Data-driven iterative learning control design using the repetitive process setting

Robert Maniarski, Wojciech Paszke, Eric Rogers

Abstract— **This paper investigates the problem of designing iterative learning control laws based on input and state measurements collected during an open-loop experiment. Disturbances corrupt the measured data, and we have a set of dynamics that could have generated the collected data. The problem considered is control law design to achieve robust convergence of the tracking error in the trial-to-trial direction. It is shown that the design problem can be formulated in a repetitive process setting, as it is required to consider the interaction between the trial-to-trial error and the transient response along the trials. The stability analysis for linear repetitive processes is used to develop conditions for the existence of a control law, and the computation of the required control law matrices is linear matrix inequality-based. Finally, comparative simulation results show the effectiveness of the new design.**

I. Introduction

Iterative learning control (ILC) applies to systems or processes that perform the same finite-duration task repeatedly. The aim is to improve the accuracy by using data from previous executions to update the control action for the next one. Early literature is covered in, e.g., [1], [2]. A literature review indicates that ILC has attracted considerable research attention since it has been used for improving the control performance in many practical problems such as industrial robotics, see, e.g., [3] and wafer stage motion systems, see, for example, [4]. Additionally, see, e.g., [5], [6], ILC can be directly applied in the chemical process industries and, in particular, batch processes.

In practical implementations, the development of ILC schemes predominantly relies on model-based strategies that incorporate the dynamics of the controlled system [1], [2]. However, recently, there has been a noticeable trend toward employing increasingly imprecise models, a trend that stems from the enhanced robustness properties of modern ILC design methodologies. Despite this trend, the fundamental objective in ILC synthesis remains the same: to ensure tracking-error convergence across successive trials and the preservation of system stability, particularly under conditions of significant model uncertainty. A widely recognized framework in the ILC literature is the two-dimensional (2D) or repetitive process representation of the learning dynamics [7], [8], [9]. This formulation enables explicit modeling of the coupling between inter-trial error evolution and trial transient behavior. Importantly, it facilitates the derivation of control design conditions in the form of Linear Matrix Inequalities (LMIs), applicable to both continuous-time and discrete-time systems. Ensuring stability along the trial axis within the repetitive process framework imposes tracking-error convergence of the controlled system.

In recent years, data-driven control synthesis has emerged as a prominent paradigm, primarily due to the widespread availability of high-capacity, low-cost digital storage technologies. These advancements facilitate the acquisition and retention of extensive datasets, which may include measurements collected during experiments on controlled objects. Unlike traditional model-based approaches, data-driven methods do not require identifying the parameters or building a model of the system first. Instead, the control algorithm is derived directly from the available measurement data, allowing the controller to be designed without an explicit modeling step.

This paper explores the application of recent advances in data-driven control (specifically those accommodating additive process noise) to the synthesis of ILC schemes. While the proposed methodology builds on existing results [10], it extends their applicability to more complex control scenarios, notably those in which ILC is formulated as a repetitive process. The adopted framework enables the construction of an ILC design procedure that guarantees robust stability of the closed-loop system in both the time and trial domains. Crucially, the controller synthesis is formulated as a convex optimization problem over LMI constraints. The effectiveness of the proposed approach is demonstrated through a numerical example, which confirms that the data-driven design achieves performance comparable to that of established benchmark methods.

Throughout this paper, the null and identity matrices with compatible dimensions are denoted by 0 and I, respectively, and the notation of $P \succ 0$ ($P \prec 0$) means that $P = P^T$ and that P is positive (negative) definite. In addition, $\rho(P)$ denotes the spectral radius of P and the superscript * denotes the complex conjugate transpose of a matrix.

The proofs of the results in this paper make use of the following result (Petersen's lemma).

Lemma 1: [10], [11] Consider matrices $\mathbf{C} \in \mathbb{R}^{n\times n}$, $\mathbf{E} \in \mathbb{R}^{n\times p}$, $\overline{\mathbf{F}} \in \mathbb{R}^{q\times q}$, $\mathbf{G} \in \mathbb{R}^{q\times n}$ with $\mathbf{C} = \mathbf{C}^\top$ and $\overline{\mathbf{F}} = \overline{\mathbf{F}}^\top \succeq$

R. Maniarski and W. Paszke are with the Institute of Automation, Electronic and Electrical Engineering, University of Zielona Góra, ul. Szafrana 2, 65-246 Zielona Góra, Poland. E-mail: {r.maniarski, w.paszke}@iee.uz.zgora.pl

E. Rogers is with School of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, UK. E-mail: etar@ecs.soton.ac.uk

This research was funded in part by National Science Centre in Poland, grant No. 2020/39/B/ST7/01487.

0, and let $\mathcal{F}$ be

$$\mathcal{F} := \{\mathbf{F} \in \mathbb{R}^{p\times q} \colon \mathbf{F}^\top \mathbf{F} \preceq \overline{\mathbf{F}}\}. \tag{1}$$

Then

$$\mathbf{C} + \mathbf{E}\mathbf{F}\mathbf{G} + \mathbf{G}^\top \mathbf{F}^\top \mathbf{E}^\top \prec 0 \text{ for all } \mathbf{F} \in \mathcal{F} \tag{2}$$

if and only if there exists a scalar $\lambda > 0$ such that

$$\mathbf{C} + \lambda \mathbf{E}\mathbf{E}^\top + \lambda^{-1}\mathbf{G}^\top \overline{\mathbf{F}}\mathbf{G} \prec 0. \tag{3}$$

II. Preliminaries

Let $t \in [0, \alpha - 1]$ be the sample index or time instant over the finite trial duration α and $k \geq 0$ be the trial number. Then discrete-time systems over a finite time interval $t \in [0, \alpha-1]$ described by the following state-space model are considered

$$\begin{aligned} x_k(t+1) =& Ax_k(t) + Bu_k(t) + d_k(t), \\ y_k(t) =& Cx_k(t), \\ x_k(0) =& x_0, \end{aligned} \tag{4}$$

where $x_k(t) \in \mathbb{R}^{n_x}$ is the state vector, $y_k(t) \in \mathbb{R}^{n_y}$ is the output vector, $u_k(t) \in \mathbb{R}^{n_u}$ is the control input vector and $d_k(t) \in \mathbb{R}^{n_x}$ is the external disturbance vector. $\{A, B\}$ are unknown system matrices with compatible dimensions and C is known output matrix.

Assumption 1: After each trial is complete, the system resets to the same initial value x_0, and there is no loss of generality in assuming that $x_0 = 0$.

Remark 1: As seen, the output equation is not subject to any noise or disturbance matrix. Also in many practical cases C directly indicates which internal state variables are observed and hence we assume that the matrix C is known.

Since the matrices $\{A, B\}$ in (4) are unknown, an experiment on the system is performed by applying an input sequence $u(t_0)$, $u(t_1)$, $\ldots$, $u(t_{N-1})$ of N samples. The state response $x(t_0)$, $x(t_1)$, $\ldots$, $x(t_{N-1})$, the shifted state response $x(t_1)$, $x(t_2)$, $\ldots$, $x(t_N)$ and the output response $y(t_0)$, $y(t_1)$, $\ldots$, $y(t_{N-1})$ are measured. Moreover, the disturbance sequence $d(t_0), d(t_1), ..., d(t_{N-1})$ affects the system's evolution and remains unknown, which implies that the collected data are noisy. This means that following data vectors are available after performing the experiment

$$\begin{aligned} U_0 &= [u(t_0) \quad u(t_1) \quad \cdots \quad u(t_{N-1})], \\ X_0 &= [x(t_0) \quad x(t_1) \quad \cdots \quad x(t_{N-1})], \\ Y_0 &= [y(t_0) \quad y(t_1) \quad \cdots \quad y(t_{N-1})], \\ X_1 &= [x(t_1) \quad x(t_2) \quad \cdots \quad x(t_N)]. \end{aligned} \tag{5}$$

Disturbances are unknown but can be collected as

$$D_0 = [d(t_0) \quad d(t_1) \quad \cdots \quad d(t_{N-1})]. \tag{6}$$

This paper uses a certain disturbance model where the disturbance sequence D_0 has bounded energy. This means that $D_0 \in \mathcal{D}$ where, for some matrix Δ

$$\mathcal{D} := \left\{ D_0 \in \mathbb{R}^{n_x \times N} : D_0 D_0^T \preceq \Delta\Delta^T \right\}. \tag{7}$$

Therefore, based on measured data the system (4) can be expressed as

$$\begin{aligned} X_1 =& AX_0 + BU_0 + D_0, \\ Y_0 =& CX_0. \end{aligned} \tag{8}$$

With data (5) and set $\mathcal{D}$ in (7), introduce the set $\mathcal{C}$ of matrices that are consistent with the data

$$\mathcal{C} := \{(A, B) : X_1 = AX_0 + BU_0 + D_0, D_0 \in \mathcal{D}\} \tag{9}$$

i.e., the set of all pairs (A, B) of dynamical matrices that could generate data X_1, X_0 and U_0 based on (4) while keeping the disturbance sequence in the set $\mathcal{D}$. Next, observe that $\mathcal{C}$ can be rewritten as

$$\begin{aligned} \mathcal{C} = \{ &(A, B) : X_1 = AX_0 + BU_0 + D_0, \\ &\begin{bmatrix} I & D_0 \end{bmatrix} \begin{bmatrix} -\Delta\Delta^\top & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} I \\ D_0^\top \end{bmatrix} \preceq 0 \Big\}. \end{aligned}$$

Furthermore, substitute $D_0 = X_1 - AX_0 - BU_0$ in the last matrix inequality and collect $\begin{bmatrix} I & A & B \end{bmatrix}$ on the left and its transpose on the right of the matrix inequality. Finally, one can find that $\mathcal{C}$ can be equivalently written as

$$\mathcal{C} = \left\{ (A, B) \colon \begin{bmatrix} I & A & B \end{bmatrix} \cdot \left[\begin{array}{c|c} \mathbf{C} & \mathbf{B}^\top \\ \hline \mathbf{B} & \mathbf{A} \end{array} \right] [\star]^\top \preceq 0 \right\} \tag{10}$$

and then

$$\mathcal{C} = \left\{ \begin{bmatrix} A & B \end{bmatrix} = Z^\top \colon \mathbf{C} + \mathbf{B}^\top Z + Z^\top \mathbf{B} + Z^\top \mathbf{A} Z \preceq 0 \right\}, \tag{11}$$

where we rearrange (A, B) as the matrix $\begin{bmatrix} A & B \end{bmatrix}$ from (10) to (11) and define

$$\left[\begin{array}{c|c} \mathbf{C} & \mathbf{B}^\top \\ \hline \mathbf{B} & \mathbf{A} \end{array} \right] := \left[\begin{array}{c|c} X_1 X_1^\top - \Delta\Delta^\top & -X_1 \begin{bmatrix} X_0 \\ U_0 \end{bmatrix}^\top \\ \hline -\begin{bmatrix} X_0 \\ U_0 \end{bmatrix} X_1^\top & \begin{bmatrix} X_0 \\ U_0 \end{bmatrix} \begin{bmatrix} X_0 \\ U_0 \end{bmatrix}^\top \end{array} \right]. \tag{12}$$

Assumption 2: Matrix $\begin{bmatrix} X_0 \\ U_0 \end{bmatrix}$ has full row rank.

The set $\mathcal{C}$ in (11) can be regarded as a matrix ellipsoid and hence enables a final simple reformulation of $\mathcal{C}$ as

$$\mathcal{C} = \left\{ \begin{bmatrix} A & B \end{bmatrix} = Z^\top : (Z - Z_{\mathrm{c}})^\top \mathbf{A} (Z - Z_{\mathrm{c}}) \preceq \mathbf{Q} \right\}. \tag{13}$$

Since $\mathbf{A} \succ 0$ one can define

$$Z_{\mathrm{c}} := -\mathbf{A}^{-1}\mathbf{B}, \quad \mathbf{Q} := \mathbf{B}^\top \mathbf{A}^{-1} \mathbf{B} - \mathbf{C}. \tag{14}$$

To derive the main result from Lemma 1, a final reformulation of $\mathcal{C}$ is needed. Let us define

$$\mathcal{E} := \left\{ Z_{\mathrm{c}} + \mathbf{A}^{-1/2} \Upsilon \mathbf{Q}^{1/2} \colon \|\Upsilon\| \leq 1 \right\}, \tag{15}$$

where Υ is an unknown matrix to be found and then for $\mathbf{A} \succ 0$ and $\mathbf{Q} \succeq 0$, $\mathcal{C} = \mathcal{E}$ (see Proposition 1 in [10] for further details). Therefore Lemma 1 allows us to eliminate the matrix Υ which will be shown when the main result will be provided.

III. Embedding ILC into a repetitive process setting

In this section, we formulate the ILC problem and introduce the LMI condition to compute the gain matrices for the assumed control law. We assume that the process is represented by (4) but no direct disturbances are present. Any disturbances arising during the measurement experiment will instead appear as uncertainty in the estimated process matrices, i.e. A and B are unknown.

Given the desired trajectory y_d, the tracking error on trial k, i.e., the difference between the desired and actual process outputs, is

$$e_k(t) = y_d(t) - y_k(t),\ t \in [0, \alpha - 1].$$

The objective of this paper is to use the tracking error to construct a control sequence $\{u_k\}_{k\geq 1}$ such that the uncertain process output y_k tracks the desired trajectory y_d as precisely as possible as $k \to \infty$. In this case, the tracking error in k converges to zero (or within some specified tolerance), and the tracking performance along the trials (in t) is improved. These requirements are represented mathematically as

$$\lim_{k\to\infty} \|e_k(\cdot)\| = 0,\ \lim_{k\to\infty} \|u_k(\cdot) - u_\infty(\cdot)\| = 0,$$

where $\|\cdot\|$ denotes the norm on the underlying function space and $u_\infty(\cdot)$ is termed the learned control. To achieve the (robust) convergence of the unknown linear iterative process (4), an ILC law is used to compute the subsequent trial input as the sum of the previous trial input plus a correction, i.e.

$$u_{k+1}(t) = u_k(t) + \Delta u_{k+1}(t), \tag{16}$$

where the update $\Delta u_{k+1}(t)$ is computed after the end of trial k and used in (16) to compute the input to trial $k+1$, using known data. In particular, suppose that

$$\Delta u_k(t-1) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} \overline{x}_{k+1}(t) \\ e_k(t) \end{bmatrix}, \tag{17}$$

where the incremental variable $\overline{x}_k(t)$ is

$$\overline{x}_{k+1}(t) = x_{k+1}(t-1) - x_k(t-1). \tag{18}$$

Since this paper uses repetitive process theory to design a control law some transformations are needed. In particular, to write the dynamics as a repetitive process observe that

$$e_{k+1}(t) - e_k(t) = -CA\overline{x}_{k+1}(t) - CB\Delta u_{k+1}(t-1). \tag{19}$$

Secondly, one can find that

$$\overline{x}_{k+1}(t+1) = A\overline{x}_{k+1}(t) + B\Delta u_k(t-1) \tag{20}$$

and hence the controlled dynamics can be written as

$$\begin{aligned} \overline{x}_{k+1}(t+1) &= (A+BK_1)\overline{x}_{k+1}(t) + BK_2 e_k(t), \\ e_{k+1}(t) &= (-CA-CBK_1)\overline{x}_{k+1}(t) + (I-CBK_2)e_k(t). \end{aligned} \tag{21}$$

The above set of equations has the form of a linear repetitive model where $\overline{x}_{k+1}(t)$ plays the role of process state vector and $e_k(t)$ plays the role of the trial profile (output) vector. Hence repetitive process stability results can be applied for computing K_1 and K_2 which ensure both trial-to-trial error convergence and along the trial performance - see [12] for further details. Several sets forms of stability have been developed for linear repetitive processes together with associated tests [12], [13]. From the practical standpoint, the most important are those which guarantee asymptotic stability or stability along the trial (pass). Asymptotic stability guarantees that a bounded initial trial profile produces a sequence of trial profiles (i.e. output signals) over the finite and fixed trial length α, whereas stability along the trial is stronger since it requires this property uniformly, i.e., for all possible values of the trial length. Hence it is not surprising that asymptotic stability is a necessary condition for stability along the trial.

The following result is a direct extension of the analysis in [13] and characterizes stability along the trial of systems described by (21).

Lemma 2: A discrete linear repetitive process described by (21) is stable along the trial if and only if there exist K_1 and K_2 such that

i) $\rho(I - CBK_2) < 1$,
ii) $\rho(A + BK_1) < 1$,
iii) all eigenvalues of

$$\begin{aligned} G(e^{j\omega}) = &((-CA-CBK_1)(e^{j\omega}I - (A+BK_1))^{-1}BK_2 \\ &+ (I-CBK_2)),\ \forall\omega \in [-\pi, \pi] \end{aligned}$$

have modulus strictly less than unity.

In terms of checking the conditions of the above result, only the third condition is a problem because it requires to check if all eigenvalues of $G(e^{j\omega})$, i.e., the characteristic loci, lie inside of the unit circle $\forall\omega \in [-\pi, \pi]$. Instead, this paper uses a sufficient condition for stability along the trial. The following result can be established where the existence of K_1 and K_2 are given in terms of LMIs.

Theorem 1: Let γ be a positive scalar satisfying $0 < \gamma \leq 1$. Then an ILC scheme described as a discrete linear repetitive process of the form (21) is stable along the trial and hence trial-to-trial error convergence occurs if there exist matrices $W_1 \succ 0$, $W_2 \succ 0$, N_1 and N_2 of compatible dimensions such that the following matrix inequality holds

$$\begin{bmatrix} -W_1 & 0 & -AW_1 - BN_1 & -BN_2 \\ 0 & -W_2 & CAW_1 + CBN_1 & -W_2 + CBN_2 \\ (\star) & (\star) & -W_1 & 0 \\ (\star) & (\star) & 0 & -\gamma^2 W_2 \end{bmatrix} \prec 0. \tag{22}$$

Furthermore, if the above inequality is feasible, the required K_1 and K_2 are given by

$$K_1 = N_1 W_1^{-1},\ K_2 = N_2 W_2^{-1}. \tag{23}$$

Proof: Assume that (22) is feasible for some $W_1 \succ 0$, $W_2 \succ 0$. Next, pre- and post-multiply the last inequality by $\operatorname{diag}\{I, I, W_1^{-1}, W_2^{-1}\}$ and then apply the Schur's complement formula to give

$$\begin{bmatrix}A+BK_1 & BK_2\\ I & 0\end{bmatrix}^T\begin{bmatrix}P_1 & 0\\ 0 & -P_1\end{bmatrix}\begin{bmatrix}A+BK_1 & BK_2\\ I & 0\end{bmatrix}$$
$$+\begin{bmatrix}-C(A+BK_1) & I-CBK_2\\ 0 & I\end{bmatrix}^T\begin{bmatrix}P_2 & 0\\ 0 & -\gamma^2P_2\end{bmatrix}$$
$$\times\begin{bmatrix}-C(A+BK_1) & I-CBK_2\\ 0 & I\end{bmatrix}\prec 0,$$

where $P_1 = W_1^{-1}$, $P_2 = W_2^{-1}$. Next apply the Kalman-Yakubovich-Popov lemma (see Theorem 2 in [14]), which yields

$$\begin{bmatrix}G(e^{j\omega})\\ I\end{bmatrix}^*\begin{bmatrix}P_2 & 0\\ 0 & -\gamma^2P_2\end{bmatrix}\begin{bmatrix}G(e^{j\omega})\\ I\end{bmatrix}\prec 0,$$

where $G(e^{j\omega})$ is defined by $iii)$ in Lemma 2. Feasibility of the above inequality guarantees that condition $iii)$ of Lemma 2 holds. Furthermore, feasibility of (22) implies that conditions $i)$ and $ii)$ of Lemma 2 hold and the proof is complete. ■

Remark 2: The scalar parameter γ can be minimized to increase the convergence rate of the tracking error.

The matrix inequality in Theorem 1 cannot be directly solved since A and B are unknown. However, by using a particular set of transformations, the inequality (22) can be converted into an appropriate form for computing K_1 and K_2 as detailed next.

IV. Data-driven design of ILC laws in repetitve process setting

Since the matrices A and B are unknown, the condition proposed in Theorem 1 poses difficulties in solving of controller design problem. To solve this problem, observe that

$$\begin{bmatrix}-AW_1-BN_1 & -BN_2\\ CAW_1+CBN_1 & -W_2+CBN2\end{bmatrix}$$
$$=\left(\begin{bmatrix}0 & 0\\ 0 & -I\end{bmatrix}+\begin{bmatrix}-I\\ C\end{bmatrix}\begin{bmatrix}A & B\end{bmatrix}\begin{bmatrix}I & 0\\ K_1 & K_2\end{bmatrix}\right)\begin{bmatrix}W_1 & 0\\ 0 & W_2\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & -W_2\end{bmatrix}+\begin{bmatrix}-I\\ C\end{bmatrix}\begin{bmatrix}A & B\end{bmatrix}\begin{bmatrix}W_1 & 0\\ N_1 & N_2\end{bmatrix}.$$

Next, introduce the notation

$$\begin{aligned}&W=\mathrm{diag}\{W_1,W_2\},\ U=\mathrm{diag}\{0,W_2\},\ \hat W=[W_1,\ 0],\\ &N=[N_1,\ N_2],\ \Lambda=\mathrm{diag}\{W_1,\ \gamma^2W_2\}\end{aligned}\tag{24}$$

and then the result of Theorem 1 can reformulated in the form that depends on the parametrization of $\mathcal{C}$ through $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ given in (10) to obtain an LMI characterization for the existence of ILC law represented by (17) for which the resulting ILC scheme is convergent. This enables the direct use of noisy experimental data for ILC controller design, as stated in the theorem below.

Theorem 2: Let γ be a positive scalar satisfying $0<\gamma\le 1$. Also, suppose the experiment has been made and data given in vectors U_0, X_0, X_1 as in (5) and satisfy the Assumption 2. Then if there exist $W_1\succ 0$, $W_2\succ 0$, N_1 and N_2 such that

$$\left[\begin{array}{c|c|c}-W-\begin{bmatrix}-I\\ C\end{bmatrix}\mathbf{C}\begin{bmatrix}-I\\ C\end{bmatrix}^T & -U & \begin{bmatrix}-I\\ C\end{bmatrix}\mathbf{B}^T\\ \hline (\star) & -\Lambda & [-\hat W^T\ -N^T]\\ \hline (\star) & (\star) & -\mathbf{A}\end{array}\right]\prec 0\tag{25}$$

holds. This implies that the inequality (22) holds too, the resulting repetitive process (21) is stable along the trial and hence trial-to-trial error convergence occurs. Moreover the required K_1 and K_2 are computed as (23).

Proof: It is sufficient to show that feasibility of (25) ensures feasibility of (22). To proceed observe the LMI (25) can be equivalently rewritten as

$$\begin{bmatrix}-W-\begin{bmatrix}-I\\ C\end{bmatrix}\mathbf{C}\begin{bmatrix}-I\\ C\end{bmatrix}^T & -U\\ (\star) & -\Lambda\end{bmatrix}$$
$$+\begin{bmatrix}\begin{bmatrix}-I\\ C\end{bmatrix}\mathbf{B}^T\\ \begin{bmatrix}-\hat W\\ -N\end{bmatrix}^T\end{bmatrix}\mathbf{A}^{-1}\begin{bmatrix}\mathbf{B}\begin{bmatrix}-I\\ C\end{bmatrix}^T & \begin{bmatrix}-\hat W\\ -N\end{bmatrix}\end{bmatrix}\prec 0.$$

Using the notation introduced in (14), the last inequality can be rewritten as

$$\begin{bmatrix}-W+\begin{bmatrix}-I\\ C\end{bmatrix}\mathbf{Q}\begin{bmatrix}-I\\ C\end{bmatrix}^T & -U+\begin{bmatrix}-I\\ C\end{bmatrix}Z_c^T\begin{bmatrix}\hat W\\ N\end{bmatrix}\\ (\star) & -\Lambda+\begin{bmatrix}\hat W\\ N\end{bmatrix}^T\mathbf{A}^{-1}\begin{bmatrix}\hat W\\ N\end{bmatrix}\end{bmatrix}\prec 0.\tag{26}$$

On the other hand, in view of (15) we know that

$$\begin{bmatrix}A & B\end{bmatrix}=\left(Z_c+\mathbf{A}^{-1/2}\Upsilon\mathbf{Q}^{1/2}\right)^T$$

and then

$$\begin{bmatrix}-AW_1-BN_1 & -BN_2\\ CAW_1+CABN_1 & -W_2+CBN_2\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & -W_2\end{bmatrix}+\begin{bmatrix}-I\\ C\end{bmatrix}\left(Z_c+\mathbf{A}^{-1/2}\Upsilon\mathbf{Q}^{1/2}\right)^T\begin{bmatrix}W_1 & 0\\ N_1 & N_2\end{bmatrix}.$$

Additional matrix manipulations yields

$$\begin{bmatrix}-W_1 & 0 & -AW_1-BN_1 & -BN_2\\ 0 & -W_2 & CAW_1+CABN_1 & -W_2+CBN_2\\ (\star) & (\star) & -W_1 & 0\\ (\star) & (\star) & 0 & -\gamma^2W_2\end{bmatrix}$$
$$=\begin{bmatrix}-W & -U+\begin{bmatrix}-I\\ C\end{bmatrix}Z_c^T\begin{bmatrix}\hat W\\ N\end{bmatrix}\\ (\star) & -\Lambda\end{bmatrix}$$
$$+\begin{bmatrix}\begin{bmatrix}-I\\ C\end{bmatrix}\mathbf{Q}^{\frac12}\\ 0\end{bmatrix}\Upsilon^T\begin{bmatrix}0 & \mathbf{A}^{-\frac12}\begin{bmatrix}\hat W\\ N\end{bmatrix}\end{bmatrix}$$
$$+\begin{bmatrix}0\\ \begin{bmatrix}\hat W\\ N\end{bmatrix}^T\mathbf{A}^{-\frac12}\end{bmatrix}\Upsilon\begin{bmatrix}\mathbf{Q}^{\frac12}\begin{bmatrix}-I\\ C\end{bmatrix}^T & 0\end{bmatrix}.$$

Finally, we use the result of Lemma 1 to obtain the inequality (26), that is, performing the elimination of the unknown matrix Υ such that $\|\Upsilon\| < 1$. Obtaining (26) is ultimately achieved by absorbing the unknown scalar λ in the matrix variables W_1, W_2, N_1, and N_2 (i.e. $W_1 = \lambda W_1$, $W_2 = \lambda W_2$, $N_1 = \lambda N_1$, and $N_2 = \lambda N_2$). ■

V. Numerical case study

The example given in this section is based on a model of a single axis of a gantry robot, which has been used to experimentally validate existing ILC algorithms. It should be emphasized that the model presented below will only be used to generate data. This means that the collected data is the only available information and the underlying system parameters remain unknown within the data-driven framework.

The modeling of each axis was carried out using the frequency response function approach. Since the axes are orthogonal, it was assumed that the interaction between them is negligible. Based on this methodology, the studies reported in [15], [16], identified the following continuous-time transfer function for one of the axes

$$G(s) = \frac{15.8869(s + 850.3)}{s(s^2 + 707.6s + 3.377 \times 10^5)}.$$

Discretization performed using the zero-order-hold method at a sampling frequency of $100[\mathrm{Hz}]$ yields the following transfer function in the z-domain

$$G(z) = \frac{0.00036482(z^2 + 0.09791z + 0.005951)}{(z - 1)(z^2 + 0.005922z + 0.0008451)}$$

and hence in (4)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.0008 & 0.0051 & 0.9941 \end{bmatrix}, \; B = \begin{bmatrix} 0.3648 \times 10^{-3} \\ 0.3984 \times 10^{-3} \\ 0.4000 \times 10^{-3} \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

The system is assumed to be subjected to non-repeating disturbances of the form

$$d(t) = 5 \times 10^{-6} \cdot \sin(2\pi \cdot \beta(t)),$$

where each state is affected independently, and $\beta(t)$ represents a realization of a uniform random variable in $[0, 1]$.

Furthermore, the collection of data X_0, X_1 and U_0 was achieved through the implementation of a uniform random variable in the range $[-0.01, 0.01]$ as the input. The measurement data collection process involved a sample length of $N = 200$ and a sampling time of 0.01 seconds. The resulting open-loop output is shown in Figure 1. The range of the input signal used for data generation is similar to that of the reference trajectory (Figure 2), which the system is intended to track using the ILC law. For the assumed interference signal, it can be observed that d satisfies $|d|^2 \leq 2.5 \times 10^{-11}$. Hence, the disturbance sequence D satisfies then the bound $DD^T \preceq 2.5 \times 10^{-11} NI$. Finally, the bound on d can be convert into $\Delta := 5 \times 10^{-6}\sqrt{N}I$.

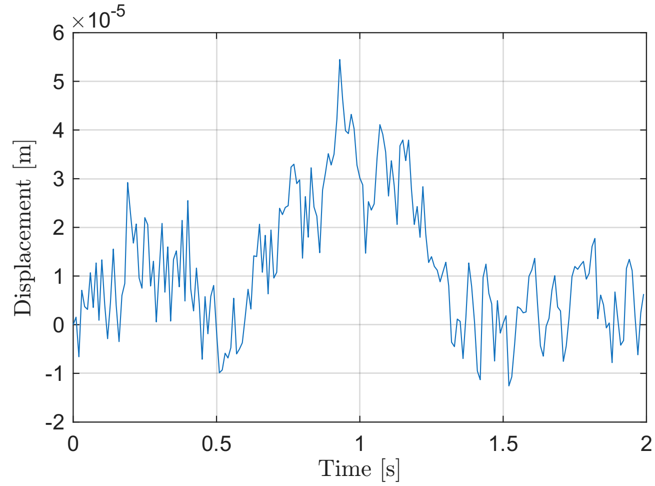

Fig. 1: Displacement of the considered axis in an open loop

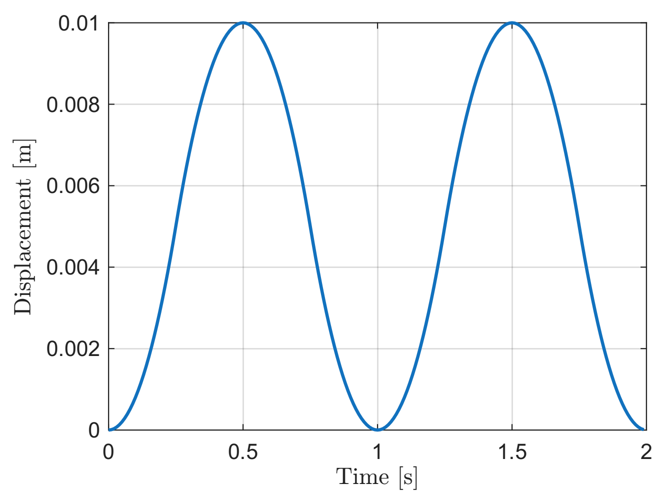

Fig. 2: Reference trajectory for the considered axis

Applying Theorem 2 for $\gamma = 0.9$, gives the ILC law matrices in (17) as

$$K_1 = [-59.1378 \;\; -367.984 \;\; -772.4510], \; K_2 = 365.4361.$$

A simulation of the controlled system was conducted over 50 trials. Upon completion of each trial, the Root Mean Square (RMS) of the tracking error was computed using

$$\mathrm{RMS}(e_k) = \sqrt{\frac{1}{200}\sum_{t=0}^{N-1} e_k^2(t)}$$

and plotted in Figure 3 as a function of the trial number. Simulations of the controlled system confirm that the system remains stable throughout each trial. Moreover, the results clearly demonstrate that the implemented control law allowed for a significant reduction in the tracking error, even in the presence of non-repetitive disturbances (see Figure 4).

VI. Conclusions and future research

This paper explores a data-driven methodology for designing ILC schemes directly from measurement data obtained

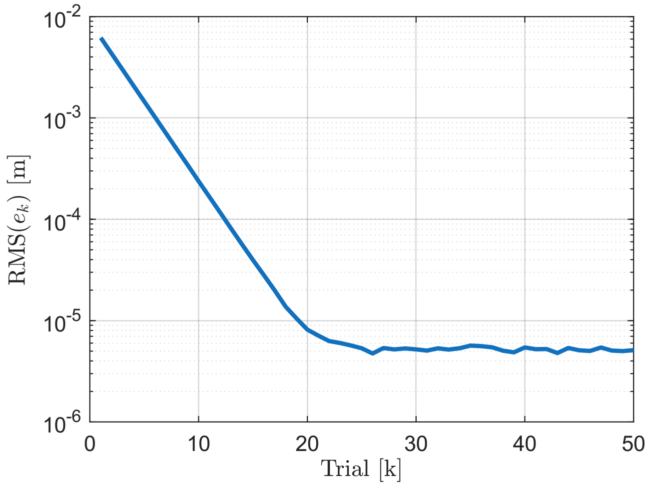

Fig. 3: RMS(e_k) values of the error over 50 trials

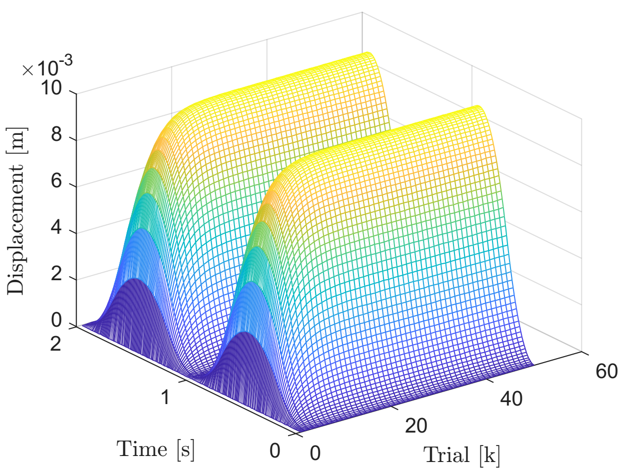

Fig. 4: Axis displacement during 50 trials

during open-loop experiments, thereby eliminating the need for explicit plant model identification. Furthermore, by reformulating the control problem to one of stability along the trial for a discrete linear repetitive process, the required ILC law matrices are derived with LMI-based computations. Application of this data-driven approach ensures that these controllers guarantee acceptable performance during the trial dynamics and that the tracking error converges trial-to-trial. A simulation study based on the gantry robot verifies the effectiveness of this design method. Future research will focus on extending this framework to design ILC laws for continuous-time systems in a repetitive process setting and on more complex control laws that rely solely on input-output measurements, without requiring access to state variables.

References

[1] H.-S. Ahn, Y.-Q. Chen, and K. L. Moore, "Iterative learning control: brief survey and categorization," *IEEE Transactions on Systems, Man and Cybernetics, Part C*, vol. 37, no. 6, pp. 1109–1121, 2007.

[2] D. A. Bristow, M. Tharayil, and A. Alleyne, "A survey of iterative learning control," *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, 2006.

[3] M. Norrlöf, "An adaptive iterative learning control algorithm with experiments on an industrial robot," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 2, pp. 245–251, 2002.

[4] M. Heertjes and T. Tso, "Nonlinear iterative learning control with applications to lithographic machinery," *Control Engineering Practice*, vol. 15, no. 12, pp. 1545–1555, 2007.

[5] Y. Wang, F. Gao, and F. Doyle, "Survey on iterative learning control, repetitive control, and run-to-run control," *Journal of Process Control*, vol. 19, no. 10, pp. 1589–1600, 2009.

[6] T. Liu, X. Z. Wang, and J. Chen, "Robust PID based indirect-type iterative learning control for batch processes with time-varying uncertainties," *Journal of Process Control*, vol. 24, pp. 95–106, 2014.

[7] E. Rogers, K. Gałkowski, W. Paszke, K. L. Moore, P. H. Bauer, L. Hładowski, and P. Dabkowski, "Multidimensional control systems: case studies in design and evaluation," *Multidimensional Systems and Signal Processing*, vol. 26, no. 4, pp. 895–939, 2015.

[8] W. Paszke, E. Rogers, K. Gałkowski, and Z. Cai, "Robust finite frequency range iterative learning control design and experimental verification," *Control Engineering Practice*, vol. 21, no. 10, pp. 1310–1320, 2013.

[9] R. Maniarski, W. Paszke, H. Tao, and E. Rogers, "Repetitive T-S fuzzy model-based iterative learning control law design," in *63rd IEEE Conference on Decision and Control (CDC)*, 2024, pp. 4137–4142.

[10] A. Bisoffi, C. De Persis, and P. Tesi, "Data-driven control via petersen's lemma," *Automatica*, vol. 145, p. 110537, 2022.

[11] I. R. Petersen, "A stabilization algorithm for a class of uncertain systems," *System & Control Letters*, vol. 8, pp. 351–357, 1987.

[12] E. Rogers, B. Chu, C. Freeman, and P. Lewin, *Iterative Learning Control Algorithms and Experimental Benchmarking*. Wiley, 2023.

[13] E. Rogers, K. Gałkowski, and D. H. Owens, *Control Systems Theory and Applications for Linear Repetitive Processes*, ser. Lecture Notes in Control and Information Sciences. Berlin, Germany: Springer-Verlag, 2007, vol. 349.

[14] A. Rantzer, "On the Kalman-Yakubovich-Popov lemma," *Systems and Control Letters*, vol. 28, no. 1, pp. 7–10, 1996.

[15] L. Hładowski, K. Gałkowski, Z. Cai, E. Rogers, C. T. Freeman, and P. L. Lewin, "A 2D systems approach to iterative learning control for discrete linear processes with zero Markov parameters," *International Journal of Control*, vol. 84, no. 7, pp. 1246–1262, 2011.

[16] J. D. Ratcliffe, J. J. Hatonen, P. L. Lewin, E. Rogers, T. J. Harte, and D. H. Owens, "P-type iterative learning control for systems that contain resonance," *International Journal of Adaptive Control and Signal Processing*, vol. 19, pp. 769–796, 2005.