, and the load factors n_x and n_z are available.

B. Extended FDI filter setup

The first component of the extended nullspace-based FDI filter of the form (13) is the EKF (14). It relies on the nonlinear flight dynamic equations without disturbances and faults to generate online state estimates $\hat{x} = [\hat{U}\ \hat{\theta}\ \hat{W}\ \hat{q}]^T$, and output estimates $\hat{y} = [\hat{V}_{\text{TAS}}\ \hat{\theta}\ \hat{\alpha}\ \hat{q}\ \hat{n}_x\ \hat{n}_z]^T$. To account for the dependence of the aircraft dynamics on air density, the measured altitude h is provided to the EKF. The Kalman gain K is computed each time step via solving (15) using the constant matrices $R_v = I_6 \cdot 10^{-6}$ and $Q_w = \text{diag}([10^{-1}\, 10^{-1}\, 10^{-3}\, 10^{-3}])$. Initialization values for $\hat{x}(0)$ are taken from the aircraft's cruise condition, while the error covariance matrix is initialized with $P(0) = I_4 \cdot 10^{-6}$. Finally, the EKF provides the output estimation error e_y to the residual generator.

The second component is the residual generator (25), whose dynamics are updated online via the nullspace-based design. The linear error dynamics (17) are obtained by evaluating the precomputed analytical expressions of the Jacobian matrices (16) and (18) each time step using the state estimate $\hat{x}$ and control input u. Numerical tools from [4] are implemented to solve the FDI problem each time step with the residual generator poles placed at -1. The resulting design comprises two residuals r_1 and r_2, where each residual is coupled to one fault component, thereby solving the fault isolation problem. Finally, the resulting extended nullspace-based FDI filter has the form (13) and is a combination of the EKF and residual generator which comprises $n_r = 6$ states in total, four originating from the EKF dynamics and two from the residual generator.

C. Simulation-based Verification

The verification is performed on the six degree-of-freedom rigid-body flight dynamics simulator of the Urban Condor X. The trajectory, depicted in Fig. 2, is selected to include altitude and speed changes to trigger the aircraft's nonlinear behavior. The maneuver starts from cruise at an altitude of 100 m with 20 m/s, followed by a steady ascending flight with a flight path angle of 3°. After reaching the new altitude the

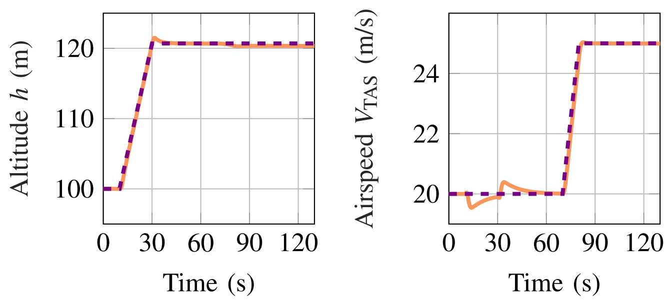

Fig. 2. Commanded (- - -) and actual (——) flight trajectory.

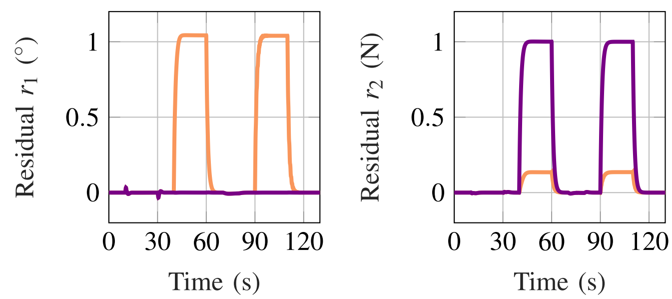

Fig. 4. Residuals for elevator bias (——) and thrust bias (——).

airpseed in increased from 20 m/s to 25 m/s. To analyze the decoupling performance of the extended nullspace-based FDI filter a fault-free flight along the flight trajectory is performed. Step-like wind disturbances are triggered in the simulation after 40 s and 90 s of flight persisting 20 s each. These disturbances model gusts with wind intensities of 1 m/s that build up with a time constant of 1 s and occur firstly in vertical and subsequently in the horizontal direction. The residuals are shown in Fig. 3 and demonstrate that disturbance decoupling is successfully achieved with residuals r_1 and r_2 that are largely unaffected by the wind.

Next, the detection performance is analyzed during the maneuver by simulating step like faults of 20 s occuring at 40 s and 90 s in flight. The simulation is performed twice, once for an elevator bias of $\eta_f = 1^\circ$ and once for a thrust bias $T_f = 1\,\text{N}$. The residuals from both simulation runs of the faulty scenario are shown in Fig. 4. They illustrate that fault detection is adequately achieved, as each residual responds almost proportionally to its corresponding fault component. Residual r_1 exhibits an excitation of approximately 1.05° in response to the actuator bias fault, whereas residual r_2 rises up to 1 N after the simulation of a thrust bias fault. Regarding fault isolation, residual r_1 shows almost no reaction to the thrust bias fault and therefore achieves decoupling successfully. In contrast, residual r_2 exhibits an excitation of around 0.14 N in the presence of the elevator bias, indicating that perfect fault isolation is not reached. This is attributed to the error introduced by approximating the operating point using the EKF estimates as mentioned in Section IV-A. The shown separation between the residual excitations of r_2 allows, however, the separation between false positives and desired detection via an adequate threshold selection.

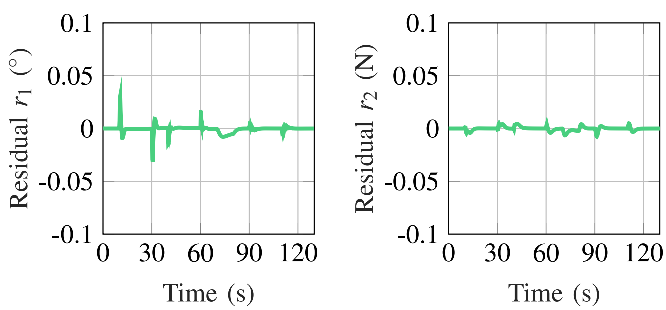

Fig. 3. Residuals in the fault-free scenario.

VI. Conclusion

The paper presents an approach to solve the nonlinear fault detection and isolation problem. An extended Kalman filter is used to track the system states within a nonlinear setting, while a nullspace based filter design fuses the estimation error signals to an appropriate residual vector. This decoupling of state estimation and residual generation provides a flexible framework which can be easily adapted in case of setup changes. The proposed method is based on online state estimation and nullspace-based residual generator design and is successfully verified in a simulation of a fixed wing UAV aircraft.

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